# Chapter 3 Problems

## Describing Pressure (§3.1)

**3.1** In units of $mH_2O$ gage, what pressure corresponds to an absolute pressure of 3 atm?

(**a**) 28 (**b**) 32 (**c**) 21 (**d**) 25 (**e**) 37

**3.2** The depth of water in an open container is 2 ft. What is the maximum absolute pressure in units of atmospheres? Answer

(**a**) 23/17 (**b**) 2/17 (**c**) 1/17 (**d**) 18/17 (**e**) 4/17

**3.3** Local atmospheric pressure is 1.0 bar. The gage pressure at point $A$ is $-9 \times 10^3$ Pa. Consider the following statements about the pressure at A:

**I.** The vacuum pressure is 1.3 psia

**II.** The gage pressure is −36.1 inches of water

**III.** The absolute pressure is $\frac{9}{100}$ bar

**IV.** The absolute pressure is 1900 psf

The true statements are: (a) all except I (b) all except III (c) II and IV (d) all except II (e) I and II

**3.4** From smallest to largest, rank order the following values of absolute pressure. Answer

**1.** 0.12 MPa

**2.** 10.3 psi

**3.** 630 psf

**4.** 17 ftH2O

**5.** 200,000 Pa

The rank order is:

**a.** (3, 5, 1, 2, 4)

**b.** (3, 4, 2, 1, 5)

**c.** (4, 2, 5, 3, 1)

**d.** (1, 3, 4, 2, 5)

**e.** (4, 3, 1, 2, 5)

SS **3.5** From memory, list the standard value of $p_{atm}$.

**a.** ________________________ kPa

**b.** ________________________ bar

**c.** ________________________ atm

**d.** ________________________ mH2O

**e.** ________________________ ftH20

**f.** ________________________ psia

**g.** ________________________ psfa

**h.** ________________________ mmHg

**i.** ________________________ inHg

SS **3.6** Apply the grid method (§1.7) to each situation. Answer

**a.** If the pressure is 15 inches of water (vacuum), what is the gage pressure in kPa?

**b.** If the pressure is 140 kPa abs, what is the gage pressure in psi?

**c.** If a gage pressure is 0.55 bar, what is absolute pressure in psi?

**d.** If a person's blood pressure is 119 mm Hg gage, what is their blood pressure in kPa abs?

**3.7** A 93-mm diameter sphere contains an ideal gas at 20°C. Apply the grid method to calculate the density in units of $kg/m^3$. SS

**a.** The gas is helium. The gage pressure is 36 in $H_2O$.

**b.** The gas is methane. The vacuum pressure is 8.8 psi.

**3.8** For the questions below, assume standard atmospheric pressure.

**a.** For a vacuum pressure of 43 kPa, what is the absolute pressure? Gage pressure? Answer

**b.** For a pressure of 15.6 psig, what is the pressure in psia?

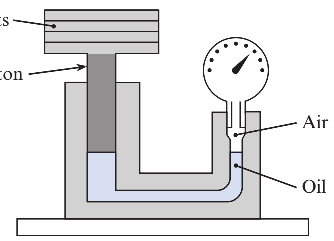
**c.** For a pressure of 190 kPa gage, what is the absolute pressure in kPa?

**d.** Give the pressure 100 psfg in psfa.

**3.9** The local atmospheric pressure is 91 kPa. A gage on an oxygen tank reads a pressure of 250 kPa gage. What is the pressure in the tank in kPa abs? SS

**3.10** (T/F) The gage pressure at a depth of 34 meters of water is about 1 bar. Answer

**3.11** The gage tester shown in the figure is used to calibrate or to test pressure gages. When the weights and the piston together weigh 132 N, the gage being tested indicates 197 kPa. If the piston diameter is 30 mm, what percentage of error exists in the gage?

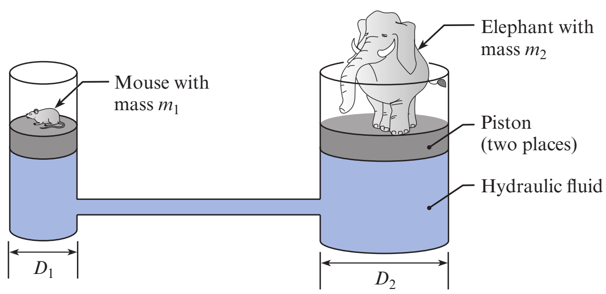



**Problem 3.11**

**3.12** As shown, a mouse can use the mechanical advantage provided by a hydraulic machine to lift up an elephant. Answer

**a.** Derive an algebraic equation that gives the mechanical advantage of the hydraulic machine shown. Assume the pistons are frictionless and massless.

**b.** A mouse can have a mass of 25 g and an elephant a mass of 7500 kg. Determine a value of $D_1$ and $D_2$ so that the mouse can support the elephant.




**Problem 3.12**

**3.13** A point is located at an elevation of 3 km in the atmosphere. In SI units, what are the properties as given by the standard atmosphere model?

**a.** $p_{atm}$ = ______________________

**b.** $T_{atm}$ = ______________________

**c.** $\rho$ = ______________________

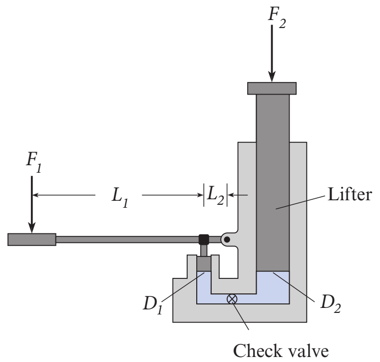
**d.** $\mu$ = ______________________

**e.** $\nu$ = ______________________

**3.14** Which equation gives the mechanical advantage of this hydraulic bottle jack? Answer

(a) $\left(\frac{D_1^2}{D_2^2}\right)\left(\frac{L_1}{L_2}\right)$ (b) $\left(\frac{D_2^2}{D_1^2}\right)\left(\frac{L_1}{L_1+L_2}\right)$ (c) $\left(\frac{D_2^2}{D_1^2}\right)\left(\frac{L_2}{L_1+L_2}\right)$

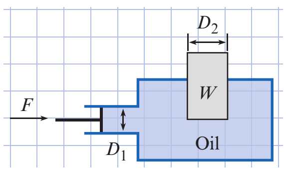
(d) $\left(\frac{D_2^2}{D_1^2}\right)\left(\frac{L_1+L_2}{L_2}\right)$ (e) $\left(\frac{D_1^2}{D_2^2}\right)\left(\frac{L_1+L_2}{L_1}\right)$




**Problem 3.14**

**3.15** What is the mechanical advantage for the pictured system? The mass of the weight $W$ is 2000 kg. The diameter $D_1$ is 100 mm. The force $F$ is 375 N

(**a**) 39 (**b**) 52 (**c**) 30 (**d**) 5 (**e**) 18




**Problem 3.15**

**3.16** Find a parked automobile for which you have information on tire pressure and weight. Measure the area of tire contact with the pavement. Next, using the weight information and tire pressure, use engineering principles to calculate the contact area. Compare your measurement with your calculation and discuss.

## The Hydrostatic Equation (§3.2)

**3.17** To derive the hydrostatic equation, which of the following must be assumed? Select all that are correct:

**a.** The specific weight is constant.

**b.** The fluid has no charged particles.

**c.** The fluid is at equilibrium.

**3.18** Write a definition of **piezometric pressure** using the standard structure of a definition.

**3.19** Write a definition of **hydrostatic conditions** using the standard structure of a definition.

**3.20** Imagine two tanks. Tank A is filled to depth $h$ with water. Tank B is filled to depth $h$ with oil. Which tank has the largest pressure? Why? Where in that tank does the largest pressure occur? Answer

**3.21** Consider Figure 3.11.

**a.** Which fluid has the larger density?

**b.** If you graphed pressure as a function of $z$ in these two layered liquids, in which fluid does the pressure change more with each incremental change in $z$?

**3.22** Apply the grid method with the hydrostatic equation ($\Delta p = \gamma \Delta z$) to each of the following cases: Answer

**a.** Predict the pressure change $\Delta p$ in kPa for an elevation change $\Delta z$ of 6.8 ft in a fluid with a density of 90 lbm/ft$^3$.

**b.** Predict the pressure change in psf for a fluid with $SG = 1.3$ and an elevation change of 22 m.

**c.** Predict pressure change in inches of water for a fluid with a density of 1.2 kg/m$^3$ and an elevation change of 2500 ft.

**d.** Predict the elevation change in millimeters for a fluid with $SG = 1.4$ that corresponds to a change in pressure of 1/6 atm.

**3.23** Using §3.2 and other resources, answer the following questions. Strive for depth, clarity, and accuracy while also combining sketches, words, and equations in ways that enhance the effectiveness of your communication.

**a.** What does hydrostatic mean? How do engineers identify whether a fluid is hydrostatic?

**b.** What are the common forms on the hydrostatic equation? Are the forms equivalent or are they different?

**c.** What is a datum? How do engineers establish a datum?

**d.** What are the main ideas of Eq. (3.10)? That is, what is the meaning of this equation?

**e.** What assumptions need to be satisfied to apply the hydrostatic equation?

**3.24** The pressure at the bottom of a lake is 20 psia. What is the depth of the lake in units of meters? Answer

(**a**) 14.1 (**b**) 5.3 (**c**) 4.7 (**d**) 3.7 (**e**) 2.3

**3.25** Apply the grid method to each of the following situations:

**a.** What is the change in air pressure in pascals between the floor and the ceiling of a room with walls that are 8 ft tall?

**b.** A diver in the ocean ($SG = 1.03$) records a pressure of 1.5 atm on her depth gage. How deep is she?

**c.** A hiker starts a hike at an elevation where the air pressure is 960 mbar, and he ascends 1240 ft to a mountain summit. Assuming the density of air is constant, what is the pressure in mbar at the summit?

**d.** Lake Pend Oreille, in northern Idaho, is one of the deepest lakes in the world, with a depth of 370 m in some locations.

This lake is used as a test facility for submarines. What is the maximum gage pressure that a submarine could experience in this lake?

**e.** A 55-m tall standpipe (a vertical pipe that is filled with water and open to the atmosphere) is used to supply water for fire-fighting. What is the maximum gage pressure in the standpipe?

**3.26** A tank that is open to the atmosphere contains 120 cm of oil ($SG = 0.8$) floating on top of 70 cm of water. The datum is situated at the oil and water interface.

What is the piezometric pressure at the bottom of the tank in units of kPa gage? Answer

(**a**) 9.4 (**b**) 11.8 (**c**) 16.3 (**d**) 19.6 (**e**) 21.4

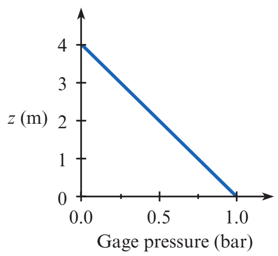
**3.27** (T/F) In a lake, the pressure in units of atmospheres at a depth $d$ is approximately equal to $d/10$, when $d$ is expressed in SI units.

**3.28** A tank that is open to the atmosphere contains water that is 34 feet deep. What is the maximum pressure in the water in units of psia? Answer

(**a**) 29 (**b**) 15 (**c**) 34 (**d**) 10 (**e**) 22

**3.29** (T/F) If you plot the hydrostatic equation with $z$ on the vertical axis and $p$ on the horizontal axis, the result is a straight line with a slope of $-1/\gamma$

**3.30** The plot shows the pressure variation in a stationary body of water on a planet named Kylerkin.

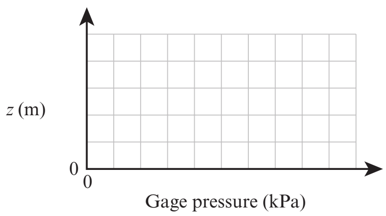



**Problem 3.30**

In units of m/s$^2$, what is the value of $g$ on Kylerkin? Answer

(**a**) 25 (**b**) 17 (**c**) 11 (**d**) 32 (**e**) 29

**3.31** An open tank is filled with a stationary liquid ($SG = 0.7$). Using the axes that follow, plot elevation in m as a function of gage pressure in kPa. The free surface, where $p_{atm}$ prevails, is situated at $z = 45$ m.



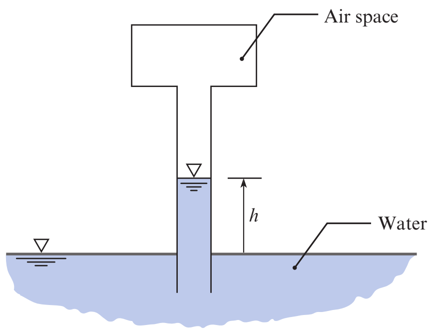

**Problem 3.31**

**3.32** The air pressure on the summit of a mountain is 700 mmHg. For this task, you can model the atmosphere as a constant density gas with $\rho = 1.2$ kg/m$^3$.

In units of meters, what is the elevation above sea level at the summit? Answer

(**a**) 160 (**b**) 490 (**c**) 200 (**d**) 680 (**e**) 540

**3.33** As shown, an air space above a long tube is pressurized to 50 kPa vacuum. Water (20°C) from a reservoir fills the tube to a height $h$. If the pressure in the air space is changed to 25 kPa vacuum, will $h$ increase or decrease, and by how much? Assume atmospheric pressure is 100 kPa.




**Problem 3.33**

**3.34** As water flows through a valve, it is cavitating. The minimum pressure inside the value is −0.3 bar.

What is the water temperature in units of Rankine? Answer

(**a**) 750 (**b**) 850 (**c**) 650 (**d**) 950 (**e**) 1050

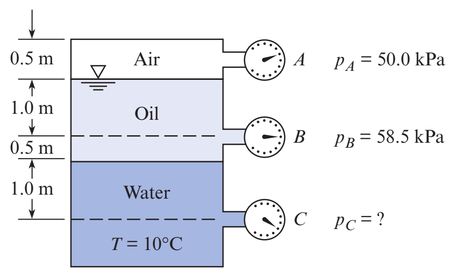
**3.35** An engineer is designing a pump to suck mud ($SG = 1.8$) up a 3-m-long vertical pipe. The engineer will model the mud as a liquid in which hydrostatic conditions apply. The pressure at the bottom of the pipe is atmospheric.

In kPa, what is the vacuum pressure at the top of the pipe?

(**a**) 53 (**b**) 37 (**c**) 29 (**d**) 24 (**e**) 18

**3.36** A field test is used to measure the density of crude oil recovered during a fracking* operation. The crude oil recovered is mixed with brine. The oil and brine mixture are placed in an open tank and allowed to separate. After separation, a 1.0-m layer of oil floats on top of 0.55 m of brine. The density of the brine is 1030 kg/m$^3$ and the pressure at the bottom of the tank is 14 kPa gage. Find the density of the oil. Answer

**3.37** For the closed tank with Bourdon-tube gages tapped into it, what is the specific gravity of the oil and the pressure reading on gage $C$?




**Problem 3.37**

*Hydraulic fracturing (or "fracking") is a method that is used to recover gas and oil. Fracking creates fractures in rocks by injecting high-pressure liquids containing particulate additives into smaller cracks and forcing the cracks to widen. The larger cracks allow more petroleum products to flow through the formation to the well. A density test as described here could be performed to determine the approximate makeup of the oil. The brine must be disposed of after fracking.

SS **3.38** A tube with an ID of 2 mm is situated in a container of mercury ($SG$ = 13.6). The contact angle is 140°. The surface tension of mercury is 490 mN · $m^{-1}$. Answer

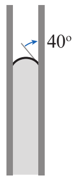
In units of millimeters, what is the capillary depression?

(**a**) 5.6 (**b**) 4.2 (**c**) 1.1 (**d**) 3.1 (**e**) 2.1

**3.39** A tube with an ID (inside diameter) of 1 mm is situated in a container of mercury ($SG$ = 13.6). The surface tension of mercury is 0.49 N/m. As shown, the angle between the tube and the mercury is 40°.

In units of millimeters, what is the capillary depression?

(**a**) 6 (**b**) 42 (**c**) 11 (**d**) 31 (**e**) 21




**Problem 3.39**

**3.40** For the given problem, which equation gives $h$? Answer

**a.** $\dfrac{2\sigma\cos\alpha}{\gamma_m d}$

**b.** $\dfrac{4\sigma\cos\alpha}{\gamma_m d}$

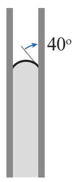
**c.** $\dfrac{2\sigma\sin\alpha}{\gamma_m d}$

**d.** $\dfrac{8\sigma\cos\alpha}{\gamma_m d}$

**e.** $\dfrac{4\sigma\sin\alpha}{\gamma_m d}$

**PROBLEM STATEMENT**

A tube with an ID (inside diameter) of $d$ = 1 mm is situated in a container of mercury. The specific weight of mercury is $\gamma_m$ = 130 kN/$m^3$. The surface tension of mercury is $\sigma$ = 0.49 N/m. As shown, the angle between the tube and the mercury is $\alpha$ = 40°. Calculate the capillary depression $h$.




**Problem 3.40**

**3.41** A liquid (SG = 0.9) is flowing in a pipe. At point $A$, the gage pressure is −12 kPa and the elevation is 7 m.

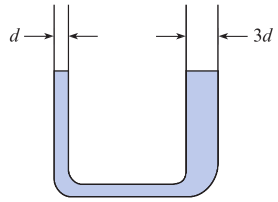
In SI units, the piezometric head at $A$ is:

(**a**) 5.6 (**b**) 2.2 (**c**) 9.9 (**d**) 1.1 (**e**) 6.8

**3.42** A closed tank contains air that is pressurized to 1.3 bar abs above a liquid ($SG$ = 0.8) that is 4 meters deep. What is the maximum pressure in the liquid in units of kPa gage? Answer

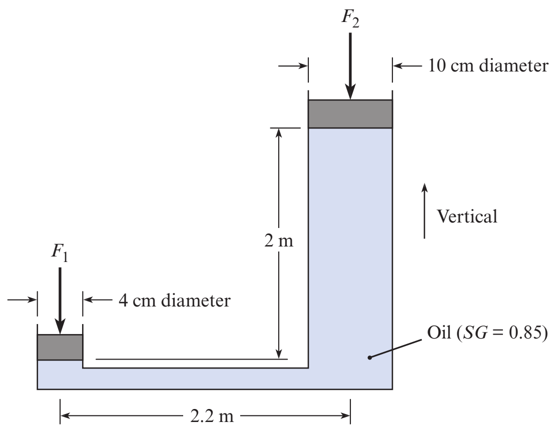
(**a**) 30 (**b**) 61 (**c**) 130 (**d**) 69 (**e**) 110

**3.43** This manometer contains water at room temperature. The glass tube on the left has an inside diameter of 1 mm ($d$ = 1.0 mm). The glass tube on the right is three times as large. For these conditions, the water surface level in the left tube will be (a) higher than the water surface level in the right tube, (b) equal to the water surface level in the right tube, or (c) less than the water surface level in the right tube. State your main reason or assumption for your choice.




**Problem 3.43**

**3.44** If a 390 N force $F_1$ is applied to the piston with the 4-cm diameter, what is the magnitude of the force $F_2$ that can be resisted by the piston with the 10-cm diameter? Neglect the weights of the pistons. Answer




**Problem 3.44**

**3.45** Regarding the hydraulic jack in Problem 3.44 which ideas were used to analyze the jack? Select all that apply: SS

**a.** pressure = (force)(area)

**b.** pressure increases linearly with depth in a fluid with a constant density

**c.** the pressure at the bottom of the 4-cm chamber is larger than the pressure at the bottom of the 10-cm chamber

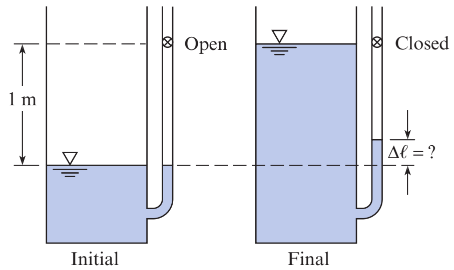
**d.** when a body is stationary, the sum of forces on the body is zero

**e.** when a body is stationary, the sum of moments on the body is zero

**f.** differential pressure = (weight/volume)(change in elevation)

**3.46** Water occupies the bottom 1.2 m of a cylindrical tank. On top of the water is 0.8 m of kerosene, which is open to the atmosphere. If the temperature is 20°C, what is the gage pressure at the bottom of the tank? Answer

**3.47** A tank with an attached manometer contains water at 20°C. The atmospheric pressure is 100 kPa. There is a stopcock located 1 m from the surface of the water in the manometer. The stopcock is closed, trapping the air in the manometer, and water is added to the tank to the level of the stopcock. Find the increase in elevation of the water in the manometer assuming the air in the manometer is compressed isothermally.




Problem 3.47

**3.48** A tank that is open to the atmosphere contains 40 cm of oil ($SG = 0.8$) floating on top of 30 cm of water.

What is the gage pressure at the bottom of the tank in units of pascals? Answer

(**a**) 4500 (**b**) 8400 (**c**) 5700 (**d**) 7600 (**e**) 6100

**3.49** A stationary body of liquid has a variable density given by $\rho = c + ah$, where $c = 1.9\,\text{slug/ft}^3$, $a = 0.01\,\text{slug/ft}^4$, and $h$ is the distance in feet measured from the free surface.

What is the pressure in psfg at a point that is 50 ft below the free surface?

(**a**) 5400 (**b**) 2700 (**c**) 3500 (**d**) 5900 (**e**) 1400

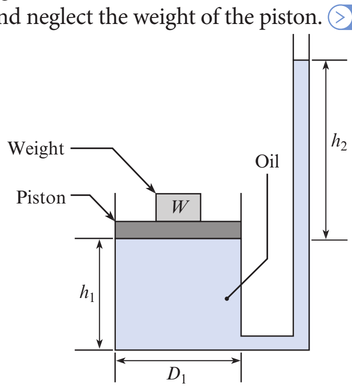
**3.50** A stationary body of liquid has a variable density given by $\rho = c + ah$, where $c = 1000\,\text{kg/m}^3$, $a = 20\,\text{kg/m}^4$, and $h$ is the distance in meters measured from the free surface.

