Chapter 2 Problems

System, State, and Property (§2.1)

2.1 Define a **system** using the standard structure of a definition.

SS **2.2** For the situation that follows, answer the following questions:

a. What system would an engineer select?

b. What is state 1 (the initial state)?

c. What is state 2 (the final state)?

d. List three examples of properties at state 1?

e. List three examples of properties at state 2?

f. List two examples of problem parameters that are not properties?

SITUATION

During a study of drag, an engineer wishes to calculate how far a tennis ball will travel. The tennis ball is launched at a speed of 55 m/s and an angle of 20° with respect to a horizontal axis. When the ball is launched, it is 1.5 m above the ground. The model will include the effects of air drag.

2.3 For each item in this list, give a simple, one-sentence definition: (a) system (b) state (c) property

Looking Up Fluid Properties (§2.2)

2.4 Where in this text can you find: Answer

a. density data for such liquids as oil and mercury?

b. specific weight data for air (at standard atmospheric pressure) at different temperatures?

c. specific gravity data for sea water and kerosene?

2.5 Regarding water and seawater:

a. Which is more dense, seawater or freshwater?

b. Find (SI units) the density of seawater (10°C, 3.3% salinity).

c. Find the same in traditional units.

d. What pressure is specified for the values in (b) and (c)?

2.6 Where in this text can you find: Answer

a. values of surface tension (σ) for kerosene and mercury?

b. values for the vapor pressure (p_v) of water as a function of temperature?

2.7 An open vat in a food processing plant contains 500 L of water at 20°C and atmospheric pressure. If the water is heated to 80°C, what will be the percentage change in its volume? If the vat has a diameter of 2 m, how much will the water level rise due to this temperature increase?

SS **2.8** Consider liquid water at atmospheric pressure over a temperature ranging from freezing to boiling.

(T/F) If $T\uparrow$, then $\rho\downarrow$. Answer

SS **2.9** (T/F) The density of water at 4°C and 1 bar is greater than the density of water at 1°C and 1 bar.

2.10 (T/F) At atmospheric pressure, a liter of water at 100°C will weigh about 0.4 N less than a liter of water at 5°C. Answer SS

2.11 The following questions relate to viscosity.

a. What are the primary dimensions of viscosity? What are five common units?

b. What is the viscosity of SAE 10W-30 motor oil at 115°F (in traditional units)?

2.12 (T/F) If the pressure of water increases, then the viscosity will also increase. Answer

2.13 Consider the following units:

I. Pa · s

II. lbf · s/ft^2

III. kg · m^{-1} · s^{-1}

IV. lbm/ft · s

V. stoke

Which choices are absolute viscosity units?

(a) All **(b)** I and II **(c)** All except V **(d)** V only **(e)** I and V

2.14 (T/F) If you have a fluid and if $\nu\downarrow$ as $T\uparrow$, then the fluid must be a liquid. Answer

2.15 When looking up values for density, absolute viscosity, and kinematic viscosity, which statement is most true for both liquids and gases? SS

a. All three of these properties vary with temperature.

b. All three of these properties vary with pressure.

c. All three of these properties vary with temperature and pressure.

2.16 Kinematic viscosity (select all that apply) Answer

a. is another name for absolute viscosity

b. is viscosity/density

c. is dimensionless because forces are canceled out

d. has dimensions of L^2/T

e. is only used with compressible fluids

2.17 What is the kinematic viscosity of air in units of m^2/s at typical room conditions?

(a) 15e-5 **(b)** 18e-5 **(c)** 18e-6 **(d)** 15e-6 **(e)** 29e-5

2.18 (T/F) At room conditions, the kinematic viscosity of air is greater than the kinematic viscosity of water. Answer

2.19 (T/F) At room conditions, the dynamic viscosity of air is greater than the dynamic viscosity of water. SS

2.20 What is the change in the viscosity and density of water between 10°C and 90°C? What is the change in the viscosity and density of air between 10°C and 90°C? Assume standard atmospheric pressure ($p = 101$ kN/m^2). Answer

2.21 The temperature of liquid water decreases from 80°C to 20°C. The percent change in absolute viscosity is SS

(a) 6 **(b)** 183 **(c)** 132 **(d)** 92 **(e)** 78

2.22 Determine the change in the kinematic viscosity of air that is heated from 10°C to 50°C. Assume standard atmospheric pressure. Answer

2.23 Find the dynamic and kinematic viscosities of kerosene, SAE 10W-30 motor oil, and water at a temperature of 50°C.

2.24 What is the ratio of the dynamic viscosity of air to that of water at standard atmospheric pressure and a temperature of 20°C? What is the ratio of the kinematic viscosity of air to that of water for the same conditions? Answer

2.25 Find the kinematic and dynamic viscosities of air and water at a temperature of 40°C and an absolute pressure of 170 kPa.

Specific Gravity, Constant Density, and the Bulk Modulus (§2.3)

2.26 (T/F) Specific gravity is defined as the density of a material divided by the density of water when the water is in the liquid state at 4°C. Answer

2.27 What is the specific weight of nitrogen in units of newtons per cubic meter at a temperature of −10°C and a pressure of 0.6 bar gage?

(a) 20 **(b)** 31 **(c)** 36 **(d)** 24 **(e)** 39

2.28 From memory, give the density of water at room conditions.

a. ______________ kg/L
b. ______________ g/L
c. ______________ g/mL
d. ______________ kg/(1000 L)
e. ______________ kg/m^3
f. ______________ slug/ft^3
g. ______________ lbm/ft^3
h. ______________ lbm/(US gallon)
i. ______________ lbm/(US quart)

2.29 From memory, give the specific weight of water at room conditions.

a. ______________ N/m^3
b. ______________ N/L
c. ______________ kN/m^3
d. ______________ lbf/ft^3
e. ______________ lbf/(US gallon)
f. ______________ lbf/(US quart)

2.30 Which statement is correct? Answer

a. *SG* (freshwater) > *SG* (sea water)
b. *SG* (concrete) > *SG* (aluminum) > *SG* (salt water)
c. *SG* (steel) > *SG* (aluminum) > *SG* (concrete) > *SG* (salt water) > *SG* (freshwater) > *SG* (oil)
d. *SG* (steel) > *SG* (concrete) > *SG* (aluminum)
e. *SG* (oil) > *SG* (freshwater)

2.31 Consider the following statements:

I. The reference temperature for *SG* is 4°C .
II. Two liquids that are miscible will not mix.
III. A block with $SG > 1$ can float on a liquid.
IV. $\rho_{oil} = SG_{oil}(62.4\ \text{lbm/ft}^3)$.
V. $\gamma_{H_2O} = \gamma_{oil}/SG_{oil}$.

Which of the statements are true? **(a)** I, IV, and V **(b)** All, but II **(c)** All **(d)** I, IV **(e)** I, II, and IV

2.32 If a liquid has a specific gravity of 1.7, what is the density in slugs per cubic feet? What is the specific weight in pounds-force per cubic feet? Answer

2.33 If you have a bulk modulus of elasticity that is a very large number, then a small change in pressure would cause

a. a very large change in volume
b. a very small change in volume

2.34 Dimensions of the bulk modulus of elasticity are Answer

a. the same as the dimensions of pressure/density
b. the same as the dimensions of pressure/volume
c. the same as the dimensions of pressure

2.35 The bulk modulus of elasticity of ethyl alcohol is 1.06×10^9 Pa. For water, it is 2.15×10^9 Pa. Which of these liquids is easier to compress?

a. ethyl alcohol
b. water

2.36 A liquid is compressed. The change in pressure is 1000 psi. The change in density is 0.02%.

What is the bulk modulus in psi? Answer

(a) 2E6 **(b)** 5E6 **(c)** 5E5 **(d)** 5E4 **(e)** 2E6

2.37 Start with the premise $E_v = -V\dfrac{\partial p}{\partial V}$ and prove that SS

$E_v = \rho\dfrac{\partial p}{\partial \rho}$. (Def: A *premise* is a statement that you believe is true.)

2.38 (T/F) Air is compressible and seawater is incompressible. Answer

2.39 (T/F) If $E_v \downarrow$, then compressibility ↑.

2.40 A pressure of 4×10^6 N/m^2 is applied to a body of water that initially filled a 4300 cm^3 volume. Estimate its volume after the pressure is applied. Answer

Pressure and Shear Stress (§2.4)

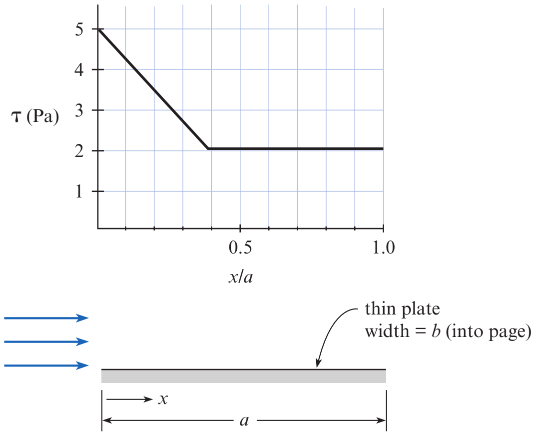
2.41 Shear stress has dimensions of

a. force/area
b. dimensionless

2.42 Water is flowing over a thin plate. The length of the plate is $a = 70$ cm. The width of the plate is $b = 20$ cm. The plot shows the shear stress acting on the top of the plate.

In units of newtons, what is the shear force acting on the top of the plate? Answer

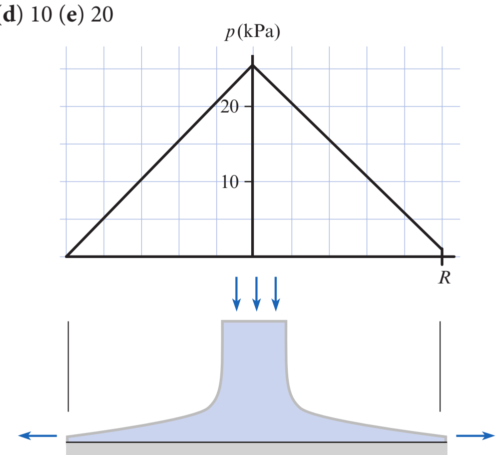
(a) 2.0 **(b)** 0.2 **(c)** 1.6 **(d)** 0.8 **(e)** 0.4



Problem 2.42

2.43 A round jet of water hits a metal plate creating the pressure distribution that is shown. The radius is $R = 40$ mm

In SI units, what is the pressure force on the plate? (**a**) 40 (**b**) 80 (**c**) 5 (**d**) 10 (**e**) 20



Problem 2.43

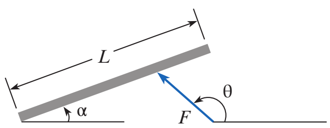
2.44 A liquid is flowing in a round pipe. The liquid exerts a constant shear stress of $\tau = 30$ Pa on the inside wall of the pipe. The pipe cross-sectional area is 700 cm^2. The pipe length is 20 m.

In SI units, what is the shear force on the pipe? Answer

(**a**) 490 (**b**) 330 (**c**) 210 (**d**) 560 (**e**) 120

SS **2.45** A flowing fluid creates a pressure and a shear stress distribution on a square plate. The resultant force is $F = 170$ N. The length of one side of the plate is $L = 70$ cm. The angles are $\alpha = 20°$ and $\theta = 140°$.

