

4) c)

$$\textcircled{1} (A \cdot B)^T = B^T \cdot A^T \quad \forall A, B \in M_{2 \times 2}$$

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

$\textcircled{2}$

$$\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} a_{11}b_{12}b_{21} & a_{11}b_{12}b_{22} \\ a_{21}b_{12}b_{21} & a_{21}b_{12}b_{22} \end{pmatrix} = A \cdot B$$

$$(A \cdot B)^T = \begin{pmatrix} a_{11}b_{12}b_{21} & a_{11}b_{12}b_{22} \\ a_{21}b_{12}b_{21} & a_{21}b_{12}b_{22} \end{pmatrix}^T = \begin{pmatrix} a_{11}b_{12}b_{21} & a_{21}b_{12}b_{21} \\ a_{11}b_{12}b_{22} & a_{21}b_{12}b_{22} \end{pmatrix}$$

$\textcircled{3}$

$$A^T = \begin{pmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{pmatrix}$$

$$B^T \cdot A^T$$

$$\begin{pmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \end{pmatrix}$$

$$\begin{pmatrix} b_{11}a_{11} + b_{21}a_{12} & b_{12}a_{11} + b_{22}a_{12} \\ b_{11}a_{21} + b_{21}a_{22} & b_{12}a_{21} + b_{22}a_{22} \end{pmatrix}$$

$\hat{=} ?$

8) a)

$$A + A^T \quad \text{sim} \quad \forall A \in M_{m \times m}$$

$$A = \begin{pmatrix} a_{11} & a_{1m} \\ a_{m1} & a_{mm} \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{1m} \\ a_{m1} & a_{mm} \end{pmatrix} + \begin{pmatrix} a_{11} & a_{1m} \\ a_{m1} & a_{mm} \end{pmatrix} = \begin{pmatrix} 2a_{11} & a_{1m} + a_{m1} \\ a_{m1} + a_{1m} & 2a_{mm} \end{pmatrix} = A + A^T$$

$$A^T = \begin{pmatrix} a_{11} & a_{m1} \\ a_{1m} & a_{mm} \end{pmatrix}$$

$$\downarrow A + A^T = (A + A^T)^T ?$$

$$(A + A^T)^T = \begin{pmatrix} 2a_{11} & a_{1m} + a_{m1} \\ a_{m1} + a_{1m} & 2a_{mm} \end{pmatrix}^T = \begin{pmatrix} 2a_{11} & a_{m1} + a_{1m} \\ a_{1m} + a_{m1} & 2a_{mm} \end{pmatrix}$$

9) c)

$$\text{Tr}(A + B) = \text{Tr}(A) + \text{Tr}(B) \quad \forall A, B \in M_{m \times m}$$

$$A = \begin{pmatrix} a_{11} & \dots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{m1} & \dots & a_{mm} \end{pmatrix}$$

$$\rightarrow \text{Tr} = a_{11} + a_{22} + \dots + a_{mm}$$

$$B = \begin{pmatrix} b_{11} & \dots & b_{1m} \\ \vdots & \ddots & \vdots \\ b_{m1} & \dots & b_{mm} \end{pmatrix}$$

$$\rightarrow \text{Tr} = b_{11} + b_{22} + \dots + b_{mm}$$

$$A + B = \begin{pmatrix} a_{11} + b_{11} & \dots & a_{1m} + b_{1m} \\ \vdots & \ddots & \vdots \\ a_{m1} + b_{m1} & \dots & a_{mm} + b_{mm} \end{pmatrix}$$

$$\rightarrow \text{Tr} = a_{11} + b_{11} + a_{22} + b_{22} + \dots + a_{mm} + b_{mm}$$