What is the pressure in kPa at a point that is 25 m below the free surface? Answer

(**a**) 240 (**b**) 310 (**c**) 280 (**d**) 90 (**e**) 480

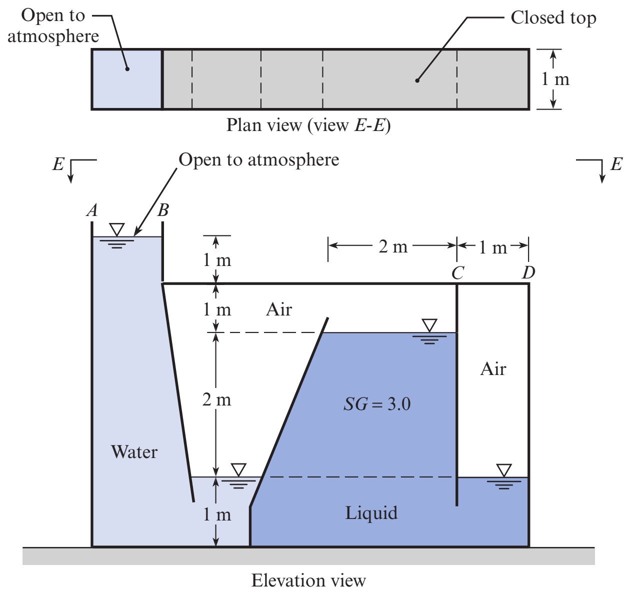
SS **3.51** As shown, a weight sits on a piston of diameter $D_1$. The piston rides on a reservoir of oil of depth $h_1$ and specific gravity $SG$. The reservoir is connected to a round tube of diameter $D_2$ and oil rises in the tube to height $h_2$. The oil in the tube is open to atmosphere. Derive an equation for the height $h_2$ in terms of the weight $W$ of the load and other relevant variables. Neglect the weight of the piston.

**3.52** As shown, a weight of mass 5 kg is situated on a piston of diameter $D_1 = 120$ mm. The piston rides on a reservoir of oil of depth $h_1 = 42$ mm and specific gravity $SG = 0.8$. The reservoir is connected to a round tube of diameter $D_2 = 5$ mm, and oil rises in the tube to height $h_2$. Find $h_2$. Assume the oil in the tube is open to atmosphere, and neglect the weight of the piston. Answer



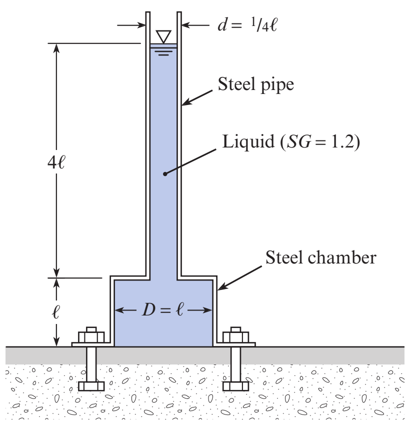

Problems 3.51, 3.52

**3.53** What is the maximum gage pressure in the odd tank shown in the figure? Where will the maximum pressure occur? What is the pressure force acting on the top ($CD$) of the last chamber on the right-hand side of the tank? Assume $T = 10°\text{C}$.



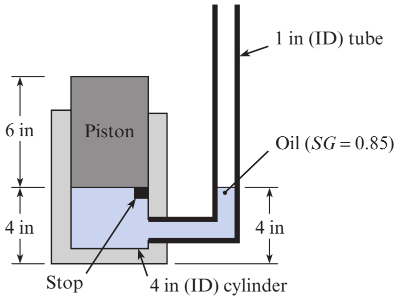

Problem 3.53

SS **3.54** The steel pipe and steel chamber shown in the figure together weigh 600 lbf. What force will have to be exerted on the chamber by all the bolts to hold it in place? The dimension $\ell$ is equal to 2.5 ft. *Note:* There is no bottom on the chamber—only a flange bolted to the floor. Answer




Problem 3.54

**3.55** The piston shown weighs 8 lbf. In its initial position, the piston is restrained from moving toward the bottom of the cylinder by means of the metal stop. Assuming there is neither friction nor leakage between piston and cylinder, what volume of oil ($SG = 0.85$) would have to be added to the 1 in. tube to cause the piston to rise 1 in. from its initial position?



Problem 3.55

**3.56** Consider an air bubble rising from the bottom of a lake. Neglecting surface tension, determine the ratio of the density of the air in the bubble at a depth of 34 ft to its density at a depth of 8 ft. Answer

## Measuring Pressure (§3.3)

**3.57** Match the following pressure-measuring devices with the correct name. The device names are: barometer, Bourdon gage, piezometer, manometer, and pressure transducer.

**a.** A U-shaped tube in which changes in pressure cause changes in relative elevation of a liquid that is usually denser than the fluid in the system measured; can be used to measure vacuum.

**b.** Typically contains a diaphragm, a strain gage, and conversion to an electric signal.

**c.** A round face with a scale to measure needle deflection, in which the needle is deflected by changes in extension of a coiled hollow tube.

**d.** A vertical tube in which a liquid rises in response to a positive gage pressure.

**e.** An instrument used to measure atmospheric pressure; can be of various designs.

SS **3.58** (T/F) A liquid filled U-tube manometer can be used to measure vacuum pressure in a gas. Answer

**3.59** What types of pressure can be measured with a piezometer or with several piezometers?

**I.** gage

**II.** vacuum

**III.** differential

**IV.** absolute

**V.** atmospheric

The correct responses are: (a) I and III (b) I only (c) I, II, and III (d) all except IV (e) all except V

SS **3.60** To measure differential pressure in a horizontal pipe, an engineer will use two piezometers placed 1 m apart. Water is flowing in the pipe. The steel pipe ID (inside diameter) is 32 cm and the OD (outside diameter) is 38 cm. The piezometers, made of acrylic, have an ID of 2 cm. The absolute pressure in the pipe at the upstream point, which is point $A$, is 110 kPa. The absolute pressure in the pipe at the downstream point, which is point $B$, is 105 kPa. Local atmospheric pressure is 1 bar.

Your task is to (a) sketch the system approximately to scale, and then to (b) label the significant features.

**3.61** A barometer, similar in design to a mercury barometer, uses a liquid with $S = 6$ and $p_v = 20$ kPa. The local atmospheric pressure is 0.9 bar. SS

What is the column height, in cm, for this barometer?

**(a)** 120 **(b)** 140 **(c)** 160 **(d)** 180 **(e)** 210

**3.62** (T/F) A pressure gauge measures gage pressure. Answer SS

## Applying the Manometer Equations (§3.3)

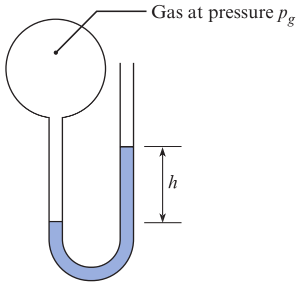
**3.63** As shown, gas at pressure $p_g$ raises a column of liquid to a height $h$. The gage pressure in the gas is given by $p_g = \gamma_{\text{liquid}} h$. Apply the grid method to each situation that follows.

**a.** The manometer uses a liquid with $SG = 1.4$. Calculate pressure in psia for $h = 2.3$ ft.

**b.** The manometer uses mercury. Calculate the column rise in mm for a gage pressure of 0.5 atm.

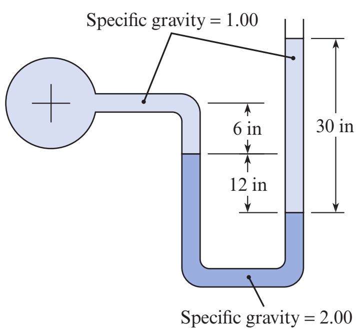
**c.** The liquid has a density of 22 lbm/ft$^3$. Calculate pressure in psfg for $h = 6$ inches.

**d.** The liquid has a density of 800 kg/m$^3$. Calculate the gage pressure in bar for $h = 2.3$ m.



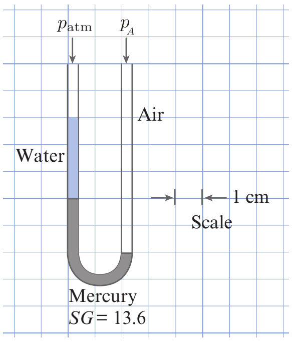
Problem 3.63

**3.64** Is the gage pressure at the center of the pipe (a) negative, (b) zero, or (c) positive? Neglect surface tension effects and state your rationale. Answer



Problem 3.64

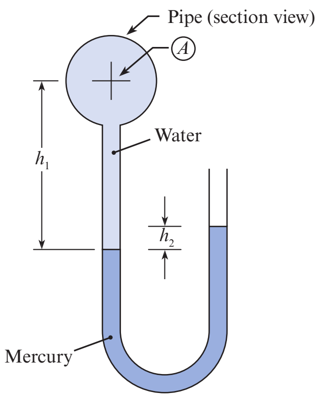
**3.65** Situation: The sketch shows a manometer. Scale: 1 grid unit = 1 cm.




**Problem 3.65**

(T/F) The air pressure is $p_A$ = 3 cmH$_2$O + 2 cmHg = 3 kPa.

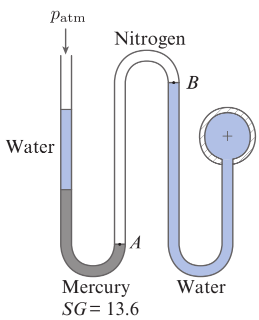
SS **3.66** Determine the gage pressure at the center of the pipe (point $A$) in pounds per square inch when the temperature is 70°F with $h_1$ = 16 in. and $h_2$ = 2 in. Answer




**Problem 3.66**

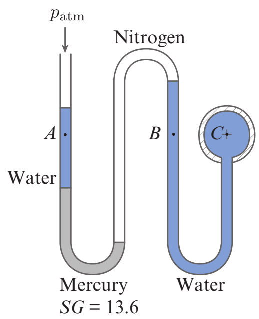
**3.67** Situation: An engineer is analyzing a manometer that contains three fluids: nitrogen, mercury, and water.

(T/F) The engineer would let $p_A = p_B$.



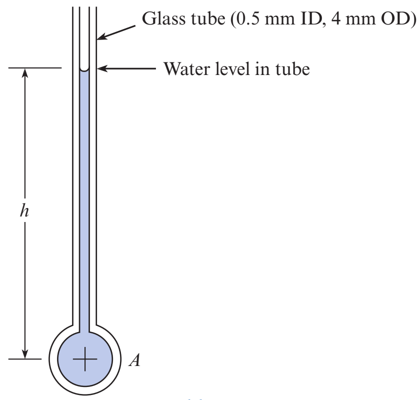

**Problem 3.67**

**3.68** (T/F) Because the sketch shows that points $A$, $B$, and $C$ are at the same elevation, the pressures at these points are equal. Thus, $p_A = p_B = p_C$. Answer



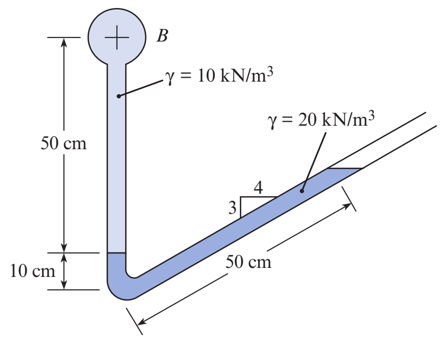

**Problem 3.68**

**3.69** Considering the effects of surface tension, estimate the gage pressure at the center of pipe $A$ for $h$ = 120 mm and $T$ = 20°C.




**Problem 3.69**

**3.70** What is the pressure at the center of pipe $B$? Answer

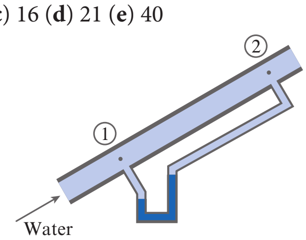



**Problem 3.70**

**3.71** The manometer deflection is 9 cm. The manometer fluid is mercury, which has a specific gravity of 13.6. Elevation 1 is 9 m, and elevation 2 is 10 m.

In kPa, the differential pressure $\Delta p = p_1 - p_2$ is

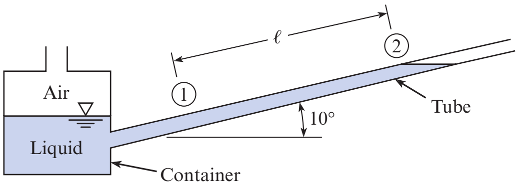
(**a**) 34 (**b**) 11 (**c**) 16 (**d**) 21 (**e**) 40




**Problem 3.71**

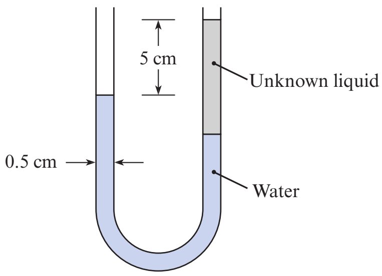
**3.72** The ratio of container diameter to tube diameter is 8. When air in the container is at atmospheric pressure, the free surface in the tube is at position 1. When the container is pressurized, the liquid in the tube moves 40 cm up the tube from position 1 to position 2. What is the container pressure that causes this deflection? The liquid density is 1200 $kg/m^3$. Answer

**3.73** The ratio of container diameter to tube diameter is 10. When air in the container is at atmospheric pressure, the free surface in the tube is at position 1. When the container is pressurized, the liquid in the tube moves 3 ft up the tube from position 1 to position 2. What is the container pressure that causes this deflection? The specific weight of the liquid is 50 $lbf/ft^3$.



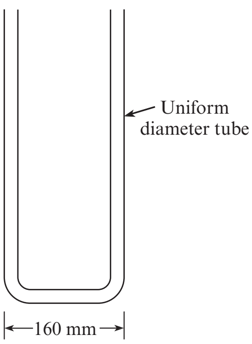

Problems 3.72, 3.73

**3.74** A device for measuring the specific weight of a liquid consists of a U-tube manometer as shown. The manometer tube has an internal diameter of 0.5 cm and originally has water in it. Exactly 2 $cm^3$ of unknown liquid is then poured into one leg of the manometer, and a displacement of 5 cm is measured between the surfaces as shown. What is the specific weight of the unknown liquid? Answer



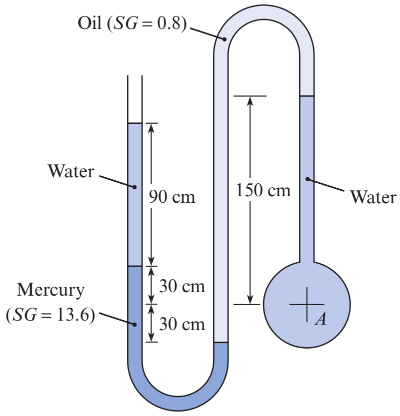

Problem 3.74

**3.75** Mercury is poured into the tube in the figure until the mercury occupies 375 mm of the tube's length. An equal volume of water is then poured into the left leg. Locate the water and mercury surfaces. Also determine the maximum pressure in the tube.



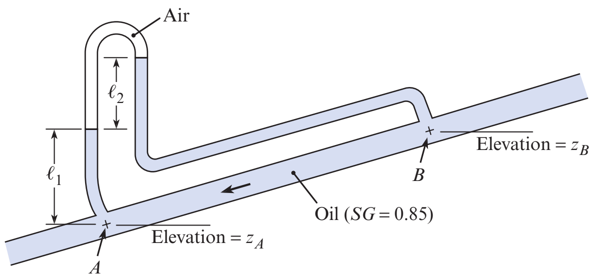

Problem 3.75

**3.76** Find the pressure at the center of pipe $A$. $T = 10°C$. Answer SS




Problem 3.76

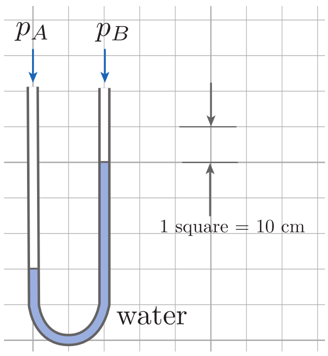
**3.77** Determine (a) the difference in pressure and (b) the difference in piezometric head between points $A$ and $B$. The elevations $z_A$ and $z_B$ are 10 m and 11 m, respectively, $\ell_1 = 1$ m, and the manometer deflection $\ell_2$ is 50 cm.




Problem 3.77

**3.78** What is the differential pressure in kPa? Answer

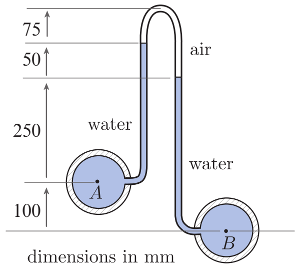
(**a**) 2.9 (**b**) 0.2 (**c**) 0.3 (**d**) 1.2 (**e**) 2.1




Problem 3.78

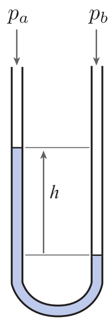
**3.79** Differential pressure is being measured between points $A$ and $B$ in two pipes. In pascals, what is $\Delta p = p_A - p_B$?

(**a**) −490 (**b**) −350 (**c**) −770 (**d**) −630 (**e**) −530



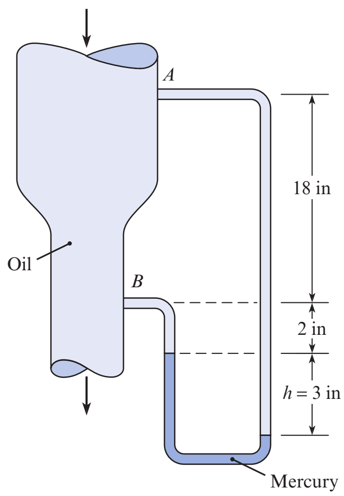
Problem 3.79

**3.80** (T/F) The column height $h$ in this manometer is related to the differential pressure by $h = \dfrac{p_a - p_b}{g\rho}$ Answer



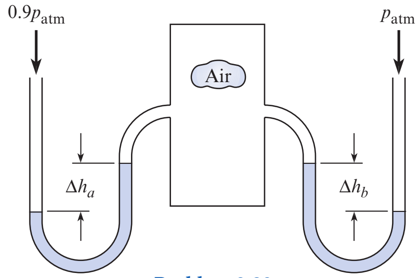
Problem 3.80

SS **3.81** A vertical conduit is carrying oil ($SG = 0.95$). A differential mercury manometer is tapped into the conduit at points $A$ and $B$. Determine the difference in pressure between $A$ and $B$ when $h = 3$ in. What is the difference in piezometric head between $A$ and $B$?



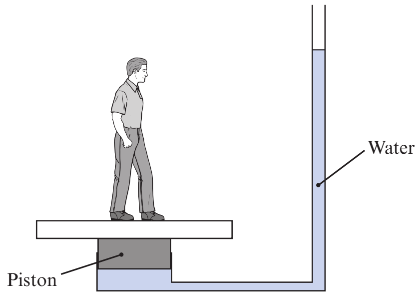
Problem 3.81

**3.82** Two water manometers are connected to a tank of air. One leg of the manometer is open to 100 kPa pressure (absolute) while the other leg is subjected to 90 kPa. Find the difference in deflection between both manometers, $\Delta h_a - \Delta h_b$. Answer



Problem 3.82

**3.83** A novelty scale for measuring a person's weight by having the person stand on a piston connected to a water reservoir and stand pipe is shown in the diagram. The level of the water in the stand pipe is to be calibrated to yield the person's weight in pounds force. When the person stands on the scale, the height of the water in the stand pipe should be near eye level so the person can read it. There is a seal around the piston that prevents leaks but does not cause a significant frictional force. The scale should function for people who weigh between 60 and 250 lbf and are between 4 and 6 feet tall. Choose the piston size and standpipe diameter. Clearly state the design features you considered. Indicate how you would calibrate the scale on the standpipe. Would the scale be linear?

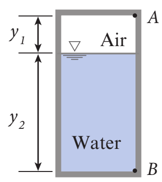


Problem 3.83

**3.84** For the given situation, (a) sketch the pressure distribution on line $AB$ and (b) describe the significant features of your sketch.

**SITUATION**

Water is contained in a closed cylindrical tank. The air above the water is pressurized to an absolute pressure of $\frac{11}{10}$ atm. The heights are $y_1 = 1$ m and $y_2 = 3$ m. The points $A$ and $B$ identify a vertical line.



Problem 3.84

**3.85** A stationary body of liquid creates a pressure distribution on a vertical flat panel.

(T/F) The pressure acting on the panel is uniform.

**3.86** A stationary body of liquid creates a pressure distribution on a horizontal flat panel. SS

(T/F) The pressure acting on the panel is uniform. Answer

**3.87** Gas inside a rectangular tank creates a pressure distribution on a vertical wall of the tank.

(T/F) This pressure distribution is uniform.

## Pressure Forces on Panels (Flat Surfaces) (§3.4)

**3.88** Using §3.4 and other resources, answer the questions below. Strive for depth, clarity, and accuracy while also combining sketches, words, and equations in ways that enhance the effectiveness of your communication.

**a.** For hydrostatic conditions, what do typical pressure distributions on a panel look like? Sketch three examples that correspond to different situations.

**b.** What is a center of pressure (CP)? What is a centroid of area?

**c.** In Eq. (3.28), what does $\bar{p}$ mean? What factors influence the value of $\bar{p}$?

**d.** What is the relationship between the pressure distribution on a panel and the resultant force?

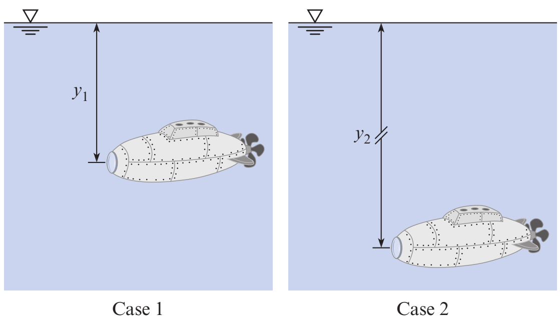
**e.** How far is the CP from the centroid of area? What factors influence this distance?

**3.89** Part 1. Consider the equation for the distance between the CP and the centroid of a submerged panel (Eq. (3.33)). In that equation, $y_{cp}$ is

**a.** the vertical distance from the water surface to the CP.

**b.** the slant distance from the water surface to the CP.

Part 2. Consider the figure shown. For case 1, the flat viewing window on the front of a submersible exploration vehicle is at a depth of $y_1$. For case 2, the submersible has moved deeper in the ocean, to $y_2$. As a result of this increased overall depth of the submersible and its window, does the spacing between the CP and centroid (a) get larger, (b) stay the same, or (c) get smaller?