What is the average pressure in Pa gage? (**a**) 170 (**b**) 340 (**c**) 420 (**d**) 520 (**e**) 300



Problem 2.45

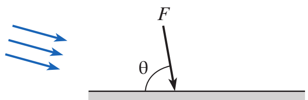
SS **2.46** (T/F) In general, pressure p is given by F_n/A, where F_n is a normal force that acts on an area A. Answer

SS **2.47** (T/F) In general, the force due to pressure F_p is given by $F_p = pA$, where p is pressure and A is area.

2.48 A flowing fluid creates a pressure and a shear stress distribution on the top of a square plate. The resultant force is $F = 220$ N. The length of one side of the plate is $L = 120$ cm. The angle is $\theta = 80°$.

In pascals, what is the average shear stress acting on the top of the plate? Answer

(**a**) 77 (**b**) 17 (**c**) 67 (**d**) 47 (**e**) 27



Problem 2.48

The Viscosity Equation (§2.5)

2.49 The dimensions of viscosity are:

(**a**) F/LT (**b**) FT^2/L (**c**) FL/T (**d**) FT/L^2 (**e**) F/L^2

2.50 Consider the following statements:

I. $[\tau] = F/L^2$

II. $[\tau] = M/L \cdot T$

III. $[\mu] = F \cdot T/L^2$

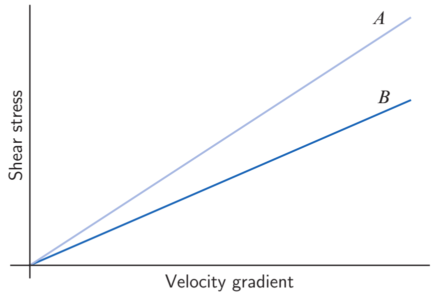
IV. $[\mu] = M/L \cdot T$

V. $[dV/dy] = L/T$

Which statements are true? Answer

(a) All (b) All except V (c) I, III (d) I, III, and IV (e) I, II, and III

2.51 (T/F) If line A is for water at temperature T_A and line B is for water at T_B, then $T_A < T_B$ SS



Problem 2.51

2.52 For the velocity gradient dV/dy Answer

a. the coordinate axis for dy is parallel to the velocity vector

b. the coordinate axis for dy is perpendicular to the velocity vector

2.53 The no-slip condition

a. only applies to ideal flow

b. only applies to rough surfaces

c. means velocity, V, is zero at the wall

d. means velocity, V, is the velocity of the wall

2.54 Which of these materials will flow (deform) with even a small shear stress applied? Answer

a. a Bingham plastic

b. a Newtonian fluid

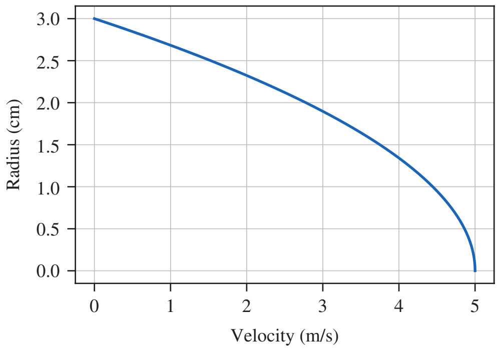
2.55 At a point in a flowing fluid, the shear stress is 3×10^{-4} psi, and the velocity gradient is 1 s^{-1}.

a. What is the viscosity in traditional units?

b. Convert this viscosity to SI units.

c. Is this fluid more or less viscous than water?

2.56 The figure shows a velocity profile in a round pipe for a fluid with a kinematic viscosity of 1×10^{-6} m^2/s and a density of 1000 kg/m^3.



Problem 2.56

The magnitude of the shear stress at the wall in SI units is Answer

(a) 0.062 **(b)** 0.0015 **(c)** 0.33 **(d)** 3.00 **(e)** 6.20

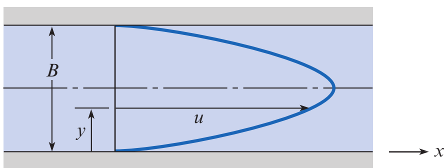
2.57 The velocity distribution for water (20°C) near a wall is given by $u = a(y/b)^{1/6}$, where $a = 10$ m/s, $b = 2$ mm, and y is the distance from the wall in mm. Determine the shear stress in the water at $y = 1$ mm.

2.58 A liquid flows between parallel boundaries as shown. The velocity distribution near the lower wall is given in the following table:

y (mm)	*V* (m/s)
0.0	0.00
1.0	1.00
2.0	1.99
3.0	2.98

a. If the viscosity of the liquid is 10^{-3} N·s/m^2, what is the maximum shear stress in the liquid?

b. Where will the minimum shear stress occur? Answer



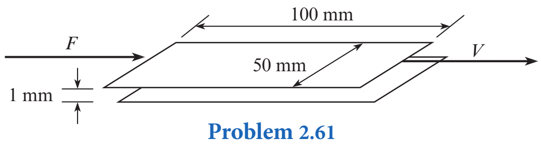
Problems 2.58, 2.59

SS **2.59** As shown, glycerin at 20°C is flowing in a channel, and the pressure gradient dp/dx is −1.2 kPa/m. What are the velocity and shear stress at a distance of 11 mm from the wall if the space B between the walls is 5.0 cm? What are the shear stress and velocity at the wall? The velocity distribution for viscous flow in a channel is

$$u = -\frac{1}{2\mu}\frac{dp}{dx}(By - y^2)$$

2.60 Two plates are separated by a 1/4 in. space. The lower plate is stationary; the upper plate moves at a velocity of 12 ft/s. Oil (SAE 10W-30, 150°F), which fills the space between the plates, has the same velocity as the plates at the surface of contact. The variation in velocity of the oil is linear. What is the shear stress in the oil? Answer

2.61 The sliding plate viscometer shown below is used to measure the viscosity of a fluid. The top plate is moving to the right with a constant velocity of V = 10 m/s in response to a force of F = 3 N. The bottom plate is stationary. What is the viscosity of the fluid? Assume a linear velocity distribution. SS

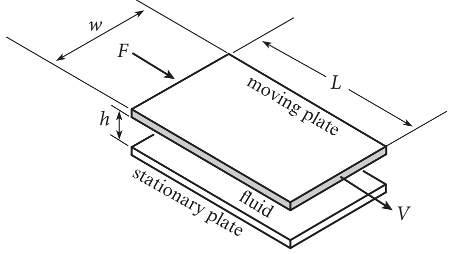


Problem 2.61

2.62 Viscosity is being measured with the sliding plate viscometer that is shown. The parameters are: $F = 5$ N, $h = 0.5$ mm, $L = 20$ cm, $w = 10$ cm, and $V = 7$ m/s.

In mPs · s, the viscosity of the fluid is Answer

(a) 2 **(b)** 84 **(c)** 27 **(d)** 18 **(e)** 8



Problem 2.62

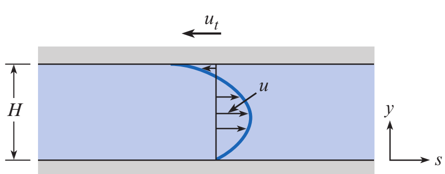
2.63 A laminar flow occurs between two horizontal parallel plates under a pressure gradient dp/ds (dp/ds is a constant and the sign of dp/ds is negative). The upper plate moves left with a speed u_t. The expression for local velocity u is given as

$$u = -\frac{1}{2\mu}\frac{dp}{ds}(Hy - y^2) - u_t\frac{y}{H}$$

a. Is the magnitude of the shear stress greater at the moving plate ($y = H$) or at the stationary plate ($y = 0$)?

b. Derive an expression for the y position of zero shear stress.

c. Derive an expression for the plate speed u_t required to make the shear stress zero at $y = 0$.



Problem 2.63

2.64 For the given situation, which equation gives the magnitude of the shear stress acting on the wall? Answer

(a) $2\mu V_o/R$ **(b)** $4\mu V_o/R$ **(c)** $\mu V_o/2R$ **(d)** $\mu V_o/4R$ **(e)** $\mu V_o/16R$

SITUATION

Statements (a) to (c) are true,

a. A Newtonian fluid is flowing in a round pipe

b. The flow is fully developed

c. The flow is laminar

Thus, the velocity profile is given by

$$V(r) = V_o(1 - (r/R)^2)$$

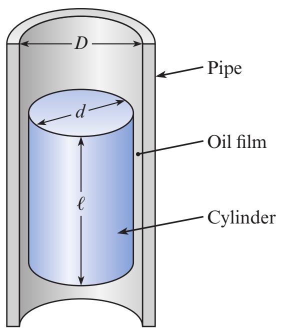
where

a. $V(r)$ = the fluid velocity at radius r (m/s)

b. V_0 = the fluid velocity at the center of the pipe which is $V(r = 0)$ (m/s)

c. R = the radius of the pipe (m)

2.65 A cylinder is falling inside a pipe that is filled with oil, as depicted in the figure. The small space between the cylinder and the pipe is lubricated with an oil film that has viscosity μ. Derive a formula for the steady rate of descent of a cylinder with weight W, diameter d, and length ℓ sliding inside a vertical smooth pipe that has inside diameter D. Assume that the cylinder is concentric with the pipe as it falls. Use the general formula to find the rate of descent of a cylinder 100 mm in diameter that slides inside a 100.5 mm pipe. The cylinder is 200-mm long and weighs 15 N. The lubricant is SAE 20W oil at 10°C.



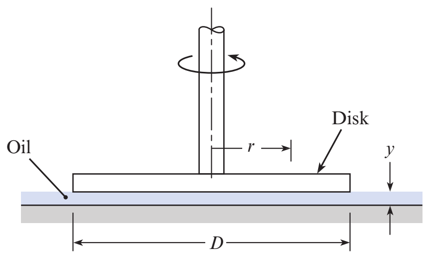
Problem 2.65

2.66 The device shown consists of a disk that is rotated by a shaft. The disk is positioned very close to a solid boundary. Between the disk and the boundary is viscous oil. Answer

a. If the disk is rotated at a rate of 1 rad/s, what will be the ratio of the shear stress in the oil at $r = 2$ cm to the shear stress at $r = 3$ cm?

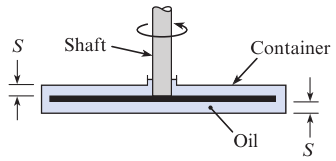
b. If the rate of rotation is 2 rad/s, what is the speed of the oil in contact with the disk at $r = 3$ cm?

c. If the oil viscosity is 0.01 N·s/m^2 and the spacing y is 2 mm, what is the shear stress for the conditions noted in part (b)?



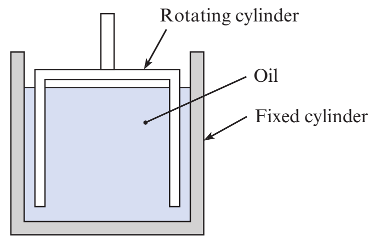
Problem 2.66

2.67 Some instruments having angular motion are damped by means of a disk connected to the shaft. The disk, in turn, is immersed in a container of oil, as shown. Derive a formula for the damping torque as a function of the disk diameter D, spacing S, rate of rotation ω, and oil viscosity μ. SS



Problem 2.67

2.68 One type of viscometer involves the use of a rotating cylinder inside a fixed cylinder. The gap between the cylinders must be very small to achieve a linear velocity distribution in the liquid. (Assume the maximum spacing for proper operation is 0.05 in.) Design a viscometer that will be used to measure the viscosity of motor oil from 50°F to 200°F.



Problem 2.68

Surface Tension (§2.6)

2.69 Surface tension (select all that apply)

a. only occurs at an interface, or surface

b. has dimensions of energy/area

c. has dimensions of force/area

d. has dimensions of force/length

e. depends on adhesion and cohesion

f. varies as a function of temperature

2.70 (T/F) Hot water is better for washing clothes because (a) the surface tension of water decreases as temperature increases, and (b) decreasing σ causes water to more readily soak into the clothing. Answer

2.71 Consider a drop of water on a horizontal solid surface. SS

a. Sketch the drop on a hydrophobic surface and label the contact angle.

b. Sketch the drop on a hydrophilic surface and label the contact angle.

c. List three examples of hydrophobic surfaces.

d. List three examples of hydrophilic surfaces.

2.72 (T/F) A capillary tube is used in water. If the diameter of this tube decreases, then the capillary rise increases linearly. SS

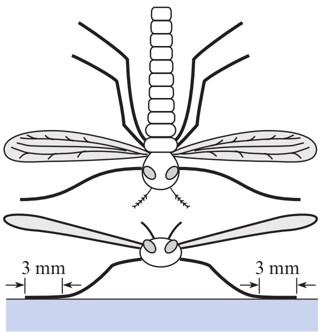
Answer

2.73 Which of the following is the formula for the gage pressure within a very small spherical droplet of water?

(a) $p = \sigma/d$ (b) $p = 4\sigma/d$ (c) $p = 8\sigma/d$

SS **2.74** A spherical soap bubble has an inside radius R, a film thickness t, and a surface tension σ. Derive a formula for the pressure within the bubble relative to the outside atmospheric pressure. What is the pressure difference for a bubble with a 4-mm radius? Assume σ is the same as for pure water. Answer

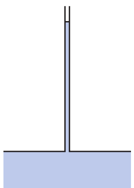
2.75 A water bug is suspended on the surface of a pond by surface tension (water does not wet the legs). The bug has six legs, and each leg is in contact with the water over a length of 3 mm. What is the maximum mass (in grams) of the bug if it is to avoid sinking?



Problem 2.75

2.76 A water column in a glass tube is used to measure the pressure in a pipe. The tube is 1/2 in. in diameter. How much of the water column is due to surface-tension effects? What would be the surface-tension effects if the tube were 1/8 in. or 1/16 in. in diameter? Answer

2.77 Calculate the maximum capillary rise of water between two vertical glass plates spaced 1 mm apart.



Problem 2.77

2.78 An engineer is designing a manometer. The capillary rise is limited to 1 mm. The fluid is kerosene. The parameters are $SG = 0.8$ and $\sigma = 0.03$ N/m.

In mm, what is the smallest diameter of tube that can be used? Answer

(**a**) 10 (**b**) 15 (**c**) 20 (**d**) 25 (**e**) 30

2.79 What is the pressure within a $d = 0.75$ mm spherical droplet of water, relative to the atmospheric pressure outside the droplet?

SS **2.80** By measuring the capillary rise of water in a tube, one can build an instrument to measure temperature. The surface tension of water varies linearly with temperature from 0.0756 N/m at 0°C to 0.0589 N/m at 100°C. Size a tube (specify diameter and length) that uses capillary rise of water to measure temperature in the range from 0°C to 100°C. Is this design for a thermometer a good idea?

2.81 Capillary rise can be used to describe how far water will rise above a water table because the interconnected pores in the soil act like capillary tubes. This means that deep-rooted plants in the desert need only grow to the top of the "capillary fringe" in order to get water; they do not have to extend all the way down to the water table.

a. Assuming that interconnected pores can be represented as a continuous capillary tube, how high is the capillary rise in a soil consisting of a silty soil, with a pore diameter of 10 μm?

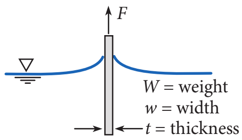
b. Is the capillary rise higher in fine sand (pore diam. approx. 0.1 mm), or in fine gravel (pore diam. approx. 3 mm)?

c. Root cells extract water from soil using capillarity. For root cells to extract water from the capillary zone, do the pores in a root need to be smaller than, or greater than, the pores in the soil? Ignore osmotic effects.

2.82 Consider a soap bubble 2 mm in diameter and a droplet of water, also 2 mm in diameter. If the value of the surface tension for the film of the soap bubble is assumed to be the same as that for water, which has the greater pressure inside it? Answer

(a) the bubble, (b) the droplet, (c) neither—the pressure is the same for both.

2.83 Surface tension can be found by measuring the force to lift a plate:



Problem 2.83

If the buoyant force on the plate is neglected, the surface tension is given by $\sigma =$

a. $\dfrac{W + F}{(w + t)}$

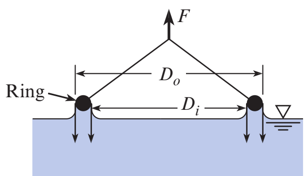
b. $\dfrac{W}{Fwt}$

c. $\dfrac{W - F}{2wt}$

d. $\dfrac{W + F}{2wt}$

e. $\dfrac{W - F}{2(w + t)}$

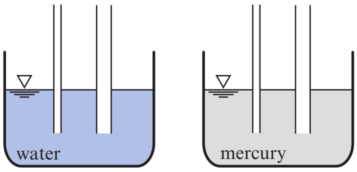
2.84 The surface tension of a liquid is being measured with a ring as shown. The ring has an outside diameter of 10 cm and an inside diameter of 9.5 cm. The mass of the ring is 10 g. The force required to pull the ring from the liquid is the weight corresponding to a mass of 16 g. What is the surface tension of the liquid (in N/m)? Answer SS



Problem 2.84

Vapor Pressure, Boiling, and Cavitation (§2.7)

2.85 Sketch the capillary rise or depression in each of the four capillary tubes. Describe the significant feature of your completed sketch.



Problem 2.85

2.86 If liquid water at 30°C is flowing in a pipe and the pressure drops to the vapor pressure, what happens in the water? Answer

a. the water begins condensing on the walls of the pipe

b. the water boils

c. the water flashes to vapor

2.87 If a pot of water boils at 90°C, what is the local atmospheric pressure in units of kPa absolute? SS

(**a**) 70 (**b**) 80 (**c**) 85 (**d**) 90 (**e**) 95

2.88 Water is at 30°C and the pressure is lowered until boiling is observed. What is the pressure? Answer

2.89 A student in the laboratory plans to exert a vacuum in the air space above the surface of water in a closed tank. She plans for the absolute pressure in the air to be 12,300 Pa. The temperature in the lab is 20°C. Will water boil under these circumstances?

2.90 The vapor pressure of water at 100°C is 101 kN/m^2. The vapor pressure of water decreases approximately linearly with decreasing temperature at a rate of 3.1 kN/m^2/°C. Calculate the boiling temperature of water at an altitude of 3000 m, where the atmospheric pressure is 69 kN/m^2 absolute. Answer

2.91 Pressure cooking involves cooking food in a sealed container at a gage pressure between 0.7 bar and 1.0 bar. The main benefit of pressure cooking is faster cooking as compared to cooking on a stove top. Why is pressure cooking faster?

CHAPTER**TWO**

Fluid Properties

CHAPTER ROAD MAP This chapter introduces ideas for idealizing real-world problems, introduces fluid properties, and presents the viscosity equation. The topics from this chapter are useful for solving all fluid problems, for example problems that are associated with open channels (Fig. 2.1).

FIGURE 2.1
This photo shows engineers observing a **flume**, which is an artificial channel for conveying water. This flume is used to study sediment transport in rivers. (Photo courtesy of Professor Ralph Budwig of the Center for Ecohydraulics Research, University of Idaho.)

LEARNING OUTCOMES

SYSTEM, STATE, AND PROPERTY (§2.1)

- Define system, boundary, surroundings, state, steady state, process, and property.

FINDING FLUID PROPERTIES (§2.2)

- Look up appropriate values of fluid properties and document your work.
- Define each of the eight common fluid properties.

DENSITY TOPICS (§2.3)

- Know the main ideas about specific gravity.
- Explain the constant density assumption and make decisions about whether or not this assumption is valid.
- Determine changes in the density of water corresponding to a pressure change or a temperature change.

STRESS (§2.4)

- Define stress, pressure, and shear stress.
- Explain how to relate stress and force.
- Describe each of the seven common fluid forces.

THE VISCOSITY EQUATION (§2.5)

- Define the velocity gradient and find values of the velocity gradient.
- Describe the no-slip condition.
- Explain the main ideas of the viscosity equation.
- Solve problems that involve the viscosity equation.
- Describe a Newtonian and non-Newtonian fluid.

SURFACE TENSION (§2.6)

- Know the main ideas about surface tension.
- Solve problems that involve surface tension.

VAPOR PRESSURE (§2.7)

- Explain the main ideas of the vapor pressure curve.
- Find the pressure at which water will boil.

2.1 System, State, and Property

The vocabulary introduced in this section is useful for solving problems. In particular, these ideas allow engineers to describe problems in ways that are precise and concrete.

A **system** is the specific entity that is being studied or analyzed by the engineer. A system can be a collection of matter, or it can be a region in space. Anything that is not part of the system is considered to be part of the **surroundings**. The **boundary** is the imaginary surface that separates the system from its surroundings. For each problem you solve, it is your job as the engineer to select and identify the system that you are analyzing.

EXAMPLE. For the flume shown in Fig. 2.1, the water that is situated inside the flume could be defined as the system. For this system, the surroundings would be the flume walls, the air above the flume, and so on. Notice that *engineers are specific about what the system is, what the surroundings are, and what boundary is.*

EXAMPLE. Suppose an engineer is analyzing the air flow from a tank being used by a SCUBA diver. As shown in Fig. 2.2, the engineer might select a system comprised of the tank and the regulator. For this system, everything that is external to the tank and regulator is the surroundings. Notice that *the system is defined with a sketch* because this is sound professional practice.

If you make a wise choice when you select a system, you increase your probability of getting an accurate solution, and you minimize the amount of work you need to do. Although the choice of system must fit the problem at hand, there are often multiple possibilities for which system to select. This topic will be revisited throughout this textbook as various kinds of systems are introduced and applied.

Systems are described by specifying numbers that characterize the system. The numbers are called properties. A **property** is a characteristic of a system that depends only on the present conditions within the system.

EXAMPLE. In Fig. 2.2, some examples of properties are as follows:

- The pressure of the air inside the tank
- The density of air inside the tank
- The weight of the system (tank plus air plus regulator)

The **state** of a system means the condition of the system as defined by specifying its properties. When a system changes from one state to another state, this is called a **process**. When the properties of a system are constant with time, the system is said to be at **steady state**.