**Problem 3.89**

SS **3.90** If a hydrostatic force acts on a flat panel, then the center of pressure is always (a) above the centroid (b) at or above the centroid (c) at the centroid (d) below the centroid (e) at or below the centroid. Answer

SS **3.91** Regarding the given statements, which of these statements are true? (a) I, II, III, IV (b) all are true (c) II, III, and IV (d) III only (e) all except V

### STATEMENTS ABOUT THE PANEL EQUATIONS

**I.** The magnitude of $\frac{I}{\bar{y}A}$ is greater than zero

**II.** The term $y_{cp} - \bar{y}$ is the vertical distance from the centroid to the CP

**III.** $[I] = L^4$

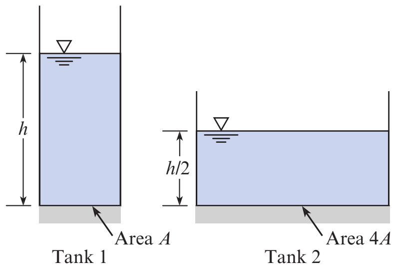
**IV.** The parameter $\bar{p}$ is the gage pressure evaluated at the depth of the CP

**V.** The centroid and the CP are coincident

**3.92** Two cylindrical tanks have bottom areas $A$ and $4A$ respectively, and are filled with water to the depths shown.

**a.** Which tank has the higher pressure at the bottom of the tank?

**b.** Which tank has the greater force acting downward on the bottom circular surface? Answer




**Problem 3.92**

**3.93** A vertical plexiglass wall on an aquarium has a height of 5 m and a width of 10 m. At state 1, the depth of water in the aquarium is 2 m. At state 2, the depth is 4 m.

What is ratio of pressure forces $F_2/F_1$ that act on the wall?

(**a**) 2 (**b**) 3 (**c**) 4 (**d**) 6 (**e**) 8

**3.94** (T/F) When a liquid exerts a pressure force on a vertical panel, both the pressure force and the location of the center of pressure depend on the shape of the panel. Answer

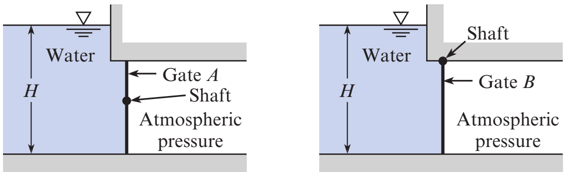
**3.95** (T/F) When a liquid exerts a pressure force on a panel, the pressure force depends on the shape of the panel. SS

**3.96** What is the force acting on the gate of an irrigation ditch if the ditch and gate are 2 ft wide, 2 ft deep, and the ditch is completely full of water? There is no water on the other side of the gate. The weather has been hot for weeks, so the water is 70°F. Answer

**3.97** An irrigation ditch is full, with slack ($V = 0$ m/s) water ($T = 5$°C) restrained by a closed gate. The ditch and gate are both 2 m wide by 1.5 m deep. Find the force acting on the gate and the location of center of pressure on the gate as measured from the bottom of the ditch. There is no water on the downstream side of the gate.

**3.98** Consider the two rectangular gates shown in the figure. They are both the same size, but gate $A$ is held in place by a horizontal shaft through its midpoint and gate $B$ is cantilevered to a shaft at its top. Now consider the torque $T$ required to hold the gates in place as $H$ is increased. Choose the valid statement(s): (a) $T_A$ increases with $H$. (b) $T_B$ increases with $H$. (c) $T_A$ does not change with $H$. (d) $T_B$ does not change with $H$. Answer

**3.99** For gate $A$, choose the statements that are valid: (a) The hydrostatic force acting on the gate increases as $H$ increases. (b) The distance between the CP on the gate and the centroid of the gate decreases as $H$ increases. (c) The distance between the CP on the gate and the centroid of the gate remains constant as $H$ increases. (d) The torque applied to the shaft to prevent the gate from turning must be increased as $H$ increases. (e) The torque applied to the shaft to prevent the gate from turning remains constant as $H$ increases.



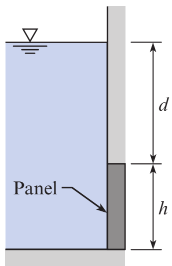

**Problems 3.98, 3.99**

**3.100** As shown, water (15°C) is in contact with a square panel; $d = 2.3$ m and $h = 2$ m. Answer

**a.** Calculate the depth of the centroid.

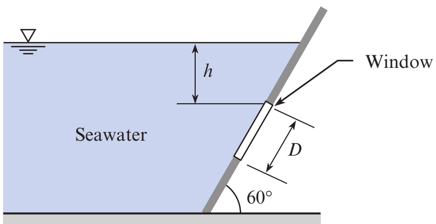
**b.** Calculate the resultant force on the panel.

**c.** Calculate the distance from the centroid to the CP.



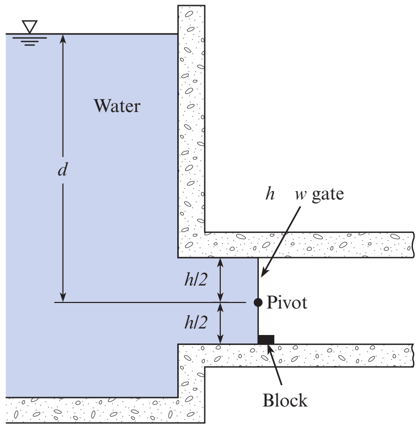

**Problem 3.100**

SS **3.101** As shown, a round viewing window of diameter $D = 0.5$ m is situated in a large tank of seawater ($SG = 1.03$). The top of the window is 1.5 m below the water surface, and the window is angled at 60° with respect to the horizontal. Find the hydrostatic force acting on the window, and locate the corresponding CP.



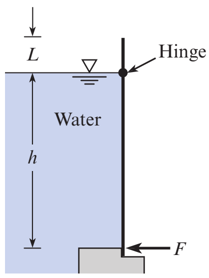

**Problem 3.101**

**3.102** Find the force of the gate on the block as shown, where $d = 12$ m, $h = 6$ m, and $w = 6$ m. Answer



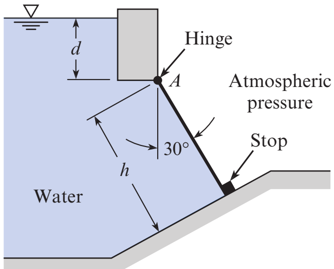

**Problem 3.102**

SS **3.103** A rectangular gate is hinged at the water line, as shown. The gate has $h = 4$ ft of its length below the waterline, $L = 1$ ft above the waterline, and is 8 ft wide. The specific weight of water is 62.4 lbf/ft$^3$. Find the force (lbf) applied at the bottom of the gate necessary to keep the gate closed.



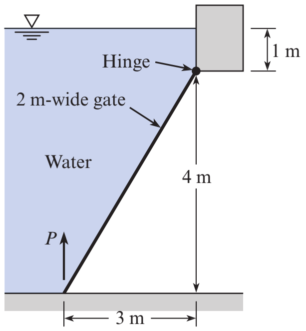

**Problem 3.103**

**3.104** The gate shown is rectangular and has dimensions height $h = 6$ m by width $b = 4$ m. The hinge is $d = 3$ m below the water surface. What is the force at point $A$? Neglect the weight of the gate. Answer



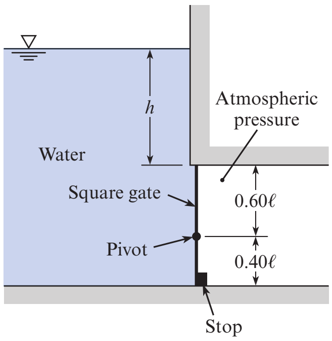

**Problem 3.104**

**3.105** Determine the force $P$ necessary to just start opening the 2-m wide gate.



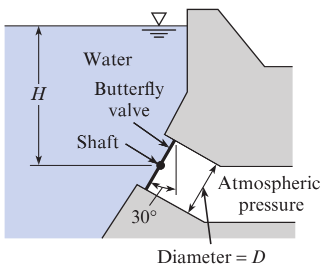

**Problem 3.105**

**3.106** The square gate shown is eccentrically pivoted so that it automatically opens at a certain value of $h$. What is that value in terms of $\ell$? Answer




**Problem 3.106**

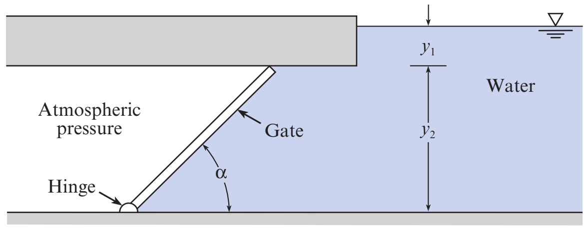
**3.107** This butterfly valve ($D = 12$ ft) is used to control the flow in a 12-ft diameter outlet pipe in a dam. In the position shown, the valve is closed. The valve is supported by a horizontal shaft through its center. The shaft is located $H = 60$ ft below the water surface. What torque would have to be applied to the shaft to hold the valve in the position shown?




**Problem 3.107**

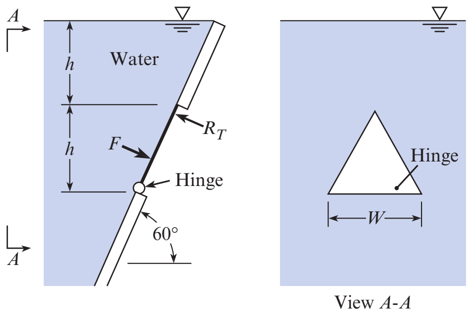
**3.108** For the gate shown, $\alpha = 45°$, $y_1 = 1$ m, and $y_2 = 4$ m. Will the gate fall or stay in position under the action of the hydrostatic and gravity forces if the gate itself weighs 150 kN and is 1.0 m wide? Assume $T = 10°\text{C}$. Use calculations to justify your answer. Answer

**3.109** For this gate, $\alpha = 45°$, $y_1 = 3$ ft, and $y_2 = 6$ ft. Will the gate fall or stay in position under the action of the hydrostatic and gravity forces if the gate itself weighs 18,000 lb and is 3 ft wide? Assume $T = 50°\text{F}$. Use calculations to justify your answer.



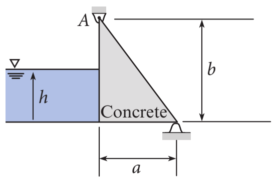

**Problems 3.108, 3.109**

**3.110** Determine the hydrostatic force $F$ on the triangular gate, which is hinged at the bottom edge and held by the reaction $R_T$ at the upper corner. Express $F$ in terms of $\gamma$, $h$, and $W$. Also determine the ratio $R_T/F$. Neglect the weight of the gate. Answer




**Problem 3.110**

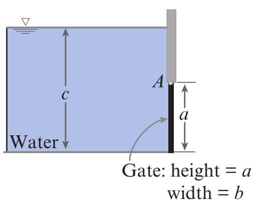
**3.111** What depth of water in meters will cause this concrete ($SG = 2.4$) gate to start to rotate about the pin at $A$? The lengths are $a = 3$ m and $b = 4$ m. Neglect friction at both of the gate supports.




**Problem 3.111**

**3.112** Water creates a load on a gate. The dimensions are $a = 6$ ft, $b = 4$ ft, and $c = 10$ ft. What is the magnitude of the torque in ft·kips that needs to be applied to the hinge at $A$ to keep this gate closed? Answer

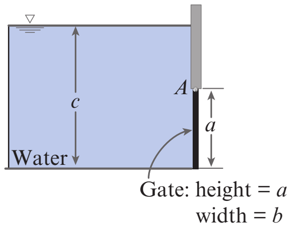
(**a**) 7 (**b**) 23 (**c**) 36 (**d**) 11 (**e**) 14




**Problem 3.112**

**3.113** Water creates a load on a gate. The dimensions are $a = 3$ m, $b = 2$ m, and $c = 5$ m. In unit of kN · m, what is the magnitude of the torque that needs to be applied to the hinge at $A$ to keep this gate closed?

(**a**) 310 (**b**) 590 (**c**) 270 (**d**) 350 (**e**) 140



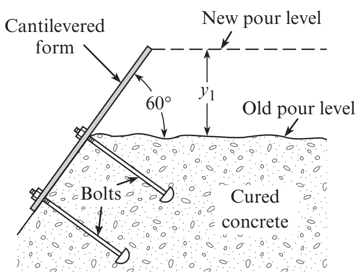

Problem 3.113

**3.114** A body of water creates a pressure force on a vertical panel. The panel has the shape of a triangle, and the 6-m wide base of the panel is aligned with the free surface. The tip of the panel is 6-m below the free surface.

In units of meters, what is the vertical distance from the free surface to the center of pressure (CP)? Answer

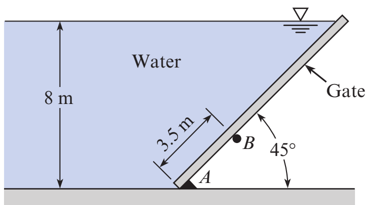
(**a**) 2.5 (**b**) 3.0 (**c**) 2.0 (**d**) 3.5 (**e**) 4.5

**3.115** In constructing dams, the concrete is poured in lifts of approximately 1.8 m ($y_1 = 1.8$ m). The forms for the face of the dam are reused from one lift to the next. The figure shows one such form, which is bolted to the already cured concrete. For the new pour, what moment will occur at the base of the form per meter of length (normal to the page)? Assume that concrete acts as a liquid when it is first poured and has a specific weight of 24 kN/m$^3$.



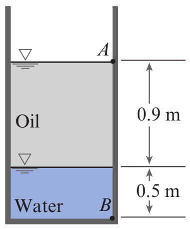

Problem 3.115

**3.116** The plane rectangular gate can pivot about the support at *B*. For the conditions given, is it stable or unstable? Neglect the weight of the gate. Justify your answer with calculations. Answer




Problem 3.116

**3.117** Oil ($SG = 0.8$) and water in an open tank cause a pressure force to act on the rectangular wall *AB*. The width of *AB* is 2.5 m.




Problem 3.117

In kN, what is the pressure force on wall *AB*?

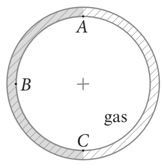
## Pressure Force on a Curved Surface (§3.5)

**3.118** Two hemispheric shells are perfectly sealed together, and the internal pressure is reduced to 25% of atmospheric pressure. The inner radius is 10.5 cm and the outer radius is 10.75 cm. The seal is located halfway between the inner and outer radius. If the atmospheric pressure is 101.3 kPa, what force is required to pull the shells apart? Answer SS

**3.119** This spherical pressure vessel contains a gas at an absolute pressure of 10 bar. The pressure vessel has a diameter of 0.6 m. The surface *ABC* is the interior surface of the left half of the pressure vessel.

In kN, what is the pressure force acting on surface *ABC*?

(**a**) 120 (**b**) 180 (**c**) 250 (**d**) 200 (**e**) 280




Problem 3.119

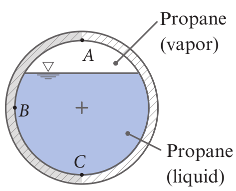
**3.120** A spherical metal tank (aka a pressure vessel) is filled with a gas at pressure $p$. This tank has an inner radius $r$ and a wall thickness $t$. The wall is thin, which generally means that the wall thickness is less than one-tenth of the radius: $t < r/10$. The average stress in the wall is defined as the ratio of force to area.

The average stress in the wall is: Answer

(**a**) $\frac{pt}{2r}$ (**b**) $\frac{pt}{4r}$ (**c**) $\frac{pr}{4t}$ (**d**) $\frac{pr}{2t}$ (**e**) $\frac{\pi pt}{2r}$

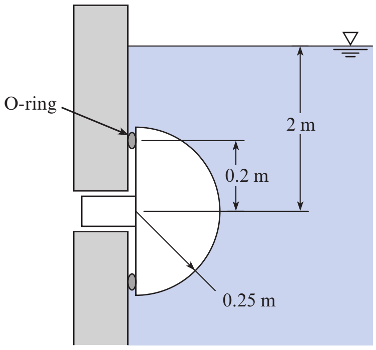
**3.121** This spherical tank contains propane. The tank diameter is 4 m. The depth of the liquid propane is 3 m. The temperature is 38°C, and the vapor pressure of the propane is 1307 kPa. The density of the liquid propane is 471 kg/m$^3$.

Sketch the pressure distribution on surface *ABC*. Describe the significant features of your sketch. Note that surface *ABC* is the curved interior surface of the left half of the tank.



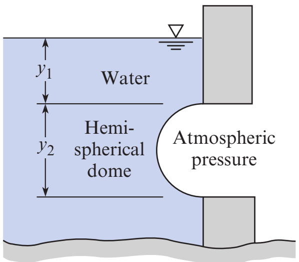

Problem 3.121

**3.122** A plug in the shape of a hemisphere is inserted in a hole in the side of a tank as shown in the figure. The plug is sealed by an O-ring with a radius of 0.2 m. The radius of the hemispherical plug is 0.25 m. The depth of the center of the plug is 2 m in fresh water. Find the horizontal and vertical forces on the plug due to hydrostatic pressure. Answer




Problem 3.122

**3.123** This dome (hemisphere) is located below the water surface as shown. Determine the magnitude and sign of the force components needed to hold the dome in place and the line of action of the horizontal component of force. Here, $y_1 = 1$ m and $y_2 = 2$ m. Assume $T = 10°C$.




Problem 3.123

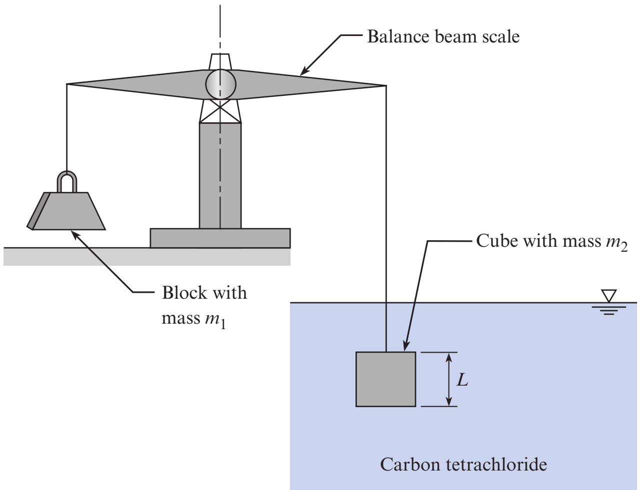
## Calculating Buoyant Forces (§3.6)

**3.124** A rock weighs 980 N in air and 609 N in water. Find its volume. Answer

**3.125** (T/F) an object is floating in water, then the location of the center of pressure will be coincident with the centroid of the object.

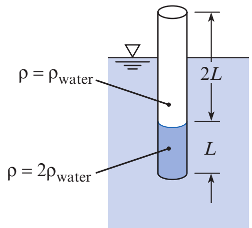
**3.126** You are at an estate sale and trying to decide whether to bid on a gold pendant that is said to be 24-carat (pure) gold. The pendant looks like gold, but you would like to check. You are permitted to make some measurements, and collect the following data: The pendant has a mass of 100 g in air and an apparent mass of 94.8 g when submerged in water. You know that the *SG* of 24-carat gold is 19.3, and the *SG* of 22-carat gold is 17.8; you decide to bid on anything that has $SG > 19.0$. Find the *SG* of the pendant, and decide whether you will bid. Answer

**3.127** As shown, a cube ($L = 94$ mm) suspended in carbon tetrachloride is exactly balanced by an object of mass $m_1 = 610$ g. Find the mass $m_2$ of the cube.




Problem 3.127

**3.128** As shown, a uniform-diameter rod is weighted at one end and is floating in a liquid. The liquid (a) is lighter than water, (b) must be water, or (c) is heavier than water. Show your work. Answer




Problem 3.128

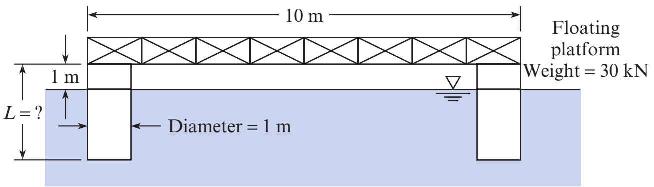
**3.129** An 150-m-long freighter weighs $300 \times 10^6$ N, and the area defined by its waterline is 2600 m$^2$. Will the ship ride higher or deeper in the water when traveling from fresh water to salt water as it leaves the harbor for the open ocean? How much (in m) will it settle or rise?

**3.130** A submerged spherical steel buoy that is 1.2 m in diameter and weighs 1800 N is to be anchored in salt water 50 m below the surface. Find the weight of scrap iron that should be sealed inside the buoy in order that the force on its anchor chain will not exceed 5 kN. Answer

**3.131** A block of material of unknown volume is submerged in water and found to weigh 300 N (in water). The same block weighs 700 N in air. Determine the specific weight and volume of the material. SS

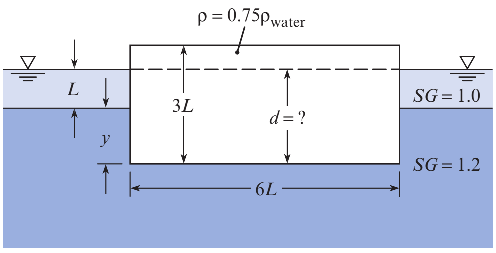
**3.132** A 1-ft diameter cylindrical tank is filled with water to a depth of 2 ft. A cylinder of wood 5 in. in diameter and 6.0 in. long is set afloat on the water. The weight of the wood cylinder is 3.5 lbf. Determine the change (if any) in the depth of the water in the tank. Answer

**3.133** The floating platform shown is supported at each corner by a hollow sealed cylinder 1 m in diameter. The platform itself weighs 30 kN in air, and each cylinder weighs 1.0 kN per meter of length. What total cylinder length $L$ is required for the platform to float 1 m above the water surface? Assume that the specific weight of the water (brackish) is 10,000 N/m$^3$. The platform is square in plan view.



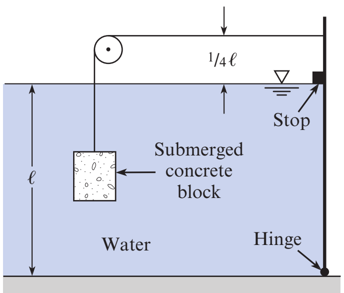

Problem 3.133

**3.134** To what depth $d$ will this rectangular block (with density 0.75 times that of water) float in the two-liquid reservoir? Answer




Problem 3.134

SS **3.135** Determine the minimum volume of concrete ($\gamma$ = 23.6 kN/m$^3$) needed to keep the gate (1 m wide) in a closed position, with $\ell$ = 2 m. Note the hinge at the bottom of the gate.




Problem 3.135

SS **3.136** A person is working in an office. This person has the size and weight of an average US adult. In SI units, the buoyant force on this person is: Answer

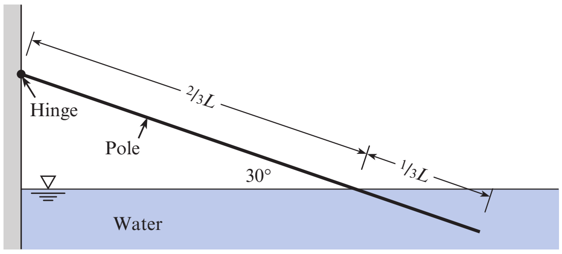
(**a**) 5 (**b**) 2 (**c**) 1 (**d**) 3 (**e**) 4

SS **3.137** A concrete ($SG$ = 2.3) weight will be attached to an object ($SG$ = 0.6) so that the object will sink in fresh water. The volume of the object is 4 m$^3$. The cost of the concrete is \$80 US dollars per cubic yard. How much will the concrete cost?