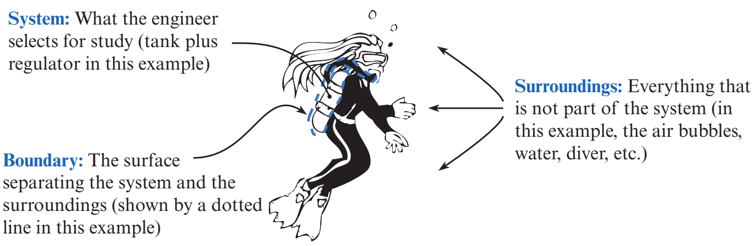
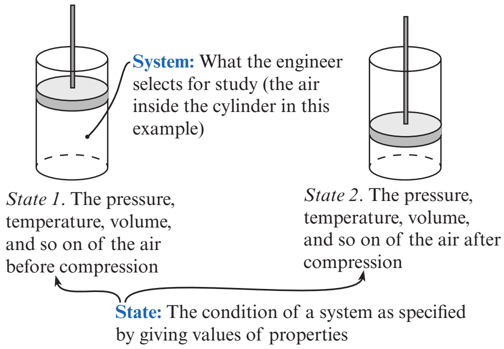


FIGURE 2.2
Example of a system, its surroundings, and the boundary.

FIGURE 2.3

Air in a cylinder being compressed by a piston. State 1 is a label for the conditions of the system prior to compression. State 2 is a label for the conditions of the system after compression.



EXAMPLE. Fig. 2.3 shows air being compressed by a piston in a cylinder. The air inside the cylinder is defined as the system. At state 1, the conditions of the system are defined by specifying properties such as pressure, temperature, and density. Similarly, state 2 is defined by specifying these same properties.

EXAMPLE. When air is compressed (Fig. 2.3), this is a process because the air (i.e., the system) has changed from one set of conditions (state 1) to another set of conditions (state 2). Engineers label processes that commonly occur. For example, an *isothermal process* is one in which the temperature of the system is held constant, and an *adiabatic process* is one in which there is no heat transfer between the system and the surroundings.

Properties are often classified into categories. Two examples of categories are as follows:

- **Kinematic properties**. These properties characterize the motion of your system. Examples include position, velocity, and acceleration.
- **Material properties**. These properties characterize the nature of the materials in your system. Examples include viscosity, density, and specific weight.

2.2 Looking Up Fluid Properties

One of the most common tasks that engineers perform is to look up material properties. This section presents ideas that help you perform this task well.

Overview of Properties

Although there are many fluid properties, there are only a few that you need often. These properties are summarized in Table 2.1. Notice that the properties are organized into three groups.

Group #1: Weight and Mass Properties. Three properties (ρ, γ, and SG) are used to characterize weight or mass. In general, you can find one of these properties and then calculate either of the other two using the following equations: $\gamma = \rho g$ and $SG = \rho/\rho_{H_2O,\,(4°C)} = \gamma/\gamma_{H_2O,\,(4°C)}$

Group #2: Properties for Characterizing Viscosity. To characterize friction-like effects in flowing fluids, engineers use viscosity, μ. Viscosity has two common synonyms: d*ynamic viscosity* and *absolute viscosity*. In addition to viscosity, engineers use another term, kinematic viscosity, which is given the symbol ν. **Kinematic viscosity** is defined by

$$\nu = \frac{\mu}{\rho} \tag{2.1}$$

An easy way to distinguish between μ and ν is to check units or dimensions because $[\mu] = M/(L \cdot T)$ and $[\nu] = L^2/T$. Regarding viscosity, we recommend that you build a physical feel for this property by finding examples that make sense to you. In this spirit, the following

TABLE 2.1 Summary of Fluid Properties

	Property	Units (SI)	Temperature Effects	Pressure Effects (common trends)	Notes
Mass and Weight Properties	**Density** (ρ): Ratio of mass to volume at a point	$\frac{kg}{m^3}$	$\rho\downarrow$ as $T\uparrow$ if the gas is free to expand	$\rho\uparrow$ as $p\uparrow$ if a gas is compressed	• *Air.* Find ρ in Table F.4 or Table A.3. • *Other Gases.* Find ρ in Table A.2. • *Caution!* Tables for gases are for $p = 1$ atm. For other pressures, find ρ using the ideal gas law.
			$\rho\downarrow$ as $T\uparrow$ for liquids	A liquid is usually idealized with ρ independent of pressure	• *Water.* Find ρ in Table F.5 or Table A.5. • *Note.* For water, $\rho\uparrow$ as $T\uparrow$ for temperatures from 0 to about 4°C. Maximum density of water is at $T \approx 4$°C. • *Other Liquids.* Find ρ in Table A.4.
	Specific Weight (γ): Ratio of weight to volume at a point	$\frac{N}{m^3}$	$\gamma\downarrow$ as $T\uparrow$ if fluid is free to expand	Gas: $\gamma\uparrow$ as $p\uparrow$ if a gas is compressed Liquid: a liquid is usually idealized with γ independent of pressure	• Use same tables as for density. • ρ and γ can be related using $\gamma = \rho g$. • *Caution!* Tables for gases are for $p = 1$ atm. For other pressures, find γ using the ideal gas law and $\gamma = \rho g$. • Typically, γ is not used for gases.
	Specific Gravity (S or SG): Ratio of (density of a liquid) to (density of water at 4°C)	none	$SG\downarrow$ as $T\uparrow$	A liquid is usually idealized with SG independent of pressure	• Find SG data in Table A.4. • SG is used for liquids, not commonly used for gases. • Density of water (at 4°C) is listed in Table F.6. • $SG = \gamma/\gamma_{H_2O,\,4°C} = \rho/\rho_{H_2O,\,4°C}$.
Properties Related to Viscosity	**Viscosity** (μ): A property that characterizes resistance to shear stress and fluid friction	$\frac{N \cdot s}{m^2}$	$\mu\uparrow$ as $T\uparrow$ for a gas	A gas is usually idealized with μ independent of pressure	• *Air:* Find μ in Table F.4, Table A.3, Fig. A.2. • *Other gases:* Find properties in Table A.2, Fig. A.2. • *Hint:* Viscosity is also known as dynamic viscosity and absolute viscosity. • *Caution!* Avoid confusing viscosity and kinematic viscosity; these are different properties.
			$\mu\downarrow$ as $T\uparrow$ for a liquid	A liquid is usually idealized with μ independent of pressure	• *Water:* Find μ in Table F.5, Table A.5, Fig. A.2. • *Other Liquids.* Find μ in Table A.4, Fig. A.2.
	Kinematic Viscosity (ν): A property that characterizes the mass and viscous properties of a fluid	$\frac{m^2}{s}$	$\nu\uparrow$ as $T\uparrow$ for a gas	$\nu\uparrow$ as $p\uparrow$ for a gas	• *Air:* Find μ in Table F.4, Table A.3, Fig. A.3. • *Other gases:* Find properties in Table A.2, Fig. A.3. • *Caution!* Avoid confusing viscosity and kinematic viscosity; these are different properties. • *Caution!* Gas tables are for $p = 1$ atm. For other pressures, look up $\mu = \mu(T)$, then find ρ using the ideal gas law, and calculate ν using $\nu = \mu/\rho$.
			$\nu\downarrow$ as $T\uparrow$ for a liquid	A liquid is usually idealized with ν independent of pressure	• *Water:* Find ν in Table F.5, Table A.5, Fig. A.3. • *Other liquids:* Find ν in Table A.4, Fig. A.3.
Miscellaneous Properties	**Surface Tension** (σ): A property that characterizes the tendency of a liquid surface to behave as a stretched membrane	$\frac{N}{m}, \frac{J}{m^2}$	$\sigma\downarrow$ as $T\uparrow$ for a liquid	A liquid is usually idealized with σ independent of pressure	• *Water:* Find σ in Fig. 2.18. • *Other liquids:* Find σ in Table A.4. • Surface tension is a property of liquids (not gases). • Surface tension is greatly reduced by contaminants or impurities.
	Vapor Pressure p_v: The pressure at which a liquid will boil	Pa	$p_v\uparrow$ as $T\uparrow$ for a liquid	Not applicable	• *Water:* Find p_v in Table A-5.
	Bulk Modulus of Elasticity E_v: A property that characterizes the compressibility of a fluid	Pa	Not presented here	Not presented here	• *Ideal gas (isothermal process):* $E_v = p =$ pressure. • *Ideal gas (adiabatic process):* $E_v = kp$; $k = c_p/c_v$. • *Water:* $E_v \approx 2.2 \times 10^9$ Pa.

are two examples that we like: **Example**. Honey has a much higher value of viscosity than does liquid water. Thus, it is harder to push a spoon through a bowl of honey than it is to push a spoon through a bowl of water. **Example**. If you try to pour motor oil out of its container on a cold day, the oil will pour very slowly because the value of viscosity is high. If you heat the motor oil up, then the value of viscosity decreases and the motor oil is easier to pour.

Group #3: Miscellaneous Properties. The last three properties (σ, p_v, and E_v) are used for specialized problems, which are described later in this chapter.

Property Variation with Temperature and Pressure

In general, the value of a fluid property varies with both temperature and pressure. These variations are summarized in the third and fourth columns of Table 2.1. The notation $\rho\downarrow$ as $T\uparrow$ is a shorthand for saying that density goes down as temperature rises. The shading is used to distinguish between gases and liquids. For example, in the row for viscosity, the text in the shaded region indicates that the viscosity of a liquid decreases with a temperature rise. Similarly, the text that is not shaded indicates that the viscosity of a gas increases with a temperature rise.

Notice that the values of many properties (e.g., density of a liquid, viscosity of a gas) can be idealized as being independent of pressure. However, every property varies with temperature.

Finding Fluid Properties

We built Table 2.1 to summarize the details needed for looking up fluid properties. For example, the last column of Table 2.1 lists locations in the text where values of properties are tabulated. In the examples that follow, the key details used to solve the problems came from Table 2.1.

EXAMPLE. What is the density of kerosene (SI units) at room conditions? **Reasoning**. (1) At room conditions, kerosene is a liquid. (2) Liquid properties can be found in Table A.4. **Conclusion**. $\rho = 814\ \text{kg/m}^3$ (20°C and 1.0 atm).

EXAMPLE. In traditional units, what is the dynamic viscosity of gasoline at 150°F? **Reasoning**. (1) Gasoline is a liquid. (2) Because the goal is to find "dynamic viscosity," note that this property is also called "viscosity" and "absolute viscosity." (3) Viscosity of liquids as a function of temperature can be found in Fig. A2.* (4) Read Fig. A.2† to find that μ is approximately 4E-6 lbf·ft/s^2. (5) *Note:* Given that the vertical scale on Fig. A.2 is hard to read, the value of μ was reported to one significant figure. **Conclusion**. μ = 4E-6 lbf·ft/s^2.

EXAMPLE. What is the specific weight of air at 20°C and 3.0 atmospheres of pressure (gage)? **Reasoning**. (1) Specific weight is related to density via $\gamma = \rho g$. (2) Density can be calculated with the IGL: $\rho = p/RT$ = (4.053E5 Pa)/(287 J/kg·K)(293.2K) = 4.817 kg/m^3. (3) Thus, $\gamma = \rho g$ = (4.817 kg/m^3)(9.807 m/s^2) = 47.2 N/m^3. **Conclusion**. γ = 47.2 N/m^3.

Quality in Documentation

Voice of the Engineer. *Document your technical work so well that you or a colleague could retrieve the work three years in the future and easily figure out what was done*. **Rationale**. (1) When you build effective documentation, this provides you with a structure that promotes good thinking. (2) In professional practice, you can use your documentation to recall the technical details months or years after a project is completed. (3) Thorough documentation helps you protect

*Fig. A-2 has a semilog scale. As an engineer, you need to be skilled at reading data from a log scale and skilled at plotting on log scales. If you have not yet gained these skills, we recommend that you ask your teacher for assistance, or consult the Internet.

†We recommend using a ruler and drawing straight pencil lines whenever you are using a log scale. This allows you to read data more accurately.

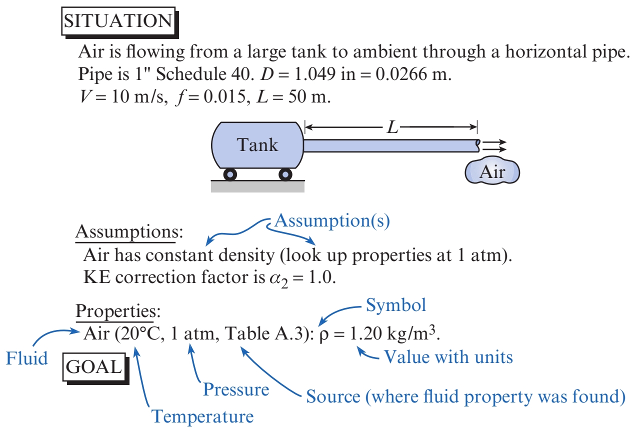


FIGURE 2.4
An example of how to document fluid properties.

your intellectual property and also helps protect your reputation (and your pocketbook) if you are involved in a legal conflict.

Most people (including us) dislike documentation, but well-crafted documentation saves abundant amounts of time and effort, so most professionals document their work well. We teach and practice a rule called the *5% rule,* which is this: *Document your technical work in real time (no rewriting allowed*) and do this so effectively that the maximum amount of extra time you need is 5% of your total time.*

Regarding quality in the documentation of fluid properties, we recommend six practices; see Fig. 2.4 for an example.

1. List the name of the fluid.
2. List the temperature and pressure at which the property was reported by the source. **Rationale**. In general, fluid properties vary with both temperature and pressure, so these values need to be listed. Also, the state (gas, liquid, or solid) depends on temperature and pressure.
3. Cite the source of the fluid property. **Rationale**. Property data are often inaccurate; thus, citing your source is a way to provide evidence that your technical work is trustworthy.
4. List relevant assumptions.
5. List the value of and the units of the fluid property.
6. Be concise; write down the minimum amount of information required to get the job done.

2.3 Specific Gravity, Constant Density, and the Bulk Modulus

This section presents three topics (specific weight, the constant density assumption, and the bulk modulus) that are related to fluid density. The first two topics are very important; the last topic is of secondary importance.

Specific Gravity

Specific gravity is useful for characterizing the density or specific weight of a material. **Specific gravity** (represented by S or SG) is defined as the ratio of the density of a material to the density of a reference material. The reference material used in this text is liquid water at 4°C. Thus,

$$SG = \frac{\rho}{\rho_{H_2O \text{ at } 4^\circ C}} \tag{2.2}$$

*This is the goal; even the best of us need to rewrite our work every now and then. What you want to avoid is getting into the habit of being sloppy and then rewriting your work.

Because $\gamma = \rho g$, Eq. (2.2) can be multiplied by g to give

$$SG = \frac{\rho}{\rho_{H_2O \text{ at } 4^\circ C}} = \frac{\gamma}{\gamma_{H_2O \text{ at } 4^\circ C}} \tag{2.3}$$

Useful Facts

- If $SG < 1$, then the material will often float on water (e.g., oil, gasoline, wood, and Styrofoam). If $SG > 1$, the material will generally sink (e.g., a piece of potato, concrete, or steel).
- If you add oil (e.g., $SG = 0.9$) to water ($SG = 1.0$), the oil will float on top of the water. This is because oil and water are *immiscible,* which means that they are not capable of being mixed. If you add alcohol (e.g., $SG = 0.8$) to water, the alcohol and water will mix; fluids that are capable of mixing are *miscible*.
- The properties ρ, SG, and γ are related. If you know one of these properties, you can easily calculate the other two by applying Eq. (2.3).
- Values of ρ and γ for water at 4°C are listed in Table F.6 (front pages).
- Values of SG for liquids are listed in Table A.4 (appendix).
- SG is commonly used for solids and liquids but is rarely used for gases. This textbook does not use SG for gases.

Recommended working knowledge:

- SG (petroleum products; e.g., gasoline or oil) $\approx$ 0.7 to 0.9
- SG (seawater) ≈ 1.03; SG (mercury) ≈ 13.6
- SG (steel) ≈ 7.8; SG (aluminum) ≈ 2.6, SG (concrete) $\approx$ 2.2 to 2.4

The Constant Density Assumption

If you can justify the assumption that a fluid has a constant density, this will make your analysis much simpler and faster. This section presents information about this assumption.

The **constant density assumption** means that you can idealize the fluid involved in your problem as if the density was constant with both position and time. Another way to state the assumption is to say that the density can be assumed to be constant even though temperature, pressure, or both are changing. To say that the constant density assumption is a "sound" or "valid" assumption means that the numbers you calculate in your problem are only impacted in a small way (e.g., by less than 5%) by this assumption.

Useful facts:

- The constant density assumption is applied to most situations in fluid mechanics. Some exceptions to watch out for are as follows:
  - Modeling the properties of air in the atmosphere; see §3.1
  - High speed flows of gases where high speed means velocity above $\approx$ Mach 0.7; see Chapter 12
  - Acoustics, which involves modeling the propagation of sound in liquids and gases
  - Water hammer, which involves surges of pressure in long pipes during transient conditions
- To characterize a density change with respect to a pressure change, engineers often use the *bulk modulus*; this topic is presented in the next subsection.
- The variation of the density of liquid water with respect to temperature is given in Table A.5.

- When flow is steady,* engineers commonly make the following assumptions:
  - **Liquids.** In general, liquids in steady flow are assumed to have constant density.
  - **Gas.** For a gas in steady flow, the density is assumed to be constant if the Mach number† is less than about 0.3.
- When fluid temperatures are changing, it is common to look up a density at an average temperature and then assume that the density is constant. **Example**. If water enters a heat exchanger at 10°C and exits at 90°C, assume that the density is constant and look up the value of density at 50°C in Table A.5.

Bulk Modulus

The bulk modulus of elasticity—which is often shortened to the **bulk modulus**—is a material property that characterizes how compressible or incompressible a material is. The symbol for the bulk modulus in this text is E_v. Other resources use the symbol K for bulk modulus. Some typical values of E_v are as follows:

Typical values of bulk modulus

Material	E_v
Air (isothermal compression)	100 kPa
Air (adiabatic compression)	140 kPa
Kerosene (liquid)	1.3 GPa
Water (liquid)	2.2 GPa
Aluminum (solid)	70 GPa
Steel (solid)	160 GPa

The term $1/E_v$ is a measure of how compressible a material is. For example, water is about 70 times more compressible than steel because the ratio of E_v values is $160/2.2 \approx 70$. Similarly, air is about 18,000 times more compressible than liquid water because $(2.2 \times 10^9)/(120 \times 10^3) \approx 18{,}000$.

The bulk modulus is defined mathematically in terms of pressure p and volume $\mathcal{V}$ by

$$E_v = -\mathcal{V}\frac{\partial p}{\partial \mathcal{V}} \tag{2.4a}$$

Since Eq. (2.4a) is true, the bulk modulus can also be defined using pressure and density by

$$E_v = \rho\frac{dp}{d\rho} \tag{2.4b}$$

The derivation of Eq. (2.4b) from Eq. (2.4a) involves letting $\mathcal{V} = m/\rho$, and then using the chain rule from calculus.

For a liquid or solid, engineers usually model E_v as a constant. For constant E_v, the definition of the derivative can be applied to show that Eqs. (2.4a) and (2.4b) simplify to

$$E_v = \frac{-\Delta p}{\Delta\mathcal{V}/\mathcal{V}} = \frac{\text{change in pressure}}{\text{fractional change in volume}} \tag{2.5a}$$

$$E_v = \frac{\Delta p}{\Delta\rho/\rho} = \frac{\text{change in pressure}}{\text{fractional change in density}} \tag{2.5b}$$

*Steady flow is defined in §4.3.
†The Mach number gives a ratio of the fluid speed to the speed of sound; see Chapter 12.

For a gas, E_v is usually modeled as nonconstant and the value of E_v depends on how the compression or expansion is performed. If the process is isothermal, the IGL (ideal gas law) can be combined with Eq. (2.4a) to show that the bulk modulus is

$$E_v = p \quad \text{(isothermal process)} \tag{2.6a}$$

where p is pressure. If the process is adiabatic (i.e., no heat transfer), then facts from thermodynamics can be used to show that the bulk modulus is

$$E_v = kp \quad \text{(adiabatic process)} \tag{2.6b}$$

where k, the specific heat ratio, is defined in §2.8. Eq. (2.6a) and (2.6b) are valid if a gas is modeled as an ideal gas, but not valid otherwise.

All materials are compressible, but solids and liquids are much less compressible than gases. For most, but not for all situations, materials in the solid or liquid phase can be accurately modeled as if the material was incompressible. While gases are easy to compress, there are many situations in which a gas can be modeled as if the gas itself is incompressible. There are no simplistic rules for deciding if a material should or should not be modeled as compressible versus incompressible.

2.4 Pressure and Shear Stress

When you understand stress, many topics in mechanics become easier. The big picture is that there are only two kinds of stress: *normal stress* and *shear stress*. In fluid mechanics, the normal stress* is nearly always just the fluid pressure; thus, the two kinds of stress are pressure and shear stress.

Definition of Stress

To define stress, we begin by noting that stress acts on material particles. For example, if you bend a beam, the material particles are deformed by normal stress (Fig. 2.5).

Thus, stress is caused by a load acting on a body. An example for a fluid body is shown in Fig. 2.6.

To build a definition of stress, we start by recognizing that the secondary dimensions of stress are force/area:

$$\text{stress} = \frac{\text{force}}{\text{area}} \tag{2.7}$$

Next, visualize the force on one face of a material particle. Resolve this force into a normal component of magnitude ΔF_n and a tangential component of magnitude ΔF_t (Fig. 2.7).

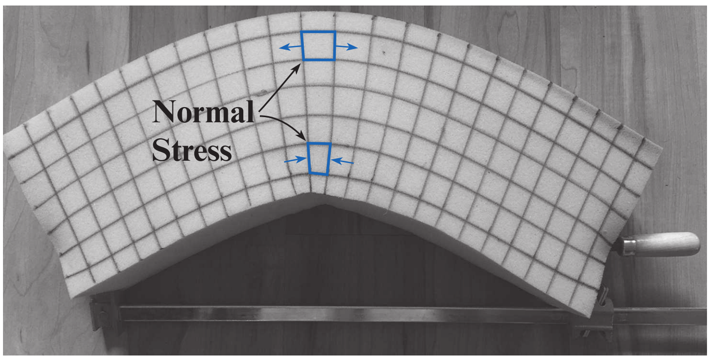


FIGURE 2.5

This figure shows our favorite way to visualize stress in a body. The method goes like this: (1) Select a *body* comprised of a *beam* made of foam. (2) Mark *material particles;* this example uses squares that are 25 mm on a side. (3) *Load* the beam; this example uses a clamp to exert a *bending moment*. (4) Observe how stress has deformed the material particles. (Photo by Donald Elger.)