(**a**) \$40 (**b**) \$130 (**c**) \$70 (**d**) \$110 (**e**) \$50

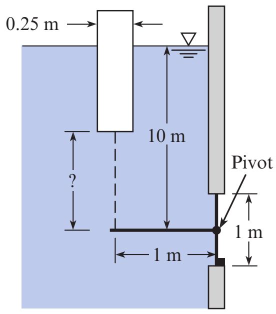
**3.138** A cylindrical container 4-ft high and 2-ft in diameter holds water to a depth of 2 ft. How much does the level of the water in the tank change when a 5-lb block of ice is placed in the container? Is there any change in the water level in the tank when the block of ice melts? Does it depend on the specific gravity of the ice? Explain all the processes. Answer

**3.139** The partially submerged wood pole is attached to the wall by a hinge as shown. The pole is in equilibrium under the action of the weight and buoyant forces. Determine the density of the wood.




Problem 3.139

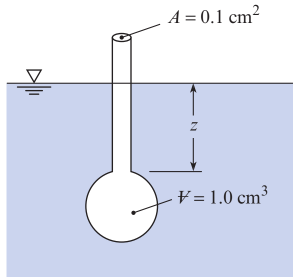
**3.140** A gate with a circular cross section is held closed by a lever 1-m long attached to a buoyant cylinder. The cylinder is 25 cm in diameter and weighs 200 N. The gate is attached to a horizontal shaft so it can pivot about its center. The liquid is water. The chain and lever attached to the gate have negligible weight. Find the length of the chain such that the gate is just on the verge of opening when the water depth above the gate hinge is 10 m. Answer




Problem 3.140

## Measuring ρ, γ, and *SG* with Hydrometers (§3.6)

**3.141** The hydrometer shown weighs 0.015 N. If the stem sinks 7.2 cm in oil ($z$ = 7.2 cm), what is the specific gravity of the oil?

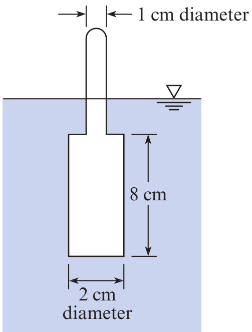



Problems 3.141

**3.142** A common commercial hydrometer for measuring the amount of antifreeze in the coolant system of an automobile engine consists of a chamber with differently colored balls. The system is calibrated to give the range of specific gravity by distinguishing between the balls that sink and those that float. The specific gravity of an ethylene glycol–water mixture varies from 1.012 to 1.065 for 10% to 50% by weight of ethylene glycol. Assume there are six balls, 1 cm in diameter each, in the chamber. What should the weight of each ball be to provide a range of specific gravities between 1.01 and 1.06 with 0.01 intervals? Answer

SS **3.143** A hydrometer with the configuration shown has a bulb diameter of 2 cm, a bulb length of 8 cm, a stem diameter of 1 cm, a length of 8 cm, and a mass of 40 g. What is the range of specific gravities that can be measured with this hydrometer?

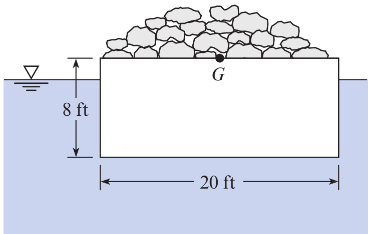
(*Hint:* Liquid levels range between bottom and top of stem.)




**Problem 3.143**

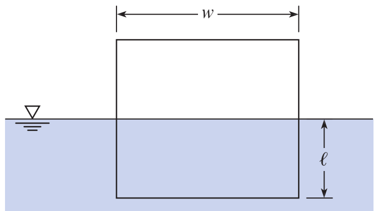
## Predicting Stability (§3.7)

**3.144** A barge 20 ft wide and 40 ft long is loaded with rocks as shown. Assume that the center of gravity of the rocks and barge is located along the centerline at the top surface of the barge. If the rocks and the barge weigh 400,000 lbf, will the barge float upright or tip over? Answer




**Problem 3.144**

**3.145** A floating body has a square cross section with side $w$ as shown in the figure. The center of gravity is at the centroid of the cross section. Find the location of the water line, $\ell/w$, where the body would be neutrally stable ($GM = 0$). If the body is floating in water, what would be the specific gravity of the body material?




**Problem 3.145**

**3.146** A cylindrical block of wood 1 m in diameter and 1 m long has a specific weight of 7500 $N/m^3$. Will it float in water with its axis vertical? Answer SS

**3.147** A cylindrical block of wood 1 m in diameter and 1 m long has a specific weight of 5000 $N/m^3$. Will it float in water with the ends horizontal?

CHAPTER**THREE**

# Fluid Statics

**CHAPTER ROAD MAP** This chapter introduces concepts related to pressure and describes how to calculate forces associated with distributions of pressure. The emphasis is on fluids in hydrostatic equilibrium. A traditional application of fluid statics is described in Fig. 3.1.

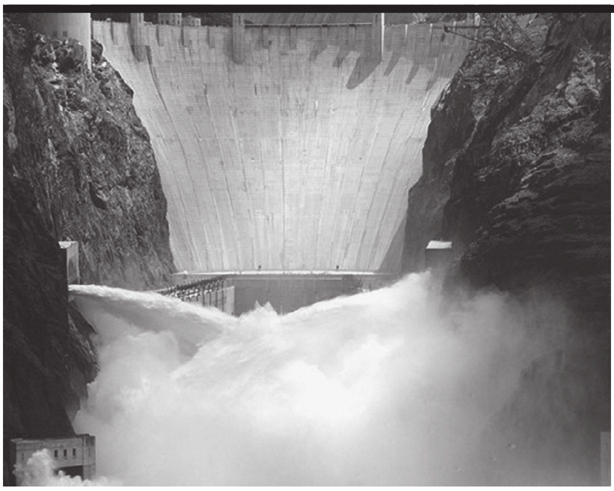


**FIGURE 3.1**
The first man-made structure to exceed the masonry mass of the Great Pyramid of Giza was Hoover Dam. The design of dams involves calculations of hydrostatic forces. (U.S. Bureau of Reclamation.)

## LEARNING OUTCOMES

**PRESSURE** (§3.1)

- Define pressure and convert pressure units.
- Describe atmospheric pressure and select an appropriate value.
- Define and apply gage, absolute, vacuum, and differential pressure.
- Know the main ideas about hydraulic machines and solve relevant problems.

**THE HYDROSTATIC EQUATIONS** (§3.2)

- Define hydrostatic equilibrium.
- Know the main ideas about the hydrostatic differential equation.
- Know the main ideas about the hydrostatic algebraic equation and solve relevant problems.

**PRESSURE MEASUREMENT** (§3.3)

- Explain how common scientific instruments work and do relevant calculations (this LO applies to the mercury barometer, piezometer, manometer, and Bourdon tube gage).

**THE PRESSURE FORCE** (§3.4)

- Define the center of pressure.
- Sketch a pressure distribution.
- Explain or apply the gage pressure rule.
- Calculate the force due to a uniform pressure distribution.
- Know the main ideas about the panel equations and be able to apply these equations.

**CURVED SURFACES** (§3.5)

- Solve problems that involve curved surfaces that are acted on by uniform or hydrostatic pressure distributions.

**BUOYANCY** (§3.6)

- Know the main ideas about buoyancy and be able to apply these ideas to solve problems.

## 3.1 Describing Pressure

Because engineers use pressure in the solution of nearly all fluid mechanics problems, this section introduces fundamental ideas about pressure.

### Pressure

**Pressure** is the ratio of the normal force due to a fluid to the area that this force acts on, in the limit as this area shrinks to zero.

$$p = \frac{\text{magnitude of normal force}}{\text{unit area}}\Bigg|_{\substack{\text{at a point}\\ \text{due to a fluid}}} = \lim_{\Delta A \to 0} \frac{\Delta F_{\text{normal}}}{\Delta A} \tag{3.1}$$

Pressure is defined at a point because pressure typically varies with each $(x, y, z)$ location in a flowing fluid.

Pressure is a scalar that produces a resultant force by its action on an area. The resultant force is normal to the area and acts in a direction toward the surface (compressive).

Pressure is caused by the molecules of the fluid interacting with the surface. For example, when a soccer ball is inflated, the internal pressure on the skin of the ball is caused by air molecules striking the wall.

Units of pressure can be organized into three categories:

- *Force per area*. The SI unit is the newtons per square meter or pascals (Pa). The traditional units include psi, which is pounds-force per square inch, and psf, which is pounds-force per square foot.
- *Liquid column height*. Sometimes pressure units give an equivalent height of a column of liquid. For example, pressure in a balloon will push a water column upward about 20 cm (Fig. 3.2). An engineer would state that the gage pressure inside the balloon is $p$ = 20 cmH2O. When a pressure unit is given as a height of a liquid column, the pressure value can be converted to other units by using the conversion ratios from Table F.1. For example, a typical pressure in a balloon is

$$p = (20\ \text{cmH2O})(9.807\ \text{Pa/mmH2O})(10\ \text{mm/1.0cm}) = 1960\ \text{Pa gage}$$

- *Atmospheres*. Sometimes pressure units are stated in terms of atmospheres where 1.0 atm is the air pressure at sea level at standard conditions. Another common unit is the bar, which is very nearly equal to 1.0 atm. (1.0 bar = $10^5$ kPa)

### Atmospheric Pressure

This subsection explains how to select an accurate value of atmospheric pressure ($p_{\text{atm}}$) because a value of $p_{\text{atm}}$ is often needed in calculations.

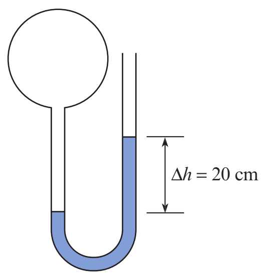
The atmosphere of the earth is an extremely thin layer of air that extends from the surface of the earth to the edge of space. The atmosphere is held in place by gravitational force. According to NASA, "if the earth were the size of a basketball, a tightly held pillowcase would represent the thickness of the atmosphere."*

If you look at data, it is evident that $p_{\text{atm}}$ is strongly influenced by elevation:†

- At London (EL = 35 m): $p_{\text{atm}}$ = 101 kPa
- At Denver, Colorado, USA (EL = 1650 m), $p_{\text{atm}}$ = 83.4 kPa
- Near the summit of Mount Everest, Nepal (EL = 8000 m): $p_{\text{atm}}$ = 35.6 kPa
- At a typical cruise altitude of a jetliner (EL = 12,190 m): $p_{\text{atm}}$ = 18.8 kPa

**FIGURE 3.2**

Pressure in a balloon causing a column of water to rise 20 cm.




*http://www.grc.nasa.gov/WWW/k-12/airplane/atmosmet.html.

†The value of atmospheric pressure is an absolute pressure. Thus, engineers commonly say that $p_{\text{atm}}$ = 101 kPa instead of saying that $p_{\text{atm}}$ = 101 kPa abs.

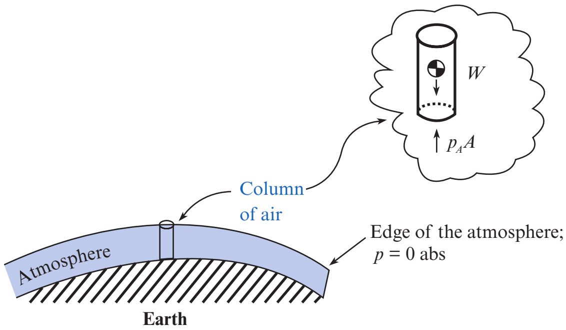



**FIGURE 3.3**

**Claim.** Atmospheric pressure ($p_{atm}$) decreases as elevation increases.

**Reasoning.** (1) Select a system comprised of air that extends from the earth's surface to the upper edge of the atmosphere. (2) Model the system as a stationary column. (3) Because the column is stationary, the forces must sum to zero. (4) Thus, statics shows that atmospheric pressure equals the weight of the column divided by the section area. (5) At a higher elevation, the fluid column is shorter and thus has less weight.

The reason that $p_{atm}$ changes with elevation is explained in Fig. 3.3.

In addition to elevation, other variables influence $p_{atm}$. As elevation increases, the average temperature of the atmosphere decreases. For example, in the Alps, the average temperature on the summit of a mountain is lower than the average temperature in a town situated in a valley. Local weather influences $p_{atm}$. Good weather is associated with higher values of atmospheric pressure and bad weather with lower values. As the atmosphere is heated during the day and cooled during the night, the atmospheric pressure varies in response to temperature changes. Fortunately, it is simple to select an appropriate value of $p_{atm}$. Three methods that we recommend are as follows:

**Method #1.** If you lack information about elevation, select the standard value of atmospheric pressure at sea level,* which is

$$p_{atm}(\text{sea level}) = 1.000 \text{ atm} = 101.3 \text{ kPa} = 14.70 \text{ psi} = 2116 \text{ psf} = 33.90 \text{ ft-H}_2\text{O}$$
$$= 760.0 \text{ mm-Hg} = 29.92 \text{ in-Hg} = 1.013 \text{ bar}$$

**Method #2.** If you have information about elevation, you can calculate a typical value of atmospheric pressure using the *standard atmosphere*. The **U.S. standard atmosphere** is a math model that gives values of parameters such as temperature, density, and pressure corresponding to average conditions. The model, developed by NASA,† is valid from the earth's surface to an elevation of 1000 km. Regarding calculations, the equations of the math model are complicated, so we recommend using the Digital Dutch online calculator.‡

**Method #3.** The most accurate way to find atmospheric pressure is to measure the value using a barometer. This method might be needed, for example, if you are processing experimental data and you want to know the exact value of atmospheric pressure at the time your data were recorded. As an alternative to using a barometer, you can look up a locally measured value on the Internet. Be careful when using the Internet as a resource, however, because many sites adjust the local atmospheric pressure to a value that the given location would have if it was situated at sea level.

**EXAMPLE.** What value of atmospheric pressure should be used for a project that will be built in Mexico City? **Reasoning.** (1) The elevation of Mexico City is 2250 m. (2) Using the U.S. standard atmosphere, as calculated with the Digital Dutch calculator,§ shows that $p_{atm} = 77.1$ kPa at an elevation of 2250 m. **Claim.** Use $p_{atm} = 77$ kPa.

## Absolute Pressure, Gage Pressure, Vacuum Pressure, and Differential Pressure

Professionals use four different pressure scales which are

1. *Absolute pressure* is pressure measured relative to absolute pressure.
2. *Gage pressure* is pressure above or below the local atmospheric pressure.

*We recommend that you add these values to your *working knowledge*. As always, memorize the approximate values not the exact values. We recommend memorizing to two to three significant digits.

†The most recent version was published in 1976.

‡http://www.digitaldutch.com/atmoscalc.

§ibid.

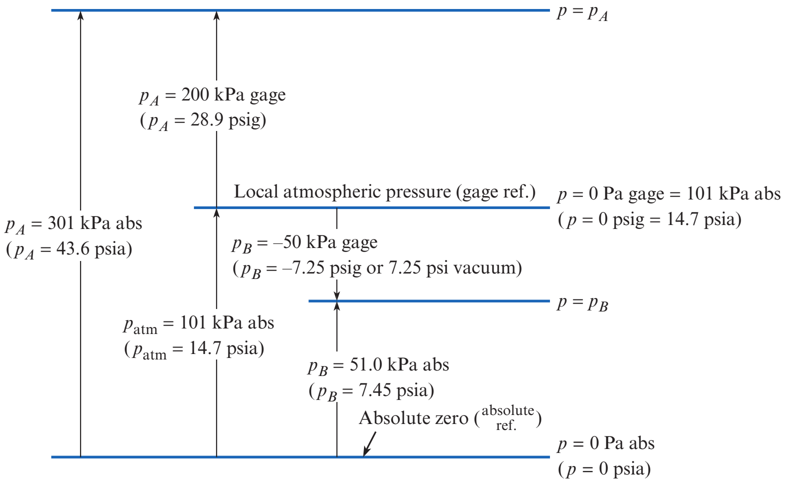



**FIGURE 3.4**

Example of pressure relations.

3. *Vacuum pressure* is the magnitude of the pressure below local atmospheric pressure.
4. *Differential pressure* is the pressure difference between two points.

Absolute pressure is referenced to regions such as outer space, where the pressure is essentially zero because the region is devoid of gas. The pressure in a perfect vacuum is called absolute zero, and pressure measured relative to this zero pressure is termed **absolute pressure**.

When pressure is measured relative to prevailing local atmospheric pressure, the pressure value is called **gage pressure**.* For example, when a tire pressure gage gives a value of 300 kPa (44 psi), this means that the absolute pressure in the tire is 300 kPa greater than local atmospheric pressure. To convert gage pressure to absolute pressure, add the local atmospheric pressure. For example, a gage pressure of 50 kPa recorded in a location where the atmospheric pressure is 100 kPa is expressed as either

$$p = 50 \text{ kPa gage} \quad \text{or} \quad p = 150 \text{ kPa abs} \tag{3.2}$$

In SI units, gage and absolute pressures are identified after the unit as shown in Eq. (3.2). In traditional units, gage pressure is identified by adding the letter *g* to the unit symbol. For example, a gage pressure of 10 pounds per square foot is designated as 10 psfg. Similarly, the letter *a* is used to denote absolute pressure. For example, an absolute pressure of 20 pounds force per square inch is designated as 20 psia.

When pressure is less than atmospheric, the pressure can be described using vacuum pressure. **Vacuum pressure** is defined as the difference between atmospheric pressure and actual pressure. Vacuum pressure is a positive number and equals the absolute value of gage pressure (which will be negative). For example, if $p_{atm} = 101$ kPa and a gage connected to a tank indicates a vacuum pressure of 31.0 kPa, this can also be stated as 70.0 kPa abs, or −31.0 kPa gage.

Fig. 3.4 provides a visual description of the three pressure scales. Notice that $p_B = 7.45$ psia is equivalent to −7.25 psig and +7.25 psi vacuum. Notice that $p_A =$ of 301 kPa abs is equivalent to 200 kPa gage. Gage, absolute, and vacuum pressure can be related using equations labeled as the "pressure equations."

$$p_{gage} = p_{abs} - p_{atm} \tag{3.3a}$$

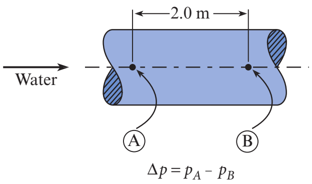
$$p_{vacuum} = p_{atm} - p_{abs} \tag{3.3b}$$

$$p_{vacuum} = -p_{gage} \tag{3.3c}$$

*There are two correct spellings used in the literature: gage pressure and gauge pressure.

**FIGURE 3.5**

An example of differential pressure for flow in a pipe. Points $A$ and $B$ are located on the centerline. The differential pressure ($\Delta p$) is the magnitude of the pressure at point $A$ minus the magnitude of the pressure at point $B$.




**EXAMPLE.** Convert 5 psi vacuum to absolute pressure in SI units.

**Solution.** First, convert vacuum pressure to absolute pressure.

$$p_{abs} = p_{atm} - p_{vacuum} = 14.7 \text{ psi} - 5 \text{ psi} = 9.7 \text{ psia.}$$

Second, convert units by applying a conversion ratio from Table F.1.

$$p = (9.7 \text{ psi})\left(\frac{101.3 \text{ kPa}}{14.7 \text{ psi}}\right) = 66{,}900 \text{ Pa absolute.}$$

**Recommendation.** It is good practice, when writing pressure units, to specify whether the pressure is absolute, gage, vacuum, or differential.

**EXAMPLE.** Suppose that the pressure in a car tire is specified as 3 bar. Find the absolute pressure in units of kPa.

**Solution.** Recognize that tire pressure is commonly specified in gage pressure. Thus, convert the gage pressure to absolute pressure.

$$p_{abs} = p_{atm} + p_{gage} = (101.3 \text{ kPa}) + (3 \text{ bar})\frac{(101.3 \text{ kPa})}{(1.013 \text{ bar})} = 401 \text{ kPa absolute}$$

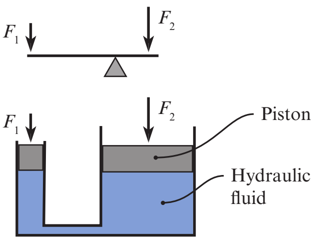
Another way to describe pressure is to use **differential pressure**, which is defined as the difference in pressure between two points and is given the symbol $\Delta p$ (Fig. 3.5).

Some useful facts about differential pressure follow.

- The points ($A$ and $B$) are typically selected so that differential pressure is positive; that is, $\Delta p > 0$.
- Differential pressure refers to the difference in pressure between two points, not to a "differential pressure" in the sense of a differential in calculus.
- The unit symbol psid stands for pounds-force per square inch differential. Similarly, psfd refers to a differential pressure.

**FIGURE 3.6**

Both the lever and hydraulic machine provide a mechanical advantage.




## Hydraulic Machines

A **hydraulic machine** uses a fluid to transmit forces or energy to assist in the performance of a human task. An example of a hydraulic machine is a hydraulic car jack in which a user can supply a small force to a handle and lift an automobile. Other examples of hydraulic machines include braking systems in cars, forklift trucks, power steering systems in cars, and airplane control systems.

The hydraulic machine provides a mechanical advantage (Fig. 3.6). **Mechanical advantage** is defined as the ratio of output force to input force:

$$(\text{mechanical advantage}) \equiv \frac{(\text{output force})}{(\text{input force})} \tag{3.4}$$

Mechanical advantage of a lever (Fig. 3.6) is found by summing moments about the fulcrum to give $F_1L_1 = F_2L_2$, where $L$ denotes the length of the lever arm.

$$(\text{mechanical advantage; lever}) \equiv \frac{(\text{output force})}{(\text{input force})} = \frac{F_2}{F_1} = \frac{L_1}{L_2} \tag{3.5}$$

To find mechanical advantage of the hydraulic machine, apply force equilibrium to each piston (Fig. 3.6) to give $F_1 = p_1A_1$ and $F_2 = p_2A_2$, where $p$ is pressure in the cylinder and $A$ is face area of the piston. Next, recognize that $p_1 = p_2$ and then solve for the mechanical advantage

$$(\text{mechanical advantage; hydraulic machine}) \equiv \frac{(\text{output force})}{(\text{input force})} = \frac{F_2}{F_1} = \frac{A_2}{A_1} = \frac{D_2^2}{D_1^2} \tag{3.6}$$

The hydraulic machine is often used to illusrate Pascal's principle. This principle states that when there is an increase in pressure at any point in a confined fluid, there is an equal increase at every other point in the container. This principle is evident when a balloon is inflated because the balloon expands evenly in all directions. The principle is also evident in the hydraulic machine (Fig. 3.7).