*There is also a viscous component to the normal stress. However, this term is seldom important, and this topic is best left to more advanced fluids books.

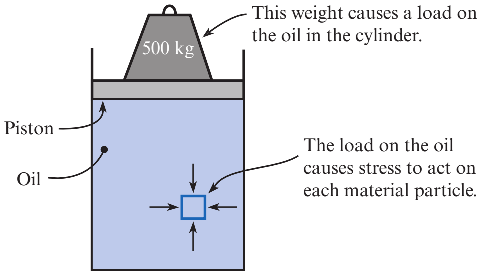


FIGURE 2.6
In this example, a load (i.e., a weight situated on piston) causes stress to act on the oil in a cylinder.

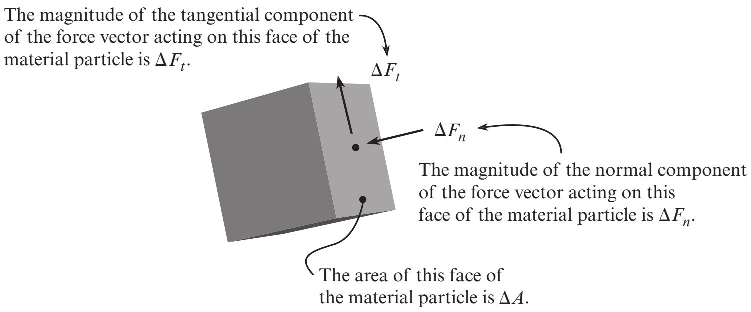


FIGURE 2.7
This sketch shows how the force on one face of a *material particle* can be resolved into a normal force and a tangential force.

Then, the pressure is defined as the ratio of normal force to area:

$$p \equiv \lim_{\Delta A \to 0} \frac{\Delta F_n}{\Delta A} \tag{2.8}$$

And, shear stress is defined as the ratio of shear force to area:

$$\tau \equiv \lim_{\Delta A \to 0} \frac{\Delta F_t}{\Delta A} \tag{2.9}$$

Summary. In mechanics, **stress** is an entity that expresses the forces that material particles exert on each other. Stress is the ratio of force to area at a point and is resolved into two components:

- **Pressure (normal stress).** The ratio of normal force to area.
- **Shear stress.** The ratio of shear force to area.

More advanced textbooks will present additional ideas about stress. For example, stress is often represented mathematically as a second-order tensor. However, these topics are beyond the scope of this text.

Relating Stress to Force

A common problem is how to relate the stress acting on an area to the associated force on the same area. The solution is to integrate the stress distribution as follows.

$$\text{force} = \int_{\text{Area}} \underbrace{\left(\frac{\text{force}}{\text{area}}\right)}_{\text{definition of stress}} dA \tag{2.10}$$

To build the details of the integration, we'll start with a pressure distribution (Fig. 2.8). To represent force as a vector quantity, we select a small area and define a unit vector (Fig. 2.9). The force on the small area is $\mathbf{dF} = -p\mathbf{n}\,dA$. To obtain the force on the body, add up the small forces ($\mathbf{F} = \Sigma\mathbf{dF}$) while letting the size of the small area (i.e., dA) go toward zero. The summation of

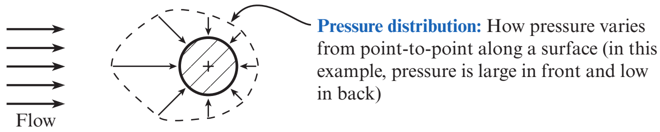


FIGURE 2.8

This figure shows the pressure distribution associated with fluid flowing over a body that has a circular shape. This can represent, for example, how pressure varies around the outside of a round pier submerged in a river.

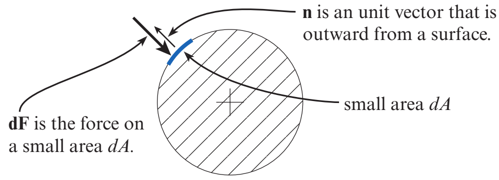


FIGURE 2.9

The pressure force on a small section of area on a cylinder.

small terms is the definition of the integral (§1.4). Thus,

$$\mathbf{F}_p = \int_A -p\mathbf{n}\, dA \tag{2.11}$$

where $\mathbf{F}_p$, called the **pressure force**, represents the net force on the area A due to the pressure distribution. Eq. (2.11) has an important special case:

$$F_p = pA \tag{2.12}$$

The reasoning to prove that Eq. (2.12) is true goes like this: (1) Assume that the area A in Eq. (2.11) represents a flat surface. (2) Assume that the pressure in Eq. (2.11) is constant so that p comes out of the integral. (3) Thus, Eq. (2.11) can be simplified like this: $F_p = p\int_A dA = pA$. Note that the unit vector was omitted because the direction of a pressure force on a flat surface is normal to the surface and directed toward the surface.

Summary. The pressure force is *always given* by the integral of pressure over area, which is $\mathbf{F}_p = \int_A -p\mathbf{n}\, dA$. Only in the special case of uniform pressure acting on a flat surface can you calculate the pressure force by using $F_p = pA$. As always, we recommend that you remember the general equation (i.e., the integral) and then derive $F_p = pA$ whenever this equation is needed.

Now we can tackle the equation for the shear force, which is represented by the symbol F_s or sometimes by F_τ. To build an equation for F_s, we can apply the same logic that was used for the pressure force. Step 1 is to start with a stress distribution (Fig. 2.10). Step 2 is to define a small area and an associated unit vector (Fig. 2.11). Step 3 is to represent the force on the small area as $\mathbf{dF} = \tau\mathbf{t}\, dA$ and then to add up the small forces by using the integral. The final result is

$$F_\tau = \int_A \tau\mathbf{t}\, dA \tag{2.13}$$

where F_τ, the **shear force**, represents the net force on the area A due to the shear stress distribution. If shear stress is constant and the area of integration is a flat surface, then Eq. (2.13) reduces to

$$F_s = \tau A \tag{2.14}$$

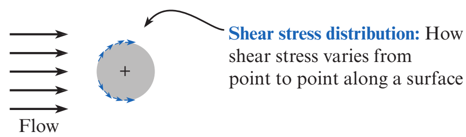


FIGURE 2.10

The image shows how shear stress varies for flow over a circular cylinder.

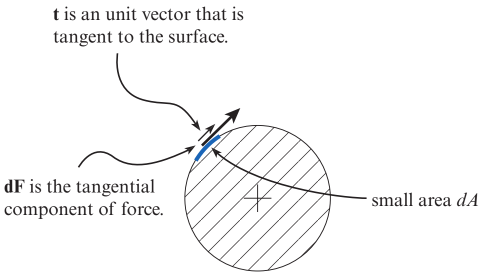


FIGURE 2.11

The shear force on a small section of area on a cylinder.

TABLE 2.2 The Seven Common Fluid Forces

#	Name	Description and Tips	Associated With
1	Pressure force	The force caused by a pressure distribution. Use gage pressure for most problems.	Pressure stress
2	Shear force (viscous force)	The force caused by a shear stress distribution. This force requires the fluid to be flowing.	Shear stress
3	Buoyant force	The force on a submerged or partially submerged body that is caused by the hydrostatic pressure distribution.	Pressure stress
4	Surface tension force	The force caused by surface tension. The common formula is $F = \sigma L$.	Forces between molecules
5	Drag force	When fluid flows over a body, the drag force is the component of the total force that is parallel to the fluid velocity.	Both pressure stress and shear stress
6	Lift force	When fluid flows over a body, the lift force is the component of the total force that is perpendicular to the fluid velocity.	Both pressure stress and shear stress (typically, the effect of shear stress is negligible as compared to pressure stress)
7	Thrust force	The force associated with propulsion, that is, the force caused by a propeller, jet engine, rocket engine, etc.	Both pressure stress and shear stress (typically, the effect of shear stress is negligible as compared to pressure stress)

Summary. The shear force is *always given* by the integral of shear stress over area, which is $\mathbf{F}_s = \int_A \tau \mathbf{t}\, dA$. Only in the special case of a uniform shear stress acting on a flat surface can you calculate the shear force by using $F_s = \tau A$.

The Seven Common Fluid Forces

In fluid mechanics, correct analysis of forces is sometimes difficult. Thus, we'd like to share an idea that we have found to be helpful: *When a force acts between Body #1 (comprised of a fluid) and Body #2 (comprised of any material including another fluid), there are seven common forces that arise. Six of the seven forces are associated with the pressure distribution, the shear stress distribution, or both.*

The seven forces are summarized in Table 2.2. Notice the descriptions and tips presented in the third column. Notice in the fourth column that all of the forces are associated with the stress distribution except for the surface tension force.

2.5 The Viscosity Equation

The viscosity equation is used to represent viscous (i.e., frictional) effects in flowing fluids. This equation is important because viscous effects influence practical matters such as energy usage, pressure drop, and the fluid dynamic drag force.

The Viscosity Equation

The viscosity equation* is

$$\tau = \mu \frac{dV}{dy} \tag{2.15}$$

The viscosity equation relates shear stress τ to viscosity μ and velocity gradient dV/dy. The viscosity equation is called *Newton's Law of Viscosity* in many references.

The Velocity Gradient

The term (dV/dy) is called the **velocity gradient**.† The variable V represents the magnitude of the velocity vector. In mechanics, **velocity** is defined as the speed and direction of travel of a material particle. Thus, when a fluid is flowing, each material particle will have a different velocity (Fig. 2.12).

The variable y in dV/dy represents position as measured from a wall. Because dV/dy is an ordinary derivative, you can analyze this term by applying your knowledge of calculus. Three methods that we recommend are as follows:

Method #1. If you have a plot of $V(y)$, find dV/dy by sketching a tangent line and then finding the slope of the tangent line by using rise over run.

Method #2. If you have a table of experimental data (e.g., V versus y data), make an estimate based on the definition of the derivative from §1.4: $dV/dy \approx \Delta V/\Delta y$.

Method #3. If you have an equation for $V(y)$, differentiate the equation using methods from calculus.

In the context of analyzing the velocity gradient, you will often need to apply the **no-slip condition**, which is this: *When fluid is in contact with a solid body, the velocity of the fluid at the point of contact is the same as the velocity of the solid body at the same point.* **Example**. When water flows in a pipe, the fluid velocity at the wall is equal to the velocity of the wall, which is zero. **Example**. When an airplane moves through the air, the fluid velocity at a point situated on the wing equals the wing velocity at this same point.

You will often see the *velocity gradient* called the *rate of strain* because one can start with the definition of strain and prove that the rate of strain of a fluid particle is given by the velocity gradient. However, this derivation is best left to advanced texts.