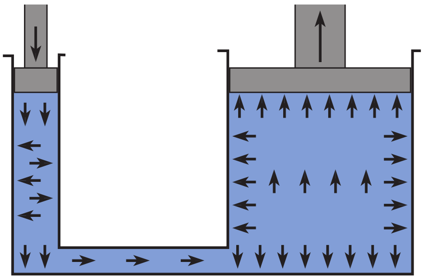



**FIGURE 3.7**

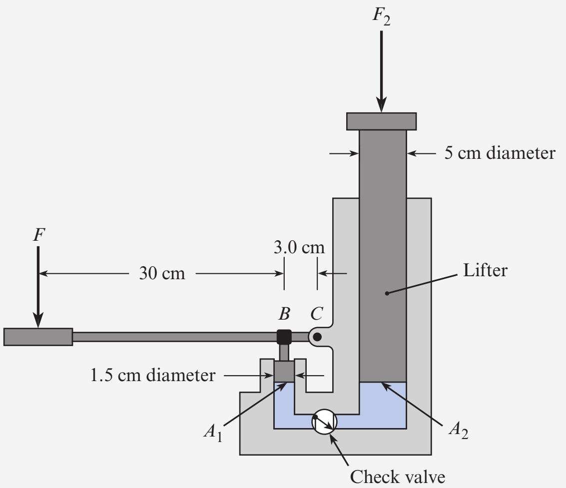
This figure shows how a hydraulic machine can be used to illustrate Pascal's principle.

## EXAMPLE 3.1

Applying Force Equilibrium to a Hydraulic Jack

### Problem Statement

A hydraulic jack has the dimensions shown. If one exerts a force $F$ of 100 N on the handle of the jack, what load, $F_2$, can the jack support? Neglect lifter weight.




### Define the Situation

A force of $F = 100$ N is applied to the handle of a jack.

**Assumption:** The weight of the lifter (see sketch) is negligible.

### State the Goal

$F_2$(N) ⬅ load that the jack can lift

### Generate Ideas and Make a Plan

Because the goal is $F_2$, apply force equilibrium to the lifter. Then, analyze the small piston and the handle. The plan is as follows:

1. Calculate force acting on the small piston by applying moment equilibrium.
2. Calculate pressure $p_1$ in the hydraulic fluid by applying force equilibrium.
3. Calculate the load $F_2$ by applying force equilibrium.

### Take Action (Execute the Plan)

1. Moment equilibrium (handle):

$$\sum M_c = 0$$

$$(0.33 \text{ m}) \times (100 \text{ N}) - (0.03 \text{ m})F_1 = 0$$

$$F_1 = \frac{0.33 \text{ m} \times 100 \text{ N}}{0.03 \text{ m}} = 1100 \text{ N}$$

2. Force equilibrium (small piston):

$$\sum F_{\text{small piston}} = p_1 A_1 - F_1 = 0$$

$$p_1 A_1 = F_1 = 1100 \text{ N}$$

Thus,

$$p_1 = \frac{F_1}{A_1} = \frac{1100 \text{ N}}{\pi d^2/4} = 6.22 \times 10^6 \text{ N/m}^2$$

3. Force equilibrium (lifter):

$$\sum F_{\text{lifter}} = F_2 - p_1 A_2 = 0$$

$$F_2 = p_1 A_2 = \left(6.22 \times 10^6 \frac{\text{N}}{\text{m}^2}\right)\left(\frac{\pi}{4} \times (0.05 \text{ m})^2\right) = \boxed{12.2 \text{ kN}}$$

Note that $p_1 = p_2$ because they are at the same elevation (this fact will be established in the next section).

### Review the Results and the Process

1. *Discussion.* The jack in this example, which combines a lever and a hydraulic machine, provides an output force of 12,200 N from an input force of 100 N. Thus, this jack provides a mechanical advantage of 122 to 1.
2. *Knowledge.* Hydraulic machines are analyzed by applying force and moment equilibrium. The force of pressure is typically given by $F = pA$.

## 3.2 The Hydrostatic Equations

This section explains how to calculate the pressure for problems in which a fluid is in hydrostatic equilibrium. There are two main results:

- The hydrostatic *differential equation*, which is applied to problems in which density varies
- The hydrostatic *algebraic equation*, which is applied to problems in which density is constant

### The Hydrostatic Condition

The equations in this section apply only if the fluid in your problem is in *hydrostatic equilibrium*. To tell if this condition applies, select a fluid particle, select a coordinate direction, and draw a free body diagram (FBD) that shows only the forces in the coordinate direction that you selected. If the acceleration of the fluid particle is zero in the coordinate direction you chose and if the only forces on the particle are the pressure force and the weight, then the hydrostatic condition applies on a plane that is parallel to your coordinate direction.

If a fluid is stationary (e.g., water in a lake as in Fig. 3.8), then the hydrostatic equation will always apply. The reason is that the acceleration of any fluid particle is zero and the only possible forces that can balance the weight of the fluid particle are the pressure force and the viscous force. However, the viscous force must be zero because of the definition of a fluid; that is, a fluid will deform continuously under the action of a viscous stress. Thus, the only force available to balance the weight of the fluid particle is the pressure force.

If a fluid is *flowing*, then the hydrostatic equation will sometimes apply (Fig 3.9). For situations similar to those shown in the figure, you can apply the hydrostatic equation $\Delta p = -\rho g \Delta z$ to points situated in a plane.

### The Hydrostatic Differential Equation (Variable Density)

This subsection shows how to derive $dp/dz = -\gamma$. This equation is important for understanding the theory and for solving problems that involve varying density.

To begin the derivation, visualize any region of static fluid (e.g., water behind a dam), isolate a cylindrical body, and then sketch an FBD, as shown in Fig. 3.10. Notice that the cylindrical

**FIGURE 3.8**

This example shows how to check to see if the *hydrostatic condition* applies. For this case, hydrostatic conditions do apply because the *weight* of the fluid particle is exactly balanced by the *pressure force*.

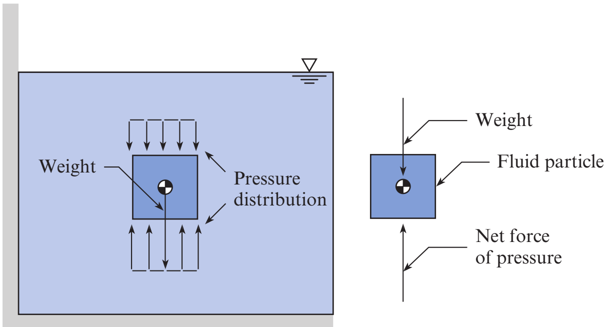
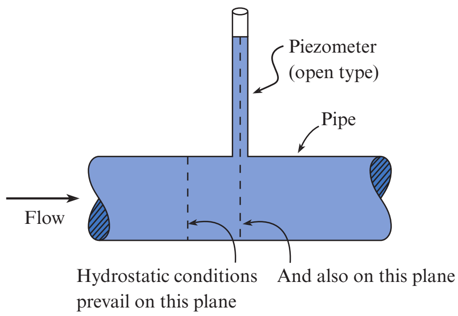



**FIGURE 3.9**

This sketch shows examples of when *hydrostatic conditions* apply to a flowing fluid. The reason why is that the pressure force balances the weight force for each fluid particle that is situated on one of the planes shown in the figure.



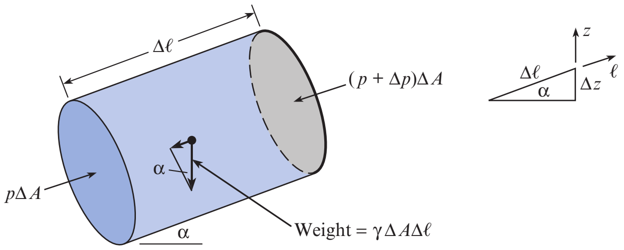



**FIGURE 3.10**
The system used to derive the hydrostatic differential equation.

body is oriented so that its longitudinal axis is parallel to an arbitrary $\ell$ direction. The body is $\Delta\ell$ long, $\Delta A$ in cross-sectional area, and inclined at an angle $\alpha$ with the horizontal. Apply force equilibrium in the $\ell$ direction:

$$\sum F_\ell = 0$$

$$F_{\text{Pressure}} - F_{\text{Weight}} = 0$$

$$p\Delta A - (p + \Delta p)\Delta A - \gamma \Delta A \Delta \ell \sin\alpha = 0$$

Simplify and divide by the volume of the body $\Delta\ell\Delta A$ to give

$$\frac{\Delta p}{\Delta \ell} = -\gamma \sin\alpha$$

From Fig. 3.10, the sine of the angle is given by

$$\sin\alpha = \frac{\Delta z}{\Delta \ell}$$

Combining the previous two equations and letting $\Delta z$ approach zero gives

$$\lim_{\Delta z \to 0} \frac{\Delta p}{\Delta z} = -\gamma$$

The final result is

$$\frac{dp}{dz} = -\gamma \qquad \text{(hydrostatic differential equation)} \qquad (3.7)$$

Eq. (3.7) means that changes in pressure correspond to changes in elevation. If one travels upward in the fluid (positive $z$ direction), the pressure decreases; if one goes downward (negative $z$), the pressure increases; if one moves along a horizontal plane, the pressure remains constant. Of course, these pressure variations are exactly what a diver experiences when ascending or descending in a lake or pool.

## The Hydrostatic Algebraic Equation (Constant Density)

Because modeling a fluid as if the density is constant is often well justified, it is useful to solve the hydrostatic differential equation for the special case of constant density. The resulting equation is called the hydrostatic algebraic equation, and we shorten this name to the *hydrostatic equation (HE)*. The hydrostatic equation is one of the most useful equations in fluid mechanics; thus, we recommend that you learn this equation well. To derive the equation, begin by integrating Eq. (3.7) for the case of constant density to give

$$p + \gamma z = p_z = \text{constant} \qquad (3.8)$$

where the term $z$ is the elevation (vertical distance) above a fixed horizontal reference plane called a datum, and $p_z$ is **piezometric pressure**. Dividing Eq. (3.8) by $\gamma$ gives

$$\frac{p_z}{\gamma} = \left(\frac{p}{\gamma} + z\right) = h = \text{constant} \tag{3.9}$$

where $h$ is the **piezometric head**. Because $h$ is constant, Eq. (3.9) can be written as

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 \tag{3.10a}$$

where the subscripts 1 and 2 identify any two points in a static fluid of constant density. Multiplying Eq. (3.10a) by $\gamma$ gives

$$p_1 + \gamma z_1 = p_2 + \gamma z_2 \tag{3.10b}$$

In Eq. (3.10b), letting $\Delta p = p_2 - p_1$ and letting $\Delta z = z_2 - z_1$ gives

$$\Delta p = -\gamma \Delta z \tag{3.10c}$$

The hydrostatic equation is given by Eqs. (3.10a), (3.10b), or (3.10c). These three equations are equivalent because any one of the equations can be used to derive the other two. The hydrostatic equation is valid for any constant density fluid in hydrostatic equilibrium.

Notice that the hydrostatic equation involves

$$\text{piezometric head} = h \equiv \left(\frac{p}{\gamma} + z\right) \tag{3.11}$$

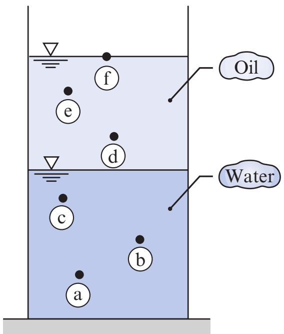
$$\text{piezometric pressure} = p_z \equiv (p + \gamma z) \tag{3.12}$$

To calculate piezometric head or piezometric pressure, an engineer identifies a specific location in a body of fluid and then uses the value of pressure and elevation at that location. Piezometric pressure and head are related by

$$p_z = h\gamma \tag{3.13}$$

**FIGURE 3.11**

Oil floating on water.




Piezometric head, $h$, a property that is widely used in fluid mechanics, characterizes hydrostatic equilibrium. When hydrostatic equilibrium prevails in a body of fluid of constant density, then $h$ will be constant at all locations. For example, Fig. 3.11 shows a container with oil floating on water. Because piezometric head is constant in the water, $h_a = h_b = h_c$. Similarly, the piezometric head is constant in the oil: $h_d = h_e = h_f$. Notice that piezometric head is not constant when density changes. For example, $h_c \neq h_d$ because points $c$ and $d$ are in different fluids with different values of density.

## Hydrostatic Equation (Working Equations)

To apply the hydrostatic equation, first check that the assumptions listed in Table 3.1 are valid. Then, select the most useful form of the hydrostatic equation. We recommend using the head form or the differential pressure form. We also recommend that you learn the meaning of the variables given in the third column because these names are used throughout fluid mechanics. For many problems, you will find the following two rules useful:

The **fluid interface rule** states that for a planar interface (e.g., Fig. 3.12) the pressure is constant across the interface (i.e., $p_1 = p_2$ at the interface). **Reasoning**. (1) The fluid interface is not moving, so $\Sigma \mathbf{F} = \mathbf{0}$. (2) Select an infinitesimally thin system so that the weight can be neglected. (3) Thus, the only forces on the interface are the pressure forces, and algebra shows that $p_1 = p_2$.

**FIGURE 3.12**

To prove the fluid interface rule (1) select an infinitesimally thin system on the interface and note that the weight of this system is negligible. (2) Apply $\Sigma F = 0$ to show that pressure is constant across the interface.

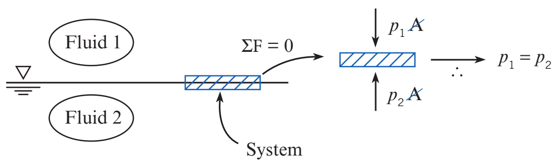


**TABLE 3.1** The Hydrostatic Equation (Working Equations and Assumptions)

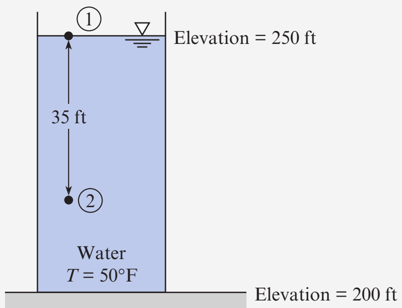
| Name (Physical Interpretation) | Equation | Variables in the Equation |
|---|---|---|
| **Head form** (the piezometric head is constant at every point) | $\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2$ Eq. (3.10a) | • $p$ = pressure (N/m$^2$) (use absolute or gage pressure; not vacuum pressure)<br>• $\gamma$ = specific weight (N/m$^3$)<br>• $p/\gamma$ = pressure head (m)<br>• $z$ = elevation or elevation head (m)<br>• $(p/\gamma + z)$ = piezometric head (m) |
| **Differential pressure form** (the differential pressure is linear with elevation change) | $\Delta p = \gamma \Delta z$ Eq. (3.10b) | • $\Delta p$ = differential pressure (N/m$^2$)<br>• $\Delta z$ = difference in elevation (m) |
| **Piezometric pressure form** (the piezometric pressure is constant at every point) | $p_1 + \gamma z_1 = p_2 + \gamma z_2$ Eq. (3.10c) | • $(p + \gamma z)$ = piezometric pressure (Pa) |
| Assumptions to check before you apply the hydrostatic equation | | **1.** You can only apply the HE to a single fluid that has constant density. For problems that have multiple fluids (e.g., oil floating on water), the HE is applied successively to each fluid.<br>**2.** You can only apply the HE if the hydrostatic condition applies. |

## EXAMPLE 3.2

Applying the Hydrostatic Equation to Find Pressure in a Tank

### Problem Statement

What is the water pressure at a depth of 35 ft in the tank shown?




### Define the Situation

Water is contained in a tank that is 50 ft deep.

**Properties:** Water (50°F, 1 atm, Table A.5): $\gamma = 62.4$ lbf/ft$^3$

### State the Goal

$p_2$ (psig) ⬅ water pressure at point 2

### Generate Ideas and Make a Plan

Apply the idea that piezometric head is constant. The plan steps are as follows:

1. Equate piezometric head at elevation 1 with piezometric head at elevation 2 (i.e., apply Eq. 3.10a).
2. Analyze each term in Eq. (3.10a).
3. Solve for the pressure at elevation 2.

### Take Action (Execute the Plan)

1. Hydrostatic equation (Eq. 3.10a):

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2$$

2. Term-by-term analysis of Eq. (3.10a) yields:
   - $p_1 = p_{atm} = 0$ psig
   - $z_1 = 250$ ft
   - $z_2 = 215$ ft
3. Combine steps 1 and 2; solve for $p_2$:

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2$$

$$0 + 250 \text{ ft} = \frac{p_2}{62.4 \text{ lbf/ft}^3} + 215 \text{ ft}$$

$$p_2 = 2180 \text{ psfg} = \boxed{15.2 \text{ psig}}$$

### Review the Solution and the Process

1. *Validation.* The calculated pressure change (15 psig) is slightly greater than 1 atm (14.7 psi). Because one atmosphere corresponds to a water column of 33.9 ft and this problem involves 35 feet of water column, the solution appears correct.
2. *Skill.* This example shows how to write down a governing equation and then analyze each term. This skill is called *term-by-term analysis*.

3. *Knowledge*. The gage pressure at the free surface of a liquid in contact with the atmosphere is zero ($p_1 = 0$ in this example).
4. *Skill*. Label a pressure as absolute or gage or vacuum. For this example, the pressure unit (psig) denotes a gage pressure.
5. *Knowledge*. The hydrostatic equation is valid when density is constant. This condition is met on this problem.

The **gas pressure change rule** states that the hydrostatic pressure change for a gas can usually be neglected. **Reasoning**. (1) The hydrostatic pressure change in a gas for a one-meter change of elevation is given by $\Delta p/\Delta z = \rho g$. (2) The given equation shows, for example, that the pressure change in air at room conditions is about 12 pascals/meter. (3) A pressure change of about 12 pascals/meter is typically negligible as compared to other relevant pressure changes. **Conclusion**. The hydrostatic pressure change in a gas can usually be neglected.

Example 3.3 shows how to find pressure by applying the idea of *constant piezometric head* to a problem involving several fluids. Notice the application of the fluid interface rule.

## EXAMPLE 3.3

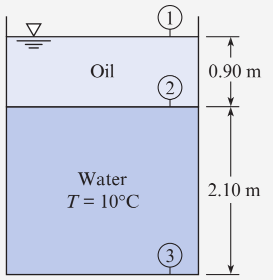
### Applying the Hydrostatic Equation to Oil and Water in a Tank

#### Problem Statement

Oil with a specific gravity of 0.80 forms a layer 0.90 m deep in an open tank that is otherwise filled with water (10°C). The total depth of water and oil is 3 m. What is the gage pressure at the bottom of the tank?

#### Problem Definition

Oil and water are contained in a tank.




**Properties:**

- Water: (10°C, 1 atm, Table A.5): $\gamma_{water} = 9810\ \text{N/m}^3$
- Oil: $\gamma_{oil} = S\gamma_{water,\,4°C} = 0.8(9810\ \text{N/m}^3) = 7850\ \text{N/m}^3$

#### State the Goal

$p_3$ (kPa gage) ⬅ pressure at bottom of the tank

#### Generate Ideas and Make a Plan

Because the goal is $p_3$, apply the hydrostatic equation to the water. Then, analyze the oil. The plan steps are as follows:

1. Find $p_2$ by applying the hydrostatic equation (3.10a).
2. Equate pressures across the oil–water interface.
3. Find $p_3$ by applying the hydrostatic equation given in Eq. (3.10a).

#### Solution

1. Hydrostatic equation (oil):

$$\frac{p_1}{\gamma_{oil}} + z_1 = \frac{p_2}{\gamma_{oil}} + z_2$$

$$\frac{0\ \text{Pa}}{\gamma_{oil}} + 3\ \text{m} = \frac{p_2}{0.8 \times 9810\ \text{N/m}^3} + 2.1\ \text{m}$$

$$p_2 = 7.063\ \text{kPa}$$

2. Oil–water interface:

$$p_2\big|_{oil} = p_2\big|_{water} = 7.063\ \text{kPa}$$

3. Hydrostatic equation (water):

$$\frac{p_2}{\gamma_{water}} + z_2 = \frac{p_3}{\gamma_{water}} + z_3$$

$$\frac{7.063 \times 10^3\ \text{Pa}}{9810\ \text{N/m}^3} + 2.1\ \text{m} = \frac{p_3}{9810\ \text{N/m}^3} + 0\ \text{m}$$

$$\boxed{p_3 = 27.7\ \text{kPa gage}}$$

#### Review

*Validation*: Because oil is less dense than water, the answer should be slightly smaller than the pressure corresponding to a water column of 3 m. From Table F.1, a water column of 10 m ≈ 1 atm. Thus, a 3 m water column should produce a pressure of about 0.3 atm = 30 kPa. The calculated value appears correct.

## 3.3 Measuring Pressure

When engineers design and conduct experiments, pressure nearly always needs to be measured. Thus, this section describes five scientific instruments for measuring pressure.

### Barometer

An instrument that is used to measure atmospheric pressure is called a **barometer**. The most common types are the mercury barometer and the aneroid barometer. A mercury barometer is made by inverting a mercury-filled tube in a container of mercury, as shown in Fig. 3.13. The pressure at the top of the mercury barometer will be the vapor pressure of mercury, which is very small: $p_v = 2.4 \times 10^{-6}$ atm at 20°C. Thus, atmospheric pressure will push the mercury up the tube to a height $h$. The mercury barometer is analyzed by applying the hydrostatic equation:

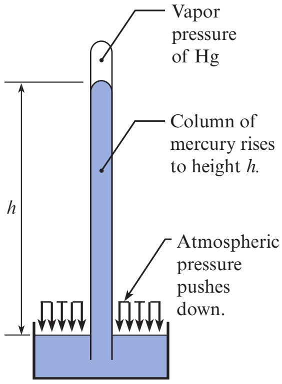
$$p_{\text{atm}} = \gamma_{\text{Hg}} h + p_v \approx \gamma_{\text{Hg}} h \tag{3.20}$$

Thus, by measuring $h$, local atmospheric pressure can be determined using Eq. (3.20).