Newtonian versus Non-Newtonian Fluids

As an engineer, you need to make decisions about whether or not an equation applies to a situation that you are analyzing. One issue in making this decision is whether a fluid can be modeled as a Newtonian fluid. To define a Newtonian fluid, imagine using air or water, setting

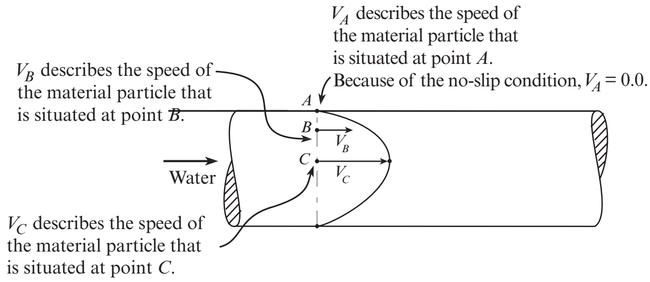


FIGURE 2.12

This sketch shows water flowing through a round pipe. A **velocity profile** is a sketch or an equation that shows how velocity varies with position.

*There is a more general form of this equation that involves partial derivatives. However, Eq. (2.15) applies to many flows of engineering interest; thus, we leave the more general form to advanced courses.

†This is called the *velocity gradient* because the gradient operator from calculus reduces to the ordinary derivative dV/dy for most simple flows.

up an experiment that involves measuring shear stress as a function of velocity gradient, and then plotting your data. You will get a straight line (Fig. 2.13) because both air and water are Newtonian fluids.

If you were to select other fluids, run experiments, and plot the data, you would find that some of the datasets do not plot the same as a Newtonian fluid (Fig. 2.14).

Fig. 2.14 shows three categories of non-Newtonian fluids. For a *shear-thinning* fluid, the viscosity of the fluid decreases as the rate of shear strain (dV/dy) increases. Some common shear-thinning fluids are ketchup, paints, and printer's ink. For a *shear-thickening* fluid, the viscosity increases with shear rate. One example of a shear-thickening fluid is a mixture of starch and water. A *Bingham plastic* acts like a solid for small values of shear stress and then behaves as a fluid at higher shear stress. Some common fluids that are idealized as Bingham plastics are mayonnaise, toothpaste, and certain muds.

In general, non-Newtonian fluids have molecules that are more complex than Newtonian fluids. Thus, if you are working with a fluid that may be non-Newtonian, consider doing some research; many of the equations and math models presented in textbooks (including this one) only apply to Newtonian fluids. To learn more about non-Newtonian fluids, watch the film entitled *Rheological Behavior of Fluids* (1) or see references (2) and (3).

Reasoning with the Viscosity Equation

Notice that the viscosity equation for a Newtonian fluid is a *linear equation*. It is a linear equation because a plot of the equation (Fig. 2.13) is a straight line. In particular, the general equation for a straight line is $y = mx + b$, where m is the slope and b is the y intercept. Because the viscosity equation is $\tau = \mu(dV/dy)$, you can see that τ is the dependent variable, μ is the slope, dV/dy is the independent variable, and 0.0 is the y intercept.

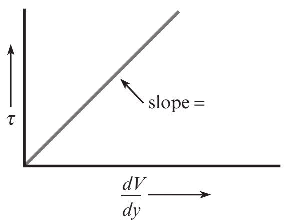


FIGURE 2.13

A fluid is defined as a **Newtonian fluid** when a plot of shear stress versus velocity gradient gives a straight line.* The slope will be equal to the value of the viscosity μ because the governing equation is $\tau = \mu(dV/dy)$.

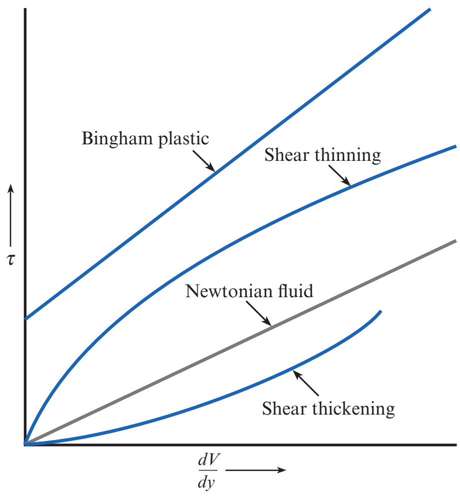


FIGURE 2.14

A **non-Newtonian fluid** is any fluid that does not follow the relationship between *shear stress* and *velocity gradient* that is followed by a Newtonian fluid.

*The curve also needs to pass through the origin to distinguish a Newtonian fluid from a Bingham plastic, which is a class of non-Newtonian fluids.

By using the viscosity equation, you can assess the magnitude of the *velocity gradient* (i.e., dV/dy) and figure out things about the magnitude of the *shear stress* τ (e.g., see Fig. 2.15). The reasoning can be represented by using arrows like this:

$$\tau\uparrow = \mu\left(\frac{dV}{dy}\uparrow\right) \tag{2.16}$$

In words, Eq. (2.16) says that if the slope (i.e., magnitude of dV/dy) increases, then the shear stress must increase. Similarly, the viscosity equation tells us that if the slope decreases, then the shear stress must decrease. And, if the slope is constant (e.g., Couette flow, which is our next topic), then the shear stress must be constant.

Couette Flow

Couette flow is used as a model for a variety of flows that involve lubrication. In Couette flow (e.g., see Fig. 2.16), a moving surface causes fluid to flow. Because of the no-slip condition, the velocity of the fluid at $y = H$ is equal to the velocity of the moving wall. Similarly, the velocity of the fluid at $y = 0$ is zero because the bottom plate is stationary. In the region between the plates, the velocity profile is linear.

When the viscosity equation is applied to Couette flow, the derivative can be replaced with a ratio because the velocity gradient is linear.

$$\tau = \mu\frac{dV}{dy} = \mu\frac{\Delta V}{\Delta y} \tag{2.17}$$

The terms on the right side of Eq. (2.17) can be analyzed as follows:

$$\tau = \mu\frac{\Delta V}{\Delta y} = \mu\frac{V_o - 0}{H - 0} = \mu\frac{V_o}{H}$$

Thus,

$$\tau\Big|_{\text{Couette Flow}} = \text{constant} = \mu\frac{V_o}{H} \tag{2.18}$$

Eq. (2.18) reveals that the shear stress at all points in a Couette flow is constant with a magnitude of $\mu V_o/H$.

FIGURE 2.15

This example shows the velocity profile associated with laminar flow in a round pipe. Notice how information about shear stress can be deduced from a velocity profile. Here, *r* is the radial position as measured from the centerline of the pipe.

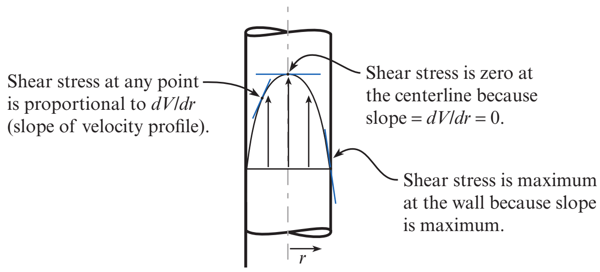
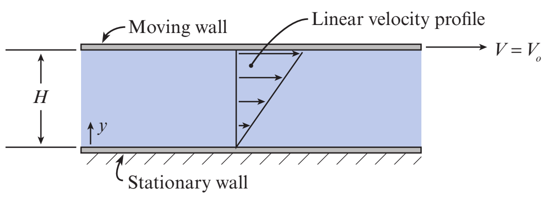


FIGURE 2.16

Couette flow is a flow that is driven by a moving wall. The velocity profile in the fluid is linear.



EXAMPLE 2.1

Applying the Viscosity Equation to Calculate Shear Stress in a Poiseuille Flow

Problem Statement

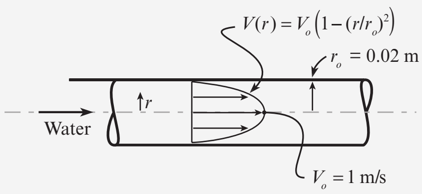
A famous solution in fluid mechanics, called Poiseuille flow, involves laminar flow in a round pipe (see Chapter 10 for details). Consider Poiseuille flow with a velocity profile in the pipe given by

$$V(r) = V_o(1 - (r/r_o)^2)$$

where r is the radial position as measured from the centerline, V_o is the velocity at the center of the pipe, and r_o is the pipe radius. Find the shear stress at the center of the pipe, at the wall, and where $r = 1$ cm. The fluid is water (15°C), the pipe diameter is 4 cm, and $V_o = 1$ m/s.

Define the Situation

Water flows in a round pipe (Poiseuille flow).



Water (15°C, 1 atm, Table A.5): $\mu = 1.14 \times 10^{-3}$ N·s/m^2.

State the Goal

Calculate the shear stress at three points:

$\tau(r = 0.00 \text{ m})$ (N/m^2) ⬅ pipe centerline

$\tau(r = 0.01 \text{ m})$ (N/m^2) ⬅ middle of the pipe

$\tau(r = 0.02 \text{ m})$ (N/m^2) ⬅ the wall

Generate Ideas and Make a Plan

Because the goal is τ, select the *viscosity equation*. Let the position variable be r instead of y.

$$\tau = -\mu \frac{dV}{dr} \quad \textbf{(a)}$$

Regarding the minus sign in Eq. (a), the y in the viscosity equation is measured from the wall. The coordinate r is in the opposite direction. The sign change occurs when the variable is changed from y to r.

To find the velocity gradient in Eq. (a), differentiate the given velocity profile.

$$\frac{dV(r)}{dr} = \frac{d}{dr}(V_o(1 - (r/r_o)^2)) = \frac{-2V_o r}{r_o^2} \quad \textbf{(b)}$$

Now, the goal can be found. **Plan**. Apply Eq. (b) to find the velocity gradient. Then, substitute into Eq. (a).