An aneroid barometer works mechanically. An aneroid is an elastic bellows that has been tightly sealed after some air was removed. When atmospheric pressure changes, this causes the aneroid to change size, and this mechanical change can be used to deflect a needle to indicate local atmospheric pressure on a scale. An aneroid barometer has some advantages over a mercury barometer because it is smaller and allows data recording over time.

**FIGURE 3.13**

A mercury barometer.

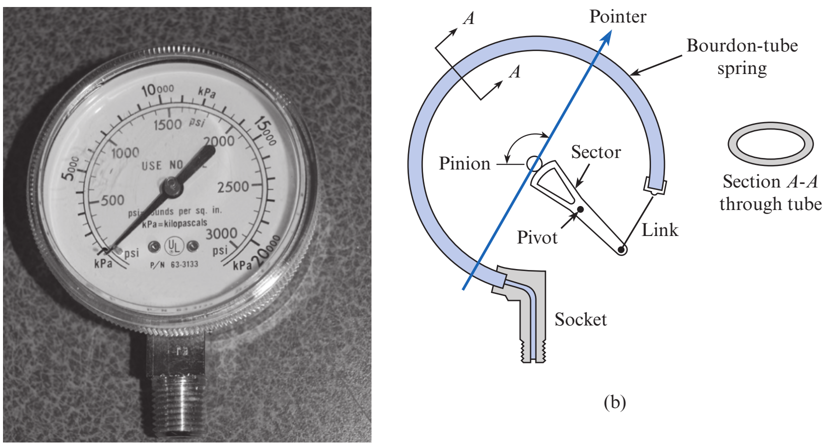



### Bourdon-Tube Gage

A **Bourdon-tube** gage,* Fig. 3.14, measures pressure by sensing the deflection of a coiled tube. The tube has an elliptical cross section and is bent into a circular arc, as shown in Fig. 3.14b. When atmospheric pressure (zero gage pressure) prevails, the tube is undeflected, and for this condition the gage pointer is calibrated to read zero pressure. When pressure is applied to the gage, the curved tube tends to straighten (much like blowing into a party favor to straighten it out), thereby actuating the pointer to read a positive gage pressure. The Bourdon-tube gage is common because it is low cost, reliable, easy to install, and available in many different pressure ranges. Bourdon-tube gages have some disadvantages: dynamic pressures may not be measured accurately; accuracy of the gage can be lower than other instruments; and the gage can be damaged by excessive pressure pulsations.

### Piezometer

A **piezometer** is a vertical tube, usually transparent, in which a liquid rises in response to a positive gage pressure. For example, Fig. 3.15 shows a piezometer attached to a pipe. Pressure




(a)

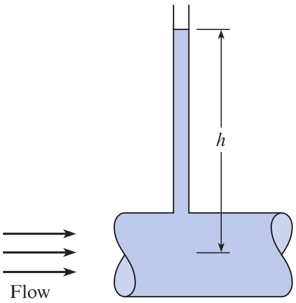
**FIGURE 3.14**

Bourdon-tube gage. (a) View of a typical gage (photo by Donald Elger). (b) Internal mechanism (schematic).

*Gage in this context means a scientific instrument. This word can also be correctly spelled as gauge.

**FIGURE 3.15**

Piezometer attached to a pipe.




in the pipe pushes the water column to a height $h$, and the gage pressure at the center of the pipe is $p = \gamma h$, which follows directly from the hydrostatic equation (3.10c). The piezometer has several advantages: simplicity, direct measurement (no need for calibration), and accuracy. However, a piezometer cannot easily be used for measuring pressure in a gas, and a piezometer is limited to low pressures because the column height becomes too large at high pressures.

## Manometer

A **manometer** (often shaped like the letter "U") is a device for measuring pressure by raising or lowering a column of liquid. For example, Fig. 3.16 shows a U-tube manometer that is being used to measure pressure in a flowing fluid. In the case shown, positive gage pressure in the pipe pushes the manometer liquid up a height $\Delta h$. To use a manometer, engineers relate the height of the liquid in the manometer to pressure, as illustrated in Example 3.4.

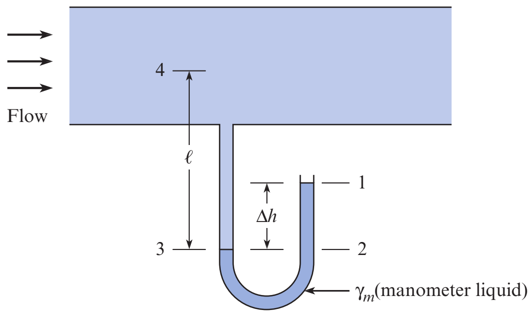
Once one is familiar with the basic principle of manometry, it is straightforward to write a single equation rather than separate equations as was done in Example 3.4. The single equation for evaluation of the pressure in the pipe of Fig 3.16 is

$$0 + \gamma_m \Delta h - \gamma \ell = p_4$$

One can read the equation in this way: Zero pressure at the open end, plus the change in pressure from point 1 to 2, minus the change in pressure from point 3 to 4, equals the

**FIGURE 3.16**

U-tube manometer.




## EXAMPLE 3.4

Pressure Measurement (U-Tube Manometer)

### Problem Statement

Water at 10°C is the fluid in the pipe of Fig. 3.16, and mercury is the manometer fluid. If the deflection $\Delta h$ is 60 cm and $\ell$ is 180 cm, what is the gage pressure at the center of the pipe?

### Define the Situation

Pressure in a pipe is being measured using a U-tube manometer.

**Properties:**

- Water (10°C), Table A.5: $\gamma = 9810\ \text{N/m}^3$
- Mercury, Table A.4: $\gamma = 133{,}000\ \text{N/m}^3$

### State the Goal

Calculate gage pressure (kPa) in the center of the pipe.

### Generate Ideas and Make a Plan

Start at point 1 and work to point 4 using ideas from Eq. (3.10c). When fluid depth increases, add a pressure change. When fluid depth decreases, subtract a pressure change.

### Take Action (Execute the Plan)

1. Calculate the pressure at point 2 using the hydrostatic equation (3.10c):

$$\begin{aligned} p_2 &= p_1 + \text{pressure increase between 1 and 2} = 0 + \gamma_m \Delta h_{12} \\ &= \gamma_m (0.6\ \text{m}) = (133{,}000\ \text{N/m}^3)(0.6\ \text{m}) \\ &= 79.8\ \text{kPa} \end{aligned}$$

2. Find the pressure at point 3:
   - The hydrostatic equation with $z_3 = z_2$ gives

$$p_3|_{\text{water}} = p_2|_{\text{water}} = 79.8\ \text{kPa}$$

   - When a fluid–fluid interface is flat, pressure is constant across the interface. Thus, at the oil–water interface

$$p_3|_{\text{mercury}} = p_3|_{\text{water}} = 79.8\ \text{kPa}$$

3. Find the pressure at point 4 using the hydrostatic equation given in Eq. (3.10c):

$$\begin{aligned} p_4 &= p_3 - \text{pressure decrease between 3 and 4} = p_3 - \gamma_w \ell \\ &= 79{,}800\ \text{Pa} - (9810\ \text{N/m}^3)(1.8\ \text{m}) \\ &= 62.1\ \text{kPa gage} \end{aligned}$$

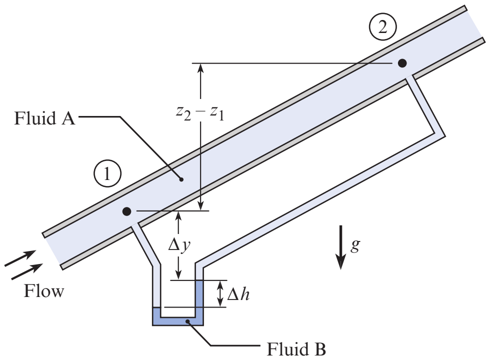



**FIGURE 3.17**
Apparatus for determining change in piezometric head corresponding to flow in a pipe.

pressure in the pipe. The main concept is that pressure increases as depth increases and decreases as depth decreases.

The general equation for the pressure difference measured by the manometer is

$$p_2 = p_1 + \sum_{\text{down}} \gamma_i h_i - \sum_{\text{up}} \gamma_i h_i \tag{3.21}$$

where $\gamma_i$ and $h_i$ are the specific weight and deflection in each leg of the manometer. It does not matter where one starts, that is, where one defines the initial point 1 and final point 2. When liquids and gases are both involved in a manometer problem, it is well within engineering accuracy to neglect the pressure changes due to the columns of gas. This is because $\gamma_{\text{liquid}} \gg \gamma_{\text{gas}}$. Example 3.5 shows how to apply Eq. (3.21) to perform an analysis of a manometer that uses multiple fluids.

Because the manometer configuration shown in Fig. 3.17 is common, it is useful to derive an equation specific to this application. To begin, apply the manometer equation (3.21) between points 1 and 2:

$$p_1 + \sum_{\text{down}} \gamma_i h_i - \sum_{\text{up}} \gamma_i h_i = p_2$$

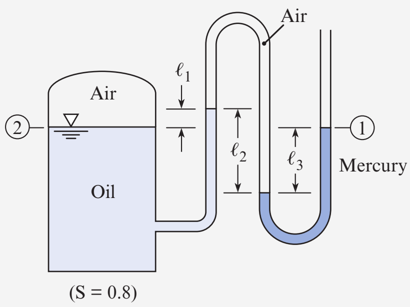
$$p_1 + \gamma_A(\Delta y + \Delta h) - \gamma_B \Delta h - \gamma_A(\Delta y + z_2 - z_1) = p_2$$

## EXAMPLE 3.5

Manometer Analysis

### Problem Statement

What is the pressure of the air in the tank if $\ell_1 = 40$ cm, $\ell_2 = 100$ cm, and $\ell_3 = 80$ cm?




### Define the Situation

A tank is pressurized with air.

**Assumptions:** Neglect the pressure change in the air column.

**Properties:**

- Oil: $\gamma_{\text{oil}} = S\gamma_{\text{water}} = 0.8 \times 9810 \text{ N/m}^3 = 7850 \text{ N/m}^3$
- Mercury, Table A.4: $\gamma = 133{,}000 \text{ N/m}^3$

### State the Goal

Find the pressure (kPa gage) in the air.

### Generate Ideas and Make a Plan

Apply the manometer equation (3.21) from location 1 to location 2.

### Take Action (Execute the Plan)

Manometer equation:

$$p_1 + \sum_{\text{down}} \gamma_i h_i - \sum_{\text{up}} \gamma_i h_i = p_2$$

$$p_1 + \gamma_{\text{mercury}} \ell_3 - \gamma_{\text{air}} \ell_2 + \gamma_{\text{oil}} \ell_1 = p_2$$

$$0 + (133{,}000 \text{ N/m}^3)(0.8 \text{ m}) - 0 + (7850 \text{ N/m}^3)(0.4 \text{ m}) = p_2$$

$$\boxed{p_2 = p_{\text{air}} = 110 \text{ kPa gage}}$$

**EXAMPLE 3.6**

Change in Piezometric Head for Pipe Flow

### Problem Statement

A differential mercury manometer is connected to two pressure taps in an inclined pipe as shown in Fig. 3.17. Water at 50°F is flowing through the pipe. The deflection of mercury in the manometer is 1 inch. Find the change in piezometric pressure and piezometric head between points 1 and 2.

### Define the Situation

Water is flowing in a pipe.

**Properties:**

- Water (50 °F): Table A.5, $\gamma_{\text{water}} = 62.4\ \text{lbf/ft}^3$.
- Mercury: Table A.4, $\gamma_{\text{Hg}} = 847\ \text{lbf/ft}^3$.

### State the Goal

Find the following:

- Change in piezometric head (ft) between points 1 and 2
- Change in piezometric pressure (psfg) between 1 and 2

### Generate Ideas and Make a Plan

1. Find difference in the piezometric head using Eq. (3.22).
2. Relate piezometric head to piezometric pressure using Eq. (3.13).

### Take Action (Execute the Plan)

1. Difference in piezometric head:

$$h_1 - h_2 = \Delta h\left(\frac{\gamma_{\text{Hg}}}{\gamma_{\text{water}}} - 1\right) = \left(\frac{1}{12}\ \text{ft}\right)\left(\frac{847\ \text{lbf/ft}^3}{62.4\ \text{lbf/ft}^3} - 1\right)$$

$$= \boxed{1.05\ \text{ft}}$$

2. Piezometric pressure:

$$p_z = h\gamma_{\text{water}}$$

$$= (1.05\ \text{ft})(62.4\ \text{lbf/ft}^3) = \boxed{65.5\ \text{psf}}$$

Simplifying gives

$$(p_1 + \gamma_A z_1) - (p_2 + \gamma_A z_2) = \Delta h(\gamma_B - \gamma_A)$$

Dividing through by $\gamma_A$ gives

$$\left(\frac{p_1}{\gamma_A} + z_1\right) - \left(\frac{p_2}{\gamma_A} + z_2\right) = \Delta h\left(\frac{\gamma_B}{\gamma_A} - 1\right)$$

Recognize that the terms on the left side of the equation are piezometric head and rewrite to give the final result:

$$h_1 - h_2 = \Delta h\left(\frac{\gamma_B}{\gamma_A} - 1\right) \tag{3.22}$$

Equation (3.22) is valid when a manometer is used to measure differential pressure. Example 3.6 shows how this equation is used.

## Summary of the Manometer Equations

These manometer equations are summarized in Table 3.2. Because the equations were derived from the hydrostatic equation, they have the same assumptions: constant fluid density and hydrostatic conditions. The process for applying the manometer equations is as follows:

**Step 1.** For measurement of pressure at a point, select Eq. (3.21). For measurement of pressure or head change between two points in a pipe, select Eq. (3.22).

**Step 2.** Select points 1 and 2 where you know information or where you want to find information.

**Step 3.** Write the general form of the manometer equation.

**Step 4.** Perform a term-by-term analysis.

**TABLE 3.2** Summary of the Manometer Equations

| Description | Equation | Terms |
|---|---|---|
| **Gage pressure analysis.** Use this equation for a manometer that is being applied to measure gage pressure (e.g., see Fig. 3.16). | $p_2 = p_1 + \sum_{\text{down}} \gamma_i h_i - \sum_{\text{up}} \gamma_i h_i$ (3.21) | $p_1$ = pressure at point 1 (N/m$^2$)<br>$p_2$ = pressure at point 2 (N/m$^2$)<br>$\gamma_i$ = specific weight of fluid $i$ (N/m$^3$)<br>$h_i$ = deflection of fluid in leg $i$ (m) |
| **Differential pressure analysis.** Use this equation for a manometer that is being applied to measure differential pressure in a pipe with a flowing fluid (e.g., see Fig. 3.17). | $h_1 - h_2 = \Delta h\left(\frac{\gamma_B}{\gamma_A} - 1\right)$ (3.22) | $h_1 = p_1/\gamma_A + z_1$ = piezometric head at point 1 (m)<br>$h_2 = p_2/\gamma_A + z_2$ = piezometric head at point 2 (m)<br>$\Delta h$ = deflection of the manometer fluid (m)<br>$\gamma_A$ = specific weight of the flowing fluid (N/m$^3$)<br>$\gamma_B$ = specific weight of the manometer fluid (N/m$^3$) |

## The Pressure Transducer

A **pressure transducer** (PT) is a device that converts pressure to an electrical signal. For example, Fig. 3.18 shows a strain–gage pressure transducer. Pressure transducers have many advantages, such as the following:

- In general, PTs have high levels of accuracy as compared to other devices, such as Bourdon-tube gages and manometers.
- A PT can be used to measure gage pressure, absolute pressure, vacuum pressure, or differential pressure.
- Most PTs can measure pressure as a function of time and can be applied to electronic data logging.
- A PT is available for almost any pressure range you want to measure.

Pressure transducers also have some disadvantages, such as the following:

- Higher costs.
- Longer setup times because they are more complicated.
- In general, PTs need to be calibrated and used carefully.

# 3.4 The Pressure Force on a Panel (Flat Surface)

Many problems require a calculation of the pressure force on a panel. Thus, this section explains how to do this calculation for two cases:

- A *uniform* pressure distribution
- A *hydrostatic* pressure distribution

A **panel** is any surface that is flat or that can be idealized as if it were flat (e.g., face of a dam, a surface on an airplane wing, or the cross section inside a pressure vessel).

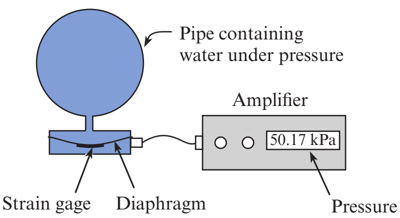



**FIGURE 3.18**

A strain gage pressure transducer operates as follows: (1) Pressure deforms a diaphragm. (2) The diaphragm deflection is sensed with a strain gage. (3) The voltage from the strain gage is amplified and then converted to a pressure value via software. (4) The pressure value is displayed.

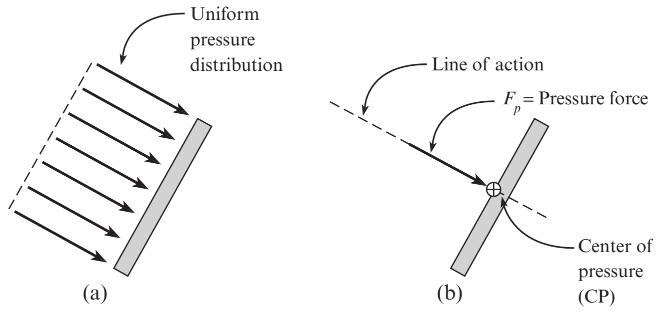



**FIGURE 3.19**

This example shows (a) a uniform pressure distribution and (b) the associated pressure force.

## The Uniform Pressure Distribution

Fig. 3.19 shows a uniform pressure distribution and the associated pressure force $\mathbf{F}_p$. The value of $F_p$ is calculated using

$$F_p = pA \tag{3.23}$$

where $p$ is the *gage pressure* and $A$ is the surface area of the panel. The pressure force acts at a location called the *center of pressure* (CP). For a uniform pressure, the CP is located at the centroid of the panel. The direction of the pressure force is normal to the panel. The reasoning for why Eq. (3.23) is true is as follows: (1) The pressure force on *any surface* is given by $\mathbf{F}_p = \int_A -p\,\mathbf{n}\,dA$. (2) Because the pressure is constant for a uniform pressure distribution, $\mathbf{F}_p = p\int_A -\mathbf{n}\,dA = pA(-\mathbf{n})$. **Conclusion**: The magnitude of $\mathbf{F}_p$ is $F_p = pA$. The direction of $\mathbf{F}_p$ is the $(-\mathbf{n})$ direction. Thus, Eq. (3.23) is true.

Some useful facts about pressure distributions follow.

- A uniform pressure distribution is commonly used to idealize the pressure distribution due to a gas and the pressure distribution due to a liquid when a panel is horizontal.
- Gage pressure (not absolute pressure) is used in Eq. (3.23) because of the *gage pressure rule*. This rule is explained in Fig 3.20.
- To analyze a pressure vessel, apply the *pressure vessel force balance method*. This method is explained in Fig. 3.21.

## The Hydrostatic Pressure Distribution

A **hydrostatic pressure distribution** (Fig. 3.22) describes the distribution of pressure when pressure varies only with elevation $z$ according to $dp/dz = -\gamma$. When hydrostatic conditions prevail, any panel that is not horizontal is subjected to a hydrostatic pressure distribution.

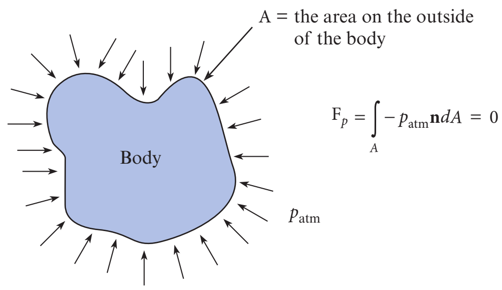



**FIGURE 3.20**

**Gage pressure rule:** When a uniform atmospheric pressure acts on a body, integrating this pressure over area shows that the net pressure force is zero. Thus, use gage pressure when analyzing the pressure force.

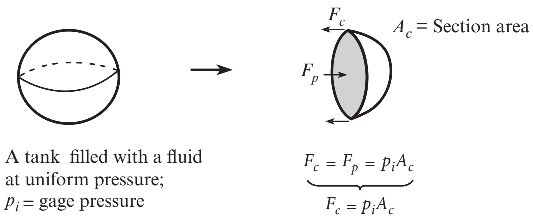



**FIGURE 3.21**

The **pressure vessel force balance** is a method for analyzing the force ($F_c$) needed to clamp a pressure vessel together. To derive an equation, take the following steps: (1) Imagine cutting the tank where it is clamped. (2) Sketch an FBD of the cut portion of the tank. (3) Balance the pressure force with the clamping force to show that $F_c = p_i A_c$.

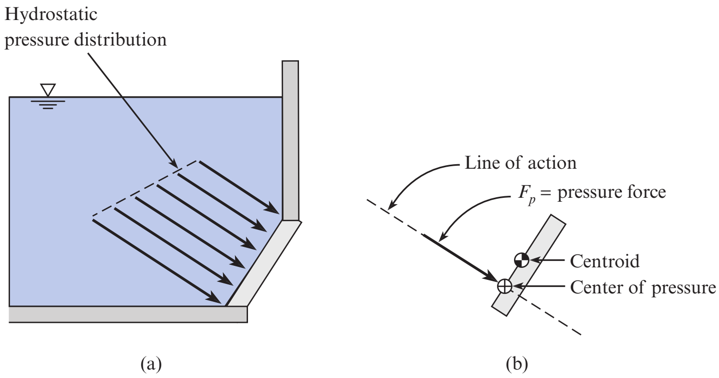



**FIGURE 3.22**

An example showing (a) a hydrostatic pressure distribution on a rectangular panel and (b) the corresponding pressure force.

A pressure force acts at a point called the **center of pressure**, which is calculated so that the torque due to the pressure force is exactly the same as the torque due to the pressure distribution. In other words, if you want to replace the pressure distribution with a statically equivalent force that acts at a point, the correct point is the center of pressure. In this text, the symbol for the CP is a circle with a plus symbol inside: ⊕.

The **centroid** of an area can be thought of as the balance point of an area (see Fig. 3.23). In general, the equations for finding the centroid are integrals such as $x_c = (\int x dA)/A$. For common shapes, the equations have been solved, and engineers look up the value. In this text, centroid formulas are presented in the appendix, Fig. A.1.

## Sketching a Pressure Distribution

As an engineer, you should be able to sketch a pressure distribution. Some guidelines are as follows: (1) draw each arrow so that its length represents the magnitude of the pressure, (2) sketch gage pressure, not absolute pressure, (3) draw each arrow so that the arrow is normal to the surface, and (4) draw each arrow to represent compression.

## Theory: Force Caused by a Hydrostatic Pressure Distribution

Next, we will show how to find the force on one face of a panel that is acted on by a hydrostatic pressure distribution. To begin, sketch a panel of arbitrary shape submerged in a liquid

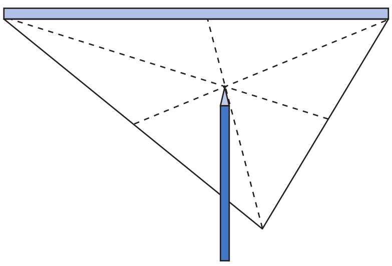


**FIGURE 3.23**

An example of the *centroid* for a triangular panel. The idea here is to (1) imagine making a model of the panel and then (2) the centroid is the point at which the model would balance on the tip of a pencil. This example assumes that the model has a uniform density and that the gravity field is uniform.

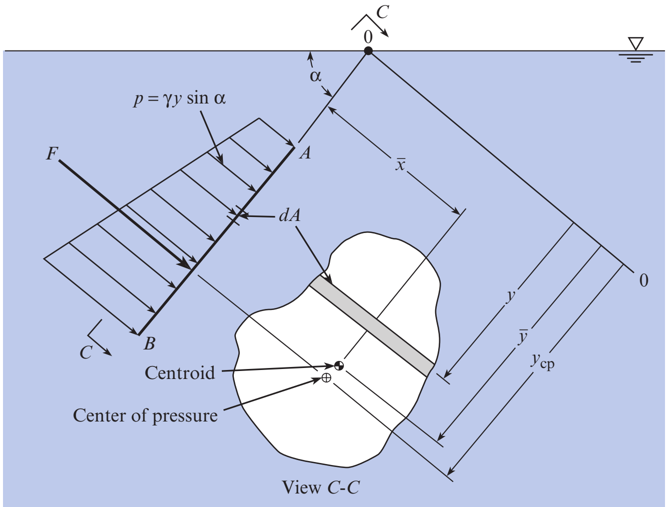



**FIGURE 3.24**
Distribution of hydrostatic pressure on a plane surface.

(Fig. 3.24). Line *AB* is the edge view of a panel. The plane of the panel intersects the horizontal liquid surface at axis 0-0 with an angle α. The distance from the axis 0-0 to the horizontal axis through the centroid of the area is given by $\bar{y}$. The distance from 0-0 to the differential area *dA* is *y*.

The force due to pressure is given by

$$F_p = \int_A p dA \tag{3.24}$$

In Eq. (3.24), the pressure can be found with the hydrostatic equation:

$$p = -\gamma \Delta z = \gamma y \sin \alpha \tag{3.25}$$

Combine Eqs. (3.24) and (3.25) to give

$$F_p = \int_A p dA = \int_A \gamma y \sin \alpha \, dA = \gamma \sin \alpha \int_A y dA \tag{3.26}$$

Because the integral on the right side of Eq. (3.26) is the first moment of the area, replace the integral by its equivalent, $\bar{y}A$. Therefore,

$$F_p = \gamma \bar{y} A \sin \alpha = (\gamma \bar{y} \sin \alpha) A \tag{3.27}$$

Apply the hydrostatic equation to show that the variables within the parentheses on the right side of Eq. (3.27) are the pressure at the centroid of the area. Thus,

$$F_p = \bar{p} A \tag{3.28}$$

Equation (3.28) shows that the hydrostatic force on a panel of arbitrary shape (e.g., rectangular, round, elliptical) is given by the product of the panel area and the pressure at the elevation of the centroid.

## Theory: The Center of Pressure for a Hydrostatic Pressure Distribution

This subsection shows how to derive an equation for the vertical location of the CP. For the panel shown in Fig. 3.24 to be in moment equilibrium, the torque due to the resultant force $F_p$ must balance the torque due to each differential force:

$$y_{cp}F_p = \int y\, dF$$

Note that $y_{cp}$ is the "*slant*" distance from the center of pressure to the surface of the liquid. The label "slant" denotes that the distance is measured in the plane that runs through the panel. The differential force $dF$ is given by $dF = p\, dA$; therefore,

$$y_{cp}F = \int_A yp\, dA$$

Also, $p = \gamma y \sin\alpha$, so

$$y_{cp}F = \int_A \gamma y^2 \sin\alpha\, dA \tag{3.29}$$

Because $\gamma$ and $\sin\alpha$ are constants,

$$y_{cp}F = \gamma \sin\alpha \int_A y^2\, dA \tag{3.30}$$

The integral on the right-hand side of Eq. (3.30) is the second moment of the area (often called the area moment of inertia). This shall be identified as $I_0$. However, for engineering applications, it is convenient to express the second moment with respect to the horizontal centroidal axis of the area. Hence by the parallel-axis theorem,

$$I_0 = \bar{I} + \bar{y}^2 A \tag{3.31}$$

Substitute Eq. (3.31) into Eq. (3.30) to give

$$y_{cp}F = \gamma \sin\alpha(\bar{I} + \bar{y}^2 A)$$

However, from Eq. (3.25), $F = \gamma\bar{y}\sin\alpha A$. Therefore,

$$y_{cp}(\gamma\bar{y}\sin\alpha A) = \gamma\sin\alpha(\bar{I} + \bar{y}^2 A) \tag{3.32}$$

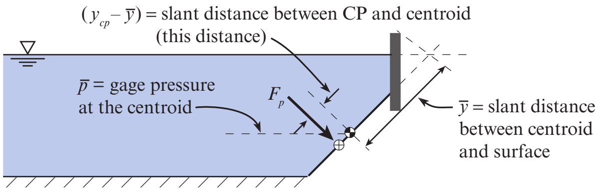
$$y_{cp} = \bar{y} + \frac{\bar{I}}{\bar{y}A}$$

$$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A} \tag{3.33}$$

In Eq. (3.33), the area moment of inertia $\bar{I}$ is taken about a horizontal axis that passes through the centroid of area. Formulas for $\bar{I}$ are presented in Fig. A.1. The slant distance $\bar{y}$ measures the length from the surface of the liquid to the centroid of the panel along an axis that is aligned with the "slant of the panel," as shown in Fig. 3.24.

Equation (3.33) shows that the CP will be situated below the centroid. The distance between the CP and the centroid depends on the depth of submersion, which is characterized by $\bar{y}$, and on the panel geometry, which is characterized by $\bar{I}/A$.

**TABLE 3.3** Summary of the Panel Equations

| Purpose of the Equation | Equation | Variables |
|---|---|---|
| Predict the magnitude of the hydrostatic force | $F_p = \bar{p}A$ (3.28) | $F_p$ = pressure force (N)<br>$\bar{p}$ = gage pressure evaluated at the depth of the centroid (Pa)<br>A = surface area of the plate ($m^2$) |
| Calculate the location of the center of pressure (CP) | $y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A}$ (3.33) | $(y_{cp} - \bar{y})$ = slant distance from the centroid to the CP (m)<br>$\bar{I}$ = area moment of inertia of the panel about its centroidal axis ($m^4$; for formulas, see Fig. A.1 in the appendix)<br>$\bar{y}$ = slant distance from the centroid to the liquid surface (m) |
| This figure defines variables |  | |
| Check these assumptions: | **1.** The problem involves only one fluid. This fluid has a constant density.<br>**2.** The pressure distribution is hydrostatic.<br>**3.** The pressure at the free surface is zero gage.<br>**4.** The panel is symmetric about an axis parallel to the slant distance. | |

Due to assumptions in the derivations, Eqs. (3.28) and (3.33) have several limitations. First, they only apply to a single fluid of constant density. Second, the pressure at the liquid surface needs to be $p = 0$ gage to correctly locate the CP. Third, Eq. (3.33) gives only the vertical location of the CP, not the lateral location.

## Panel Force Working Equations (Summary)

In Table 3.3, we have summarized information that is useful for applying the panel equations. Notice that this table gives the equations, the variables, and the main assumptions. These equations are applied in Examples 3.7 and 3.8.

### EXAMPLE 3.7

Hydrostatic Force Due to Concrete

**Problem Statement**

Determine the force acting on one side of a concrete form 2.44 m high and 1.22 m wide (8 ft by 4 ft) that is used for pouring a basement wall. The specific weight of concrete is 23.6 kN/m$^3$ (150 lbf/ft$^3$).

**Define the Situation**

Concrete in a liquid state acts on a vertical surface.

The vertical wall is 2.44 m high and 1.22 m wide

**Assumptions:** Freshly poured concrete can be represented as a liquid.

**Properties:** Concrete: $\gamma = 23.6$ kN/m$^3$

**State the Goal**

Find the resultant force (kN) acting on the wall.

**Plan**

Apply the panel equation (3.28).

**Solution**

1. Panel equation:

$$F = \bar{p}A$$

2. Term-by-term analysis:
   - $\bar{p}$ = pressure at depth of the centroid

$$\bar{p} = (\gamma_{\text{concrete}})(z_{\text{centroid}}) = (23.6 \text{ kN/m}^3)(2.44/2 \text{ m})$$
$$= 28.79 \text{ kPa}$$

   - $A$ = area of panel

$$A = (2.44 \text{ m})(1.22 \text{ m}) = 2.977 \text{ m}^2$$

3. Resultant force:

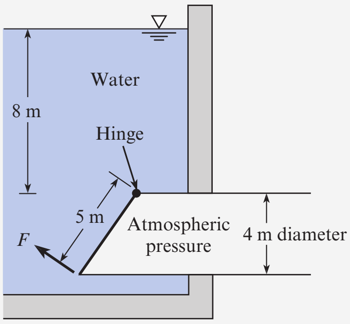
$$F = \bar{p}A = (28.79 \text{ kPa})(2.977 \text{ m}^2) = \boxed{85.7 \text{ kN}}$$

**EXAMPLE 3.8**

Force to Open an Elliptical Gate

### Problem Statement

An elliptical gate covers the end of a pipe 4 m in diameter. If the gate is hinged at the top, what normal force $F$ is required to open the gate when water is 8 m deep above the top of the pipe and the pipe is open to the atmosphere on the other side? Neglect the weight of the gate.




### Define the Situation

Water pressure is acting on an elliptical gate.

**Properties:** Water (10°C): Table A.5, $\gamma = 9810\ \text{N/m}^3$

**Assumptions:**

1. Neglect the weight of the gate.
2. Neglect friction between the bottom on the gate and the pipe wall.

### State the Goal

F(N) ⬅ force needed to open gate

### Generate Ideas and Make a Plan

1. Calculate resultant hydrostatic force using $F = \bar{p}A$.
2. Find the location of the center of pressure using Eq. (3.33).
3. Draw an FBD of the gate.
4. Apply moment equilibrium about the hinge.

### Take Action (Execute the Plan)

1. Hydrostatic (resultant) force:
   - $\bar{p}$ = pressure at depth of the centroid

   $$\bar{p} = (\gamma_{\text{water}})(z_{\text{centroid}}) = (9810\ \text{N/m}^3)(10\ \text{m}) = 98.1\ \text{kPa}$$

   - $A$ = area of elliptical panel (using Fig. A.1 to find formula)

   $$A = \pi ab = \pi(2.5\ \text{m})(2\ \text{m}) = 15.71\ \text{m}^2$$

   - Calculate resultant force:

   $$F_p = \bar{p}A = (98.1\ \text{kPa})(15.71\ \text{m}^2) = \boxed{1.54\ \text{MN}}$$

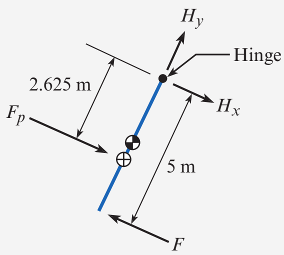
2. Center of pressure:
   - $\bar{y} = 12.5$ m, where $\bar{y}$ is the slant distance from the water surface to the centroid
   - Area moment of inertia $\bar{I}$ of an elliptical panel using a formula from Fig. A.1:

   $$\bar{I} = \frac{\pi a^3 b}{4} = \frac{\pi(2.5\ \text{m})^3(2\ \text{m})}{4} = 24.54\ \text{m}^4$$

   - Finding center of pressure:

   $$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A} = \frac{25.54\ \text{m}^4}{(12.5\ \text{m})(15.71\ \text{m}^2)} = 0.125\ \text{m}$$

3. FBD of the gate:




4. Moment equilibrium:

$$\sum M_{\text{hinge}} = 0$$

$$1.541 \times 10^6\ \text{N} \times 2.625\ \text{m} - F \times 5\ \text{m} = 0$$

$$F = \boxed{809\ \text{kN}}$$

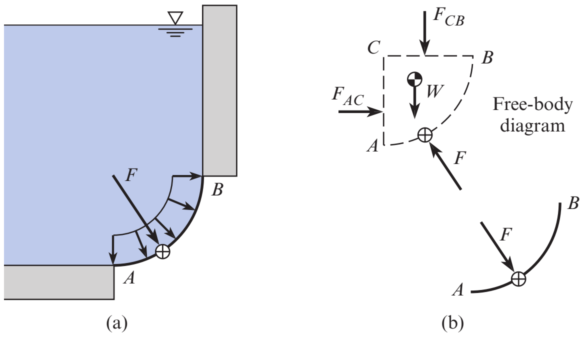
## 3.5 Calculating the Pressure Force on a Curved Surface

As engineers, we calculate pressure forces on curved surfaces when we are designing components such as tanks, pipes, and curved gates. Thus, this topic is described in this section.

Consider the curved surface $AB$ in Fig. 3.25a. The goal is to represent the pressure distribution with a resultant force that passes through the center of pressure. One approach is to integrate the pressure force along the curved surface and find the equivalent force. However, it is easier to sum forces for the free body shown in the upper part of Fig. 3.25b. The lower sketch in Fig. 3.25b shows how the force acting on the curved surface relates to the force

**FIGURE 3.25**

(a) Pressure distribution and equivalent force. (b) Free body diagram and action–reaction force pair.




$F$ acting on the free body. Using the FBD and summing forces in the horizontal direction shows that

$$F_x = F_{AC} \tag{3.34}$$

The line of action for the force $F_{AC}$ is through the center of pressure for side $AC$.

The vertical component of the equivalent force is

$$F_y = W + F_{CB} \tag{3.35}$$

where $W$ is the weight of the fluid in the free body, and $F_{CB}$ is the force on the side $CB$.

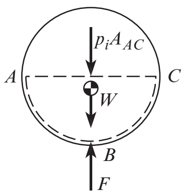
The force $F_{CB}$ acts through the centroid of surface $CB$, and the weight acts through the center of gravity of the free body. The line of action for the vertical force may be found by summing the moments about any convenient axis.

Example 3.9 illustrates how curved surface problems can be solved by applying equilibrium concepts together with the panel force equations.

The central idea of this section is that *forces on curved surfaces may be found by applying equilibrium concepts to systems comprised of the fluid in contact with the curved surface*. Notice how equilibrium concepts are used in each of the situations discussed ahead.

**FIGURE 3.26**

Pressurized spherical tank showing forces that act on the fluid inside the marked region.




Consider a sphere holding a gas pressurized to a gage pressure $p_i$, as shown in Fig. 3.26. The indicated forces act on the fluid in volume $ABC$. Applying equilibrium in the vertical direction gives

$$F = p_i A_{AC} + W$$

Because the specific weight for a gas is quite small, engineers usually neglect the weight of the gas:

$$F = p_i A_{AC} \tag{3.36}$$

Another example is finding the force on a curved surface submerged in a reservoir of liquid, as shown in Fig. 3.27a. If atmospheric pressure prevails above the free surface and on the outside of surface $AB$, then force caused by atmospheric pressure cancels out, and equilibrium gives

$$F = \gamma \forall_{ABCD} = W\downarrow \tag{3.37}$$

Hence, the force on surface $AB$ equals the weight of liquid above the surface, and the arrow indicates that the force acts downward.

Now consider the situation in which the pressure distribution on a thin, curved surface comes from the liquid underneath, as shown in Fig. 3.27b. If the region above the surface, volume $abcd$, were filled with the same liquid, then the pressure acting at each point on the upper surface of $ab$ would equal the pressure acting at each point on the lower surface. In other words, there would be no net force on the surface. Thus, the equivalent force on surface $ab$ is given by

$$F = \gamma \forall_{abcd} = W\downarrow \tag{3.38}$$

where $W$ is the weight of liquid needed to fill a volume that extends from the curved surface to the free surface of the liquid.

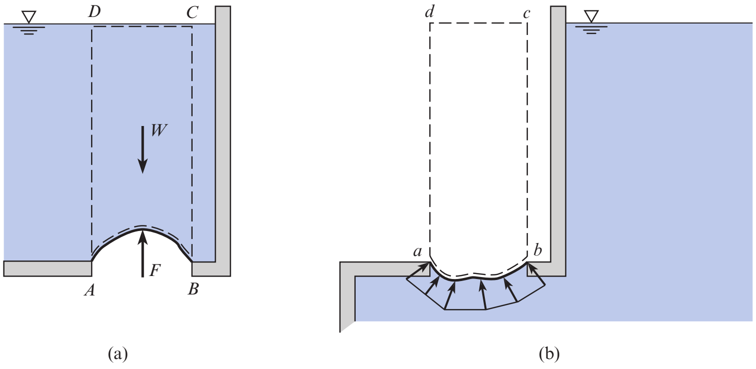



**FIGURE 3.27**

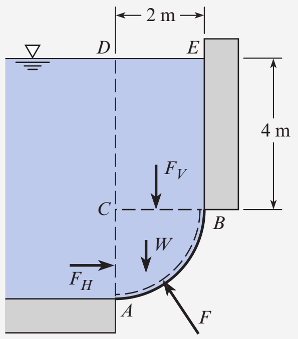
Curved surface with (a) liquid above and (b) liquid below. In (a), arrows represent forces acting on the liquid. In (b), arrows represent the pressure distribution on surface *ab*.

## EXAMPLE 3.9

Hydrostatic Force on a Curved Surface

### Problem Statement

Surface *AB* is a circular arc with a radius of 2 m and a width of 1 m into the paper. The distance *EB* is 4 m. The fluid above surface *AB* is water, and atmospheric pressure prevails on the free surface of the water and on the bottom side of surface *AB*. Find the magnitude and line of action of the hydrostatic force acting on surface *AB*.




### Define the Situation

**Situation:** A body of water is contained by a curved surface.

**Properties:** Water (10°C): Table A.5, $\gamma = 9810\ \text{N/m}^3$

### State the Goal

Find:

1. Hydrostatic force (in newtons) on the curved surface *AB*
2. Line of action of the hydrostatic force

### Generate Ideas and Make a Plan

Apply equilibrium concepts to the body of fluid *ABC*:

1. Find the horizontal component of *F* by applying Eq. (3.34).
2. Find the vertical component of *F* by applying Eq. (3.35).
3. Find the line of action of *F* by finding the lines of action of components and then using a graphical solution.

### Take Action (Execute the Plan)

1. Force in the horizontal direction:

$$F_x = F_H = \bar{p}A = (5\ \text{m})(9810\ \text{N/m}^3)(2 \times 1\ \text{m}^2)$$
$$= 98.1\ \text{kN}$$

2. Force in the vertical direction:
   - Vertical force on side *CB*:

$$F_V = \bar{p}_0 A = 9.81\ \text{kN/m}^3 \times 4\ \text{m} \times 2\ \text{m} \times 1\ \text{m} = 78.5\ \text{kN}$$

   - Weight of the water in volume *ABC*:

$$W = \gamma V\!\!\!\!-_{ABC} = (\gamma)(\tfrac{1}{4}\pi r^2)(w)$$
$$= (9.81\ \text{kN/m}^3) \times (0.25 \times \pi \times 4\ \text{m}^2)(1\ \text{m}) = 30.8\ \text{kN}$$

   - Summing forces:

$$F_y = W + F_V = 109.3\ \text{kN}$$

3. Line of action (horizontal force):

$$y_{cp} = \bar{y} + \frac{\bar{I}}{\bar{y}A} = (5\ \text{m}) + \left(\frac{1 \times 2^3/12}{5 \times 2 \times 1}\text{m}\right)$$
$$y_{cp} = 5.067\ \text{m}$$

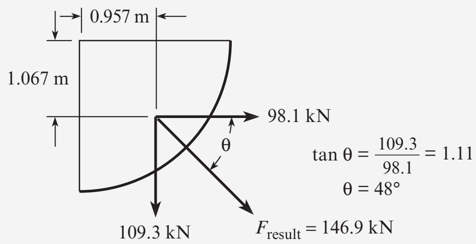
4. The line of action ($x_{cp}$) for the vertical force is found by summing moments about point *C*:

$$x_{cp}F_y = F_V \times 1\ \text{m} + W \times \bar{x}_w$$

The horizontal distance from point *C* to the centroid of the area *ABC* is found using Fig. A.1: $\bar{x}_W = 4r/3\pi = 0.849$ m. Thus,

$$x_{cp} = \frac{78.5\ \text{kN} \times 1\ \text{m} + 30.8\ \text{kN} \times 0.849\ \text{m}}{109.3\ \text{kN}} = 0.957\ \text{m}$$

5. The resultant force that acts on the curved surface is shown in the following figure:



## 3.6 Calculating Buoyant Forces

Engineers calculate buoyant forces for applications such as the design of ships, sediment transport in rivers, and fish migration. Buoyant forces are sometimes significant in problems involving gases (e.g., a weather balloon). This section describes how to calculate the buoyant force on an object.

A **buoyant force** is defined as an upward force (with respect to gravity) on a body that is totally or partially submerged in a fluid, either a liquid or gas. Buoyant forces are caused by the hydrostatic pressure distribution.

### The Buoyant Force Equation

To derive an equation, consider a body *ABCD* submerged in a liquid of specific weight $\gamma$ (Fig. 3.28). The sketch on the left shows the pressure distribution acting on the body. As shown by Eq. (3.38), pressures acting on the lower portion of the body create an upward force equal to the weight of liquid needed to fill the volume above surface *ADC*. The upward force is

$$F_{\text{up}} = \gamma(\forall_b + \forall_a)$$

where $\forall_b$ is the volume of the body (i.e., volume *ABCD*) and $\forall_a$ is the volume of liquid above the body (i.e., volume *ABCFE*). As shown by Eq. (3.37), pressures acting on the top surface of the body create a downward force equal to the weight of the liquid above the body:

$$F_{\text{down}} = \gamma\forall_a$$

Subtracting the downward force from the upward force gives the net or buoyant force $F_B$ acting on the body:

$$F_B = F_{\text{up}} - F_{\text{down}} = \gamma\forall_b \quad \textbf{(3.39)}$$

Hence, the net force or buoyant force ($F_B$) equals the weight of liquid that would be needed to occupy the volume of the body.

Consider a body that is floating as shown in Fig. 3.29. The marked portion of the object has a volume $\forall_D$. Pressure acts on curved surface *ADC,* causing an upward force equal to the weight of liquid that would be needed to fill volume $\forall_D$. The buoyant force is given by

$$F_B = F_{\text{up}} = \gamma\forall_D \quad \textbf{(3.40)}$$

Hence, the buoyant force equals the weight of liquid that would be needed to occupy the volume $\forall_D$. This volume is called the displaced volume. Comparison of Eqs. (3.39) and (3.40) shows that one can write a single equation for the buoyant force:

$$F_B = \gamma\forall_D \quad \textbf{(3.41a)}$$

In Eq. (3.41a), $\forall_D$ is the volume that is displaced by the body. If the body is totally submerged, the displaced volume is the volume of the body. If a body is partially submerged, the displaced volume is the portion of the volume that is submerged.

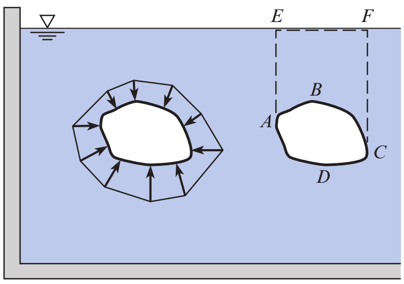



**FIGURE 3.28**

Two views of a body immersed in a liquid.

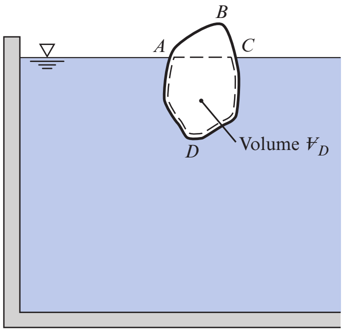



**FIGURE 3.29**
A body partially submerged in a liquid.

Eq. (3.41b) is only valid for a single fluid of uniform density. The general principle of buoyancy is called **Archimedes' principle**:

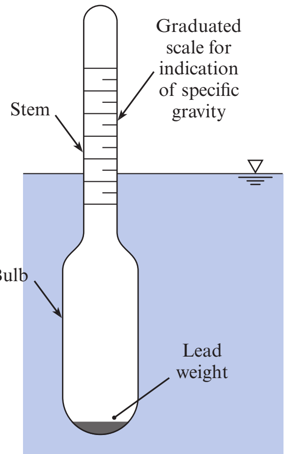
$$\text{(buoyant force)} = F_B = \text{(weight of the displaced fluid)} \quad \textbf{(3.41b)}$$

The buoyant force acts at a point called the center of buoyancy, which is located at the center of gravity of the displaced fluid.

## The Hydrometer

A **hydrometer** (Fig. 3.30) is an instrument for measuring the specific gravity of a liquid. It is typically made of a glass bulb that is weighted on one end so the hydrometer floats in an upright position. A stem of constant diameter is marked with a scale, and the specific weight of the liquid is determined by the depth at which the hydrometer floats. The operating principle of the hydrometer is buoyancy. In a heavy liquid (i.e., high $\gamma$), the hydrometer will float more shallowly because a lesser volume of the liquid must be displaced to balance the weight of the hydrometer. In a light liquid, the hydrometer will float deeper.

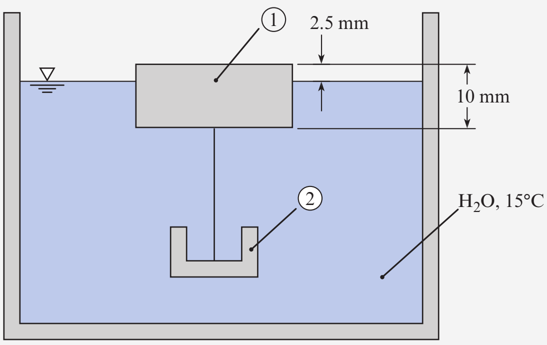
**FIGURE 3.30**
Hydrometer




## EXAMPLE 3.10
Buoyant Force on a Metal Part

### Problem Statement

A metal part (object 2) is hanging by a thin cord from a floating wood block (object 1). The wood block has a specific gravity $S_1 = 0.3$ and dimensions of $50 \times 50 \times 10$ mm. The metal part has a volume of 6600 mm$^3$. Find the mass $m_2$ of the metal part and the tension $T$ in the cord.




### Define the Situation

A metal part is suspended from a floating block of wood.

**Properties:**

- Water (15°C): Table A.5, $\gamma = 9800$ N/m$^3$
- Wood: $S_1 = 0.3$

### State the Goal

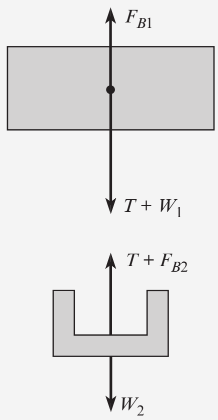
- Find the mass (in grams) of the metal part.
- Calculate the tension (in newtons) in the cord.

### Generate Ideas and Make a Plan

1. Draw FBDs of the block and the part.
2. Apply equilibrium to the block to find the tension.
3. Apply equilibrium to the part to find the weight of the part.
4. Calculate the mass of the metal part using $W = mg$.

### Take Action (Execute the Plan)

1. FBDs:




2. Force equilibrium (vertical direction) applied to block:

$$T = F_{B1} - W_1$$

- Buoyant force $F_{B1} = \gamma \forall_{D1}$, where $\forall_{D1}$ is the submerged volume:

$$F_{B1} = \gamma \forall_{D1}$$
$$= (9800\ \text{N/m}^3)(50 \times 50 \times 7.5\ \text{mm}^3)(10^{-9}\ \text{m}^3/\text{mm}^3)$$
$$= 0.184\ \text{N}$$

- Weight of the block:

$$W_1 = \gamma S_1 \forall_1$$
$$= (9800\ \text{N/m}^3)(0.3)(50 \times 50 \times 10\ \text{mm}^3)(10^{-9}\ \text{m}^3/\text{mm}^3)$$
$$= 0.0735\ \text{N}$$

- Tension in the cord:

$$T = (0.184 - 0.0735) = \boxed{0.110\ \text{N}}$$

3. Force equilibrium (vertical direction) applied to metal part:

- Buoyant force:

$$F_{B2} = \gamma \forall_2 = (9800\ \text{N/m}^3)(6600\ \text{mm}^3)(10^{-9}) = 0.0647\ \text{N}$$

- Equilibrium equation:

$$W_2 = T + F_{B2} = (0.110\ \text{N}) + (0.0647\ \text{N})$$

4. Mass of metal part:

$$m_2 = W_2/g = \boxed{17.8\ \text{g}}$$

### Review the Solution and the Process

*Discussion.* Notice that tension in the cord (0.11 N) is less than the weight of the metal part (0.18 N). This result is consistent with the common observation that an object will weigh less in water than in air.

*Tip.* When solving problems that involve buoyancy, draw an FBD.

## 3.7 Predicting Stability of Immersed and Floating Bodies

Engineers need to calculate whether an object will tip over or remain in an upright position when placed in a liquid (e.g., for the design of ships and buoys). Thus, stability is presented in this section.

### Immersed Bodies

When a body is completely immersed in a liquid, its stability depends on the relative positions of the center of gravity of the body and the centroid of the displaced volume of fluid, which is called the **center of buoyancy**. If the center of buoyancy is above the center of gravity (see Fig. 3.31a), any tipping of the body produces a righting couple, and consequently the body is stable. Alternatively, if the center of gravity is above the center of buoyancy, any tipping produces

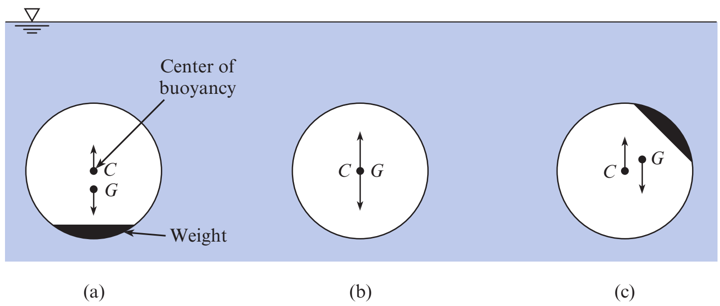



**FIGURE 3.31**
Conditions of stability for immersed bodies. (a) Stable. (b) Neutral. (c) Unstable.

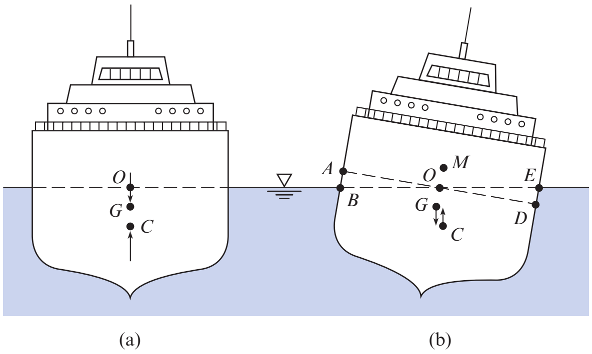



**FIGURE 3.32**
Ship stability relations.

an overturning moment, thus causing the body to rotate through 180° (see Fig. 3.31c). If the center of buoyancy and center of gravity are coincident, the body is neutrally stable—that is, it lacks a tendency for righting itself or for overturning (see Fig. 3.31b).

## Floating Bodies

The question of stability is more involved for floating bodies than for immersed bodies because the center of buoyancy may take different positions with respect to the center of gravity, depending on the shape of the body and the position in which it is floating. For example, consider the cross section of a ship shown in Fig. 3.32a. Here, the center of gravity *G* is above the center of buoyancy *C*. Therefore, at first glance, it would appear that the ship is unstable and could flip over. However, notice the position of *C* and *G* after the ship has taken a small angle of heel. As shown in Fig. 3.32b, the center of gravity is in the same position, but the center of buoyancy has moved outward from the center of gravity, thus producing a righting moment. A ship having such characteristics is stable.

The reason for the change in the center of buoyancy for the ship is that part of the original buoyant volume, as shown by the wedge shape *AOB*, is transferred to a new buoyant volume *EOD*. Because the buoyant center is at the centroid of the displaced volume, it follows that for this case the buoyant center must move laterally to the right. The point of intersection of the lines of action of the buoyant force before and after heel is called the *metacenter* (*M*), and the distance *GM* is called the *metacentric height*. If *GM* is positive—that is, if *M* is above *G*—the ship is stable; however, if *GM* is negative, the ship is unstable. Quantitative relations involving these basic principles of stability are presented in the next paragraph.

Consider the ship shown in Fig. 3.33, which has taken a small angle of heel $\alpha$. First, evaluate the lateral displacement of the center of buoyancy, $CC'$; then, it will be easy by simple trigonometry to solve for the metacentric height *GM* or to evaluate the righting moment. Recall that the center of buoyancy is at the centroid of the displaced volume. Therefore, resort to the fundamentals of centroids to evaluate the displacement $CC'$. From the definition of the centroid of a volume,

$$\bar{x}\,\forall = \Sigma x_i \Delta \forall_i \qquad (3.42)$$

where $\bar{x} = CC'$, which is the distance from the plane about which moments are taken to the centroid of $\forall$; $\forall$ is the total volume displaced; $\Delta\forall_i$ is the volume increment; and $x_i$ is the moment arm of the increment of volume.

Take moments about the plane of symmetry of the ship. Recall from mechanics that volumes to the left produce negative moments and volumes to the right produce positive moments. For the right side of Eq. (3.42), write terms for the moment of the submerged volume about the plane of symmetry. A convenient way to do this is to consider the moment of the volume before heel, subtract the moment of the volume represented by the wedge *AOB*,

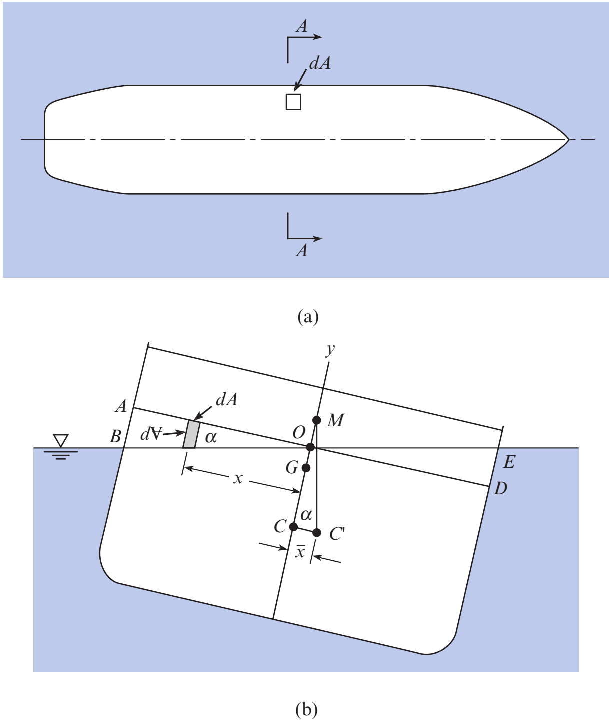



**FIGURE 3.33**
(a) A plan view of a ship. (b) Section A-A of the ship.

and add the moment represented by the wedge *EOD*. In a general way, this is given by the following equation:

$$\bar{x}\,\forall = \text{moment of } \forall \text{ before heel} - \text{moment of } \forall_{AOB} + \text{moment of } \forall_{EOD} \qquad \textbf{(3.43)}$$

Because the original buoyant volume is symmetrical with $y$-$y$, the moment for the first term on the right is zero. Also, the sign of the moment of $\forall_{AOB}$ is negative; therefore, when this negative moment is subtracted from the right-hand side of Eq. (3.43), the result is

$$\bar{x}\forall = \sum x_i \Delta\forall_{iAOB} + \sum x_i \Delta\forall_{iEOD} \qquad \textbf{(3.44)}$$

Now, express Eq. (3.44) in integral form:

$$\bar{x}\,\forall = \int_{AOB} x\,d\forall + \int_{EOD} x\,d\forall \qquad \textbf{(3.45)}$$

However, it may be seen from Fig. 3.33b that $d\forall$ can be given as the product of the length of the differential volume, $x \tan\alpha$, and the differential area, $dA$. Consequently, Eq. (3.45) can be written as

$$\bar{x}\,\forall = \int_{AOB} x^2 \tan\alpha\, dA + \int_{EOD} x^2 \tan\alpha\, dA$$

Here, $\tan\alpha$ is a constant with respect to the integration. Also, because the two terms on the right-hand side are identical except for the area over which integration is to be performed, combine them as follows:

$$\bar{x}\,\forall = \tan\alpha \int_{A_{\text{waterline}}} x^2\, dA \qquad \textbf{(3.46)}$$

The second moment, or moment of inertia of the area defined by the waterline, is given the symbol $I_{00}$, and the following is obtained:

$$\bar{x}\,\mathcal{V} = I_{00}\tan\alpha$$

Next, replace $\bar{x}$ by $CC'$ and solve for $CC'$:

$$CC' = \frac{I_{00}\tan\alpha}{\mathcal{V}}$$

From Fig. 3.33b,

$$CC' = CM\tan\alpha$$

Thus, eliminating $CC'$ and $\tan\alpha$ yields

$$CM = \frac{I_{00}}{\mathcal{V}}$$

However,

$$GM = CM - CG$$

Therefore, the *metacentric height* is

$$GM = \frac{I_{00}}{\mathcal{V}} - CG \qquad \textbf{(3.47)}$$

Equation (3.47) is used to determine the stability of floating bodies. As already noted, if $GM$ is positive, the body is stable; if $GM$ is negative, the body is unstable.

Note that for small angles of heel $\alpha$, the righting moment or overturning moment is given as follows:

$$\text{RM} = \gamma\,\mathcal{V}\,GM\alpha \qquad \textbf{(3.48)}$$

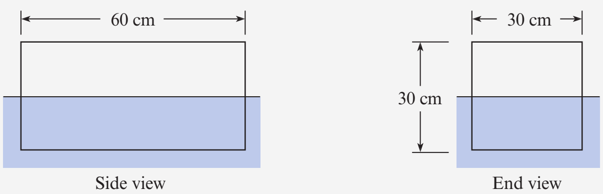
However, for large angles of heel, direct methods of calculation based on these same principles would have to be employed to evaluate the righting or overturning moment.

## EXAMPLE 3.11

Stability of a Floating Block

### Problem Statement

A block of wood 30 cm square in cross section and 60 cm long weighs 318 N. Will the block float with sides vertical as shown?




### Define the Situation

A block of wood is floating in water.

### State the Goal

Determine the stable configuration of the block of wood.

### Generate Ideas and Make a Plan

1. Apply force equilibrium to find the depth of submergence.
2. Determine if the block is stable about the long axis by applying Eq. (3.47).
3. If the block is not stable, repeat steps 1 and 2.

### Take Action (Execute the Plan)

1. Equilibrium (vertical direction):

$$\sum F_y = 0$$

$$-\text{weight} + \text{buoyant force} = 0$$

$$-318\text{ N} + 9810\text{ N/m}^3 \times 0.30\text{ m} \times 0.60\text{ m} \times d = 0$$

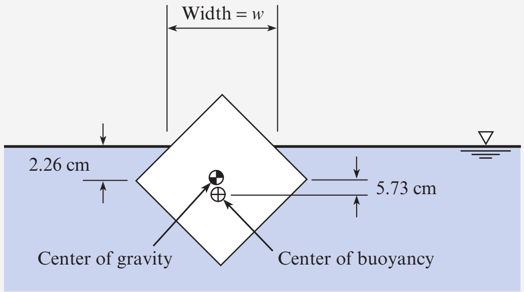
$$d = 0.18\text{ m} = 18\text{ cm}$$

2. Stability (longitudinal axis):

$$GM = \frac{I_{00}}{\mathcal{V}} - CG = \frac{\frac{1}{12} \times 60 \times 30^3}{18 \times 60 \times 30} - (15 - 9)$$

$$= 4.167 - 6 = -1.833\text{ cm}$$

Because the metacentric height is negative, the block is not stable about the longitudinal axis. Thus, a slight disturbance will make it tip to the orientation shown below. Note: calculations to find the dimensions (2.26 and 5.73 cm) are not shown in this example.




3. Equilibrium (vertical direction):

$$-\text{weight} + \text{buoyant force} = 0$$

$$-(318\ \text{N}) + (9810\ \text{N/m}^3)(V_D) = 0$$

$$V_D = 0.0324\ \text{m}^3$$

4. Find the dimension $w$:

(Displaced volume)

= (Block volume) − (Volume above the waterline)

$$V_D = 0.0324\ \text{m}^3 = (0.3^2)(0.6)\ \text{m}^3 - \frac{w^2}{4}(0.6\ \text{m})$$

$$w = 0.379\ \text{m}$$

5. Moment of inertia at the waterline:

$$I_{00} = \frac{bh^3}{12} = \frac{(0.6\ \text{m})(0.379\ \text{m})^3}{12} = 0.00273\ \text{m}^4$$

6. Metacentric height:

$$GM = \frac{I_{00}}{V} - CG = \frac{0.00273\ \text{m}^4}{0.0324\ \text{m}^3} - 0.0573\ \text{m} = 0.027\ \text{m}$$

Because the metacentric height is positive, the block will be stable in this position.

## 3.8 Summarizing Key Knowledge

### Pressure

- *Pressure p* is the ratio of (magnitude of normal force due to a fluid) to (area) at a point.
  - Pressure always acts to compress the material that is in contact with the fluid exerting the pressure.
  - Pressure is a scalar not a vector.
- Engineers express pressure with gage pressure, absolute pressure, vacuum pressure, and differential pressure.
  - Absolute pressure is measured relative to absolute zero.
  - Gage pressure gives the magnitude of pressure relative to atmospheric pressure.

$$p_{\text{abs}} = p_{\text{atm}} + p_{\text{gage}}$$

  - Vacuum pressure gives the magnitude of the pressure below atmospheric pressure.

$$p_{\text{vacuum}} = p_{\text{atm}} - p_{\text{abs}}$$

  - Differential pressure ($\Delta p$) gives the difference in pressure between two points (e.g., $A$ and $B$).

### Hydrostatic Equilibrium

- A *hydrostatic condition* means that the weight of each fluid particle is balanced by the net pressure force.
- The weight of a fluid causes pressure to increase with increasing depth, giving the *hydrostatic differential equation*. The equations that are used in hydrostatics are derived from this equation. The hydrostatic differential equation is

$$\frac{dp}{dz} = -\gamma = -\rho g$$

- If density is constant, the hydrostatic differential equation can be integrated to give the hydrostatic equation. The meaning (i.e., physics) of the hydrostatic equation is that piezometric head (or piezometric pressure) is constant everywhere in a static body of fluid.

$$\frac{p}{\gamma} + z = \text{constant}$$

## Pressure Distributions and Forces Due to Pressure

- A fluid in contact with a surface produces a *pressure distribution*, which is a mathematical or visual description of how the pressure varies along the surface.
- A pressure distribution is often represented as a statically equivalent force $\mathbf{F}_p$ acting at the *center of pressure* (CP).
- A *uniform pressure distribution* means that the pressure is the same at every point on a surface. Pressure distributions due to gases are typically idealized as uniform pressure distributions.
- A *hydrostatic pressure distribution* means that the pressure varies according to $dp/dz = -\gamma$.

## Force on a Flat Surface

- For a panel subjected to a hydrostatic pressure distribution, the hydrostatic force is

$$F_p = \bar{p}A$$

- This hydrostatic force
  - Acts *at* the centroid of area for a uniform pressure distribution.
  - Acts *below* the centroid of area for a hydrostatic pressure distribution. The slant distance between the center of pressure and the centroid of area is given by

$$y_{\text{cp}} - \bar{y} = \frac{I}{\bar{y}A}$$

## Hydrostatic Forces on a Curved Surface

- When a surface is curved, one can find the pressure force by applying force equilibrium to a free body comprised of the fluid in contact with the surface.

## The Buoyant Force

- The *buoyant force* is the pressure force on a body that is partially or totally submerged in a fluid.
- The magnitude of the buoyant force is given by

$$\text{Buoyant force} = F_B = \text{weight of the displaced fluid}$$

- The center of buoyancy is located at the center of gravity of the displaced fluid. The direction of the buoyant force is opposite the gravity vector.
- When the buoyant force is due to a single fluid with constant density, the magnitude of the buoyant force is

$$F_B = \gamma\forall_D$$

## Hydrodynamic Stability

- Hydrodynamic stability means that if an object is displaced from equilibrium, then there is a moment that causes the object to return to equilibrium.
- The criteria for stability are as follows:
  - *Immersed object*. The body is stable if the center of gravity is below the center of buoyancy.
  - *Floating object*. The body is stable if the metacentric height is positive.