Take Action (Execute the Plan)

1. Viscosity equation ($r = 0$ m):

$$\left.\frac{dV(r)}{dr}\right|_{r=0\text{ m}} = \frac{-2V_o(0 \text{ m})}{r_o^2} = \frac{-2(1 \text{ m/s})(0 \text{ m})}{(0.02 \text{ m})^2} = 0.0 \text{ s}^{-1}$$

$$\tau(r = 0 \text{ m}) = -\mu \left.\frac{dV(r)}{dr}\right|_{r=0\text{ m}}$$

$$= (1.14 \times 10^{-3} \text{ N} \cdot \text{s/m}^2)(0.0 \text{ s}^{-1})$$

$$= \boxed{0.0 \text{ N/m}^2}$$

2. Viscosity equation ($r = 0.01$ m):

$$\left.\frac{dV(r)}{dr}\right|_{r=0.01\text{ m}} = \frac{-2V_o(0.01 \text{ m})}{r_o^2}$$

$$\frac{-2(1 \text{ m/s})(0.01 \text{ m})}{(0.02 \text{ m})^2} = -50 \text{ s}^{-1}$$

Next, calculate shear stress:

$$\tau(r = 0.01 \text{ m}) = -\mu \left.\frac{dV(r)}{dr}\right|_{r=0.01\text{ m}}$$

$$= (1.14 \times 10^{-3} \text{ N} \cdot \text{s/m}^2)(50 \text{ s}^{-1})$$

$$= \boxed{0.0570 \text{ N/m}^2}$$

3. Viscosity equation ($r = 0.02$ m):

$$\left.\frac{dV(r)}{dr}\right|_{r=0.02\text{ m}} = \frac{-2V_o(0.02 \text{ m})}{r_o^2}$$

$$= \frac{-2(1 \text{ m/s})(0.02 \text{ m})}{(0.02 \text{ m})^2} = -100 \text{ s}^{-1}$$

Next, calculate shear stress:

$$\tau(r = 0.02 \text{ m}) = -\mu \left.\frac{dV(r)}{dr}\right|_{r=0.02\text{ m}}$$

$$= (1.14 \times 10^{-3} \text{ N} \cdot \text{s/m}^2)(100 \text{ s}^{-1})$$

$$= \boxed{0.114 \text{ N/m}^2}$$

Review the Solution and the Process

1. *Tip*. On most problems, including this example, carrying and canceling units is useful, if not critical.
2. *Notice*. Shear stress varies with location. For this example, τ is zero on the centerline of the flow and nonzero everywhere else. The maximum value of shear stress occurs at the wall of the pipe.
3. *Notice*. For flow in a round pipe, the viscosity equation has a minus sign and uses the position coordinate r.

$$\tau = -\mu \frac{dV}{dr}$$

EXAMPLE 2.2

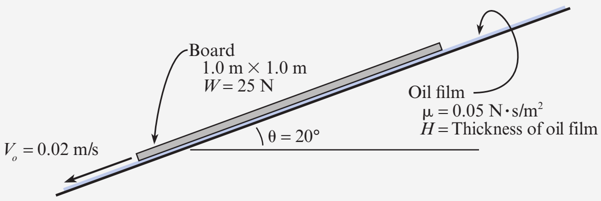
Applying the Viscosity Equation to Couette Flow

Problem Statement

A board 1 m by 1 m that weighs 25 N slides down an inclined ramp (slope = 20°) with a constant velocity of 2.0 cm/s. The board is separated from the ramp by a thin film of oil with a viscosity of 0.05 N·s/m^2. Assuming that the oil can be modeled as a Couette flow, calculate the space between the board and the ramp.

Define the Situation

A board slides down an oil film on a inclined plane.



Assumptions. (1) Couette flow. (2) Board has constant speed.

State the Goal

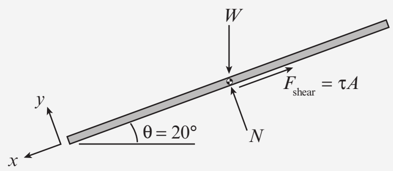
H(mm) ⬅ thickness of the film of oil

Generate Ideas and Make a Plan

Because the goal is H, apply the *viscosity equation* (Eq. (2.18)):

$$H = \mu \frac{V_o}{\tau} \quad \textbf{(a)}$$

To find the shear stress τ in Eq. (a), draw a *Free Body Diagram* (FBD) of the board. In the FBD, W is the weight, N is the normal force, and F_{shear} is shear force. Because shear stress is constant with x, the shear force can be expressed as $F_{shear} = \tau A$.



Because the board moves at constant speed, the forces are in balance. Thus, apply *force equilibrium.*

$$\Sigma F_x = 0 = W \sin\theta - \tau A \quad \textbf{(b)}$$

Rewrite Eq. (b) as

$$\tau = (W \sin\theta)/A \quad \textbf{(c)}$$

Eq. (c) can be solved for τ. The plan is as follows:

1. Calculate τ using force equilibrium (Eq. (c)).
2. Calculate H using the shear stress equation (Eq. (a)).

Take Action (Execute the Plan)

1. Force equilibrium:

$$\tau = (W\sin\theta)/A = (25\ \text{N})(\sin 20°)/(1.0\ \text{m}^2) = 8.55\ \text{N/m}^2$$

2. Shear stress equation:

$$H = \mu\frac{V_o}{\tau} = (0.05\ \text{N}\cdot\text{s/m}^2)\frac{(0.02\ \text{m/s})}{(8.55\ \text{N/m}^2)} = \boxed{0.117\ \text{mm}}$$

Review the Solution and the Process

1. H is about 12% of a millimeter; this is quite small.
2. *Tip*. Solving this problem involved drawing an FBD. The FBD is useful for most problems involving Couette flow.

2.6 Surface Tension*

Engineers need to be able to predict and characterize surface tension effects because they affect many industrial problems. Some examples of surface tension effects include the following:

- *Wicking*. Water will wick into a paper towel. Ink will wick into paper. Polypropylene, an excellent fiber for cold-weather aerobic activity, wicks perspiration away from the body.
- *Capillary rise*. A liquid will rise in a small-diameter tube. Water will rise in soil.
- *Capillary instability*. A liquid jet will break up into drops.
- *Drop and bubble formation*. Water on a leaf beads up. A leaky faucet drips. Soap bubbles form.
- *Excess pressure*. The pressure inside a water drop is higher than ambient pressure. The pressure inside a vapor bubble during boiling is higher than ambient pressure.
- *Walking on water*. The water strider, an insect, can walk on water. Similarly, a metal paper clip or a metal needle can be positioned to float (through the action of surface tension) on the surface of water.

*The authors acknowledge and thank Dr. Eric Aston for his feedback and inputs on this section. Dr. Aston is a chemical engineering professor at the University of Idaho.

- *Detergents.* Soaps and detergents improve the cleaning of clothes because they lower the surface tension of water so that the water can more easily wick into the pores of the fabric.

Many experiments have shown that the surface of liquid behaves like a stretched membrane. The material property that characterizes this behavior is **surface tension**, σ (sigma). Surface tension can be expressed in terms of force:

$$\text{surface tension}(\sigma) = \frac{\text{force along an interface}}{\text{length of the interface}} \tag{2.19}$$

Surface tension can also be expressed in terms of energy:

$$\text{surface tension}(\sigma) = \frac{\text{energy required to increase the surface area of a liquid}}{\text{unit area}} \tag{2.20}$$

From Eq. (2.19), the unit of surface tension is newton per meter (N/m). Surface tension typically has a magnitude ranging from 1 to 100 mN/m. The unit of surface tension can also be joule per meter squared (J/m^2) because

$$\frac{\text{N}}{\text{m}} = \frac{\text{N} \cdot \text{m}}{\text{m} \cdot \text{m}} = \frac{\text{J}}{\text{m}^2}$$

The physical mechanism of surface tension is based on **cohesive force**, which is the attractive force between like molecules. Because liquid molecules attract one another, molecules in the interior of a liquid (see Fig. 2.17) are attracted equally in all directions. In contrast, molecules at the surface are pulled toward the center because they have no liquid molecules above them. This pull on surface molecules draws the surface inward and causes the liquid to seek to minimize surface area. This is why a drop of water draws into a spherical shape.

Surface tension of water decreases with temperature (see Fig. 2.18) because thermal expansion moves the molecules farther apart, and this reduces the average attractive force between molecules (i.e., cohesive force goes down). Surface tension is strongly affected by the presence of contaminants or impurities. For example, adding soap to water decreases surface tension. The reason is that impurities concentrate on the surface, and these molecules decrease the average attractive force between the water molecules. As shown in Fig. 2.18, surface tension of water at 20°C is $\sigma = 0.0728 \approx 0.073$ N/m. This value is used in many of the calculations in this text.

In Fig. 2.18, surface tension is reported for an interface of air and water. It is common practice to report surface tension data based on the materials that were used during the measurement of the surface tension data.

To learn more about surface tension, we recommend the online film entitled *Surface Tension in Fluid Mechanics* (5) and Shaw's book (6).

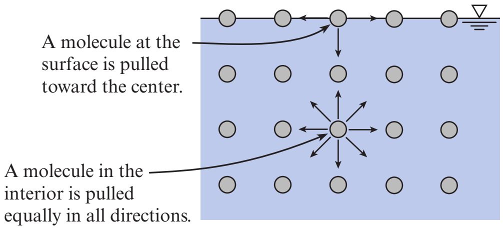


FIGURE 2.17
Forces between molecules in a liquid.

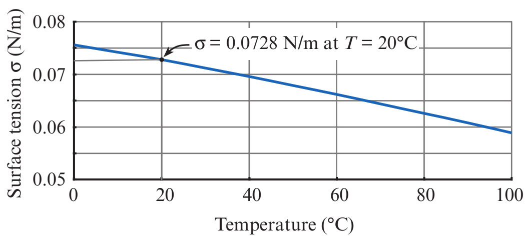


FIGURE 2.18
Surface tension of water for a water/air interface.

Example Problems

Most problems involving surface tension are solved by drawing an FBD and applying force equilibrium. The force due to surface tension, from Eq. (2.19), is

$$F_\sigma = \sigma L \tag{2.21}$$

where L is the length of a line that lies along the surface of the liquid. The use of force equilibrium to solve problems is illustrated in Examples 2.3 and 2.4.

Adhesion and Capillary Action

When a drop of water is placed on glass, the water will wet the glass (see Fig. 2.19) because water is strongly attracted to the glass. This attractive force pulls the water outward as shown. The force between dissimilar surfaces is called **adhesion** (see Fig. 2.19b). Water will "wet out" on a surface when *adhesion is greater than cohesion*.

EXAMPLE 2.3

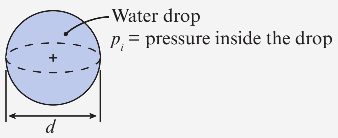
Applying Force Equilibrium to Calculate the Pressure Rise Inside a Water Droplet

Problem Statement

The pressure inside a water drop is higher than the pressure of the surroundings. Derive a formula for this pressure rise. Then, calculate the pressure rise for a 2-mm-diameter water drop. Use $\sigma = 73$ mN/m.

Define the Situation

Pressure inside a water drop is larger than ambient pressure.
$d = 0.002$ m, $\sigma = 73$ mN/m.

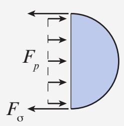


State the Goal

1. Derive an equation for p_i.
2. Calculate p_i in pascals.

Generate Ideas and Make a Plan

Because pressure is involved in a force balance, draw an FBD of the drop.



Force due to pressure = Force due to surface tension

$$F_p = F_\sigma \tag{a}$$

From Eq. (2.19), the surface tension force is σ times the length of the interface:

$$F_\sigma = \sigma L = \sigma \pi d \tag{b}$$

The pressure force is pressure times area:

$$F_p = p_i \frac{\pi d^2}{4} \tag{c}$$

Combine Eqs. (a) to (c):

$$p_i \frac{\pi d^2}{4} = \sigma \pi d \tag{d}$$

Solve for pressure:

$$\boxed{p_i = \frac{4\sigma}{d}} \tag{e}$$

The first goal (equation for pressure) has been attained. The next goal (value of pressure) can be found by substituting values into Eq. (e).

Take Action (Execute the Plan)

$$p_i = \frac{4\sigma}{d} = \frac{4(0.073\text{ N/m})}{(0.002\text{ m})} = \boxed{146\text{ Pa gage}}$$

Review the Results and the Process

1. *Notice.* The answer is expressed as gage pressure. Gage pressure in this context is the pressure rise above ambient.
2. *Physics.* The pressure rise inside a liquid drop is a consequence of the membrane effect of surface tension. One way to visualize this is make an analogy with a balloon filled with air. The pressure inside the balloon pushes outward against the membrane force of the rubber skin. In the same way, the pressure inside a liquid drop pushes outward against the membrane like force of surface tension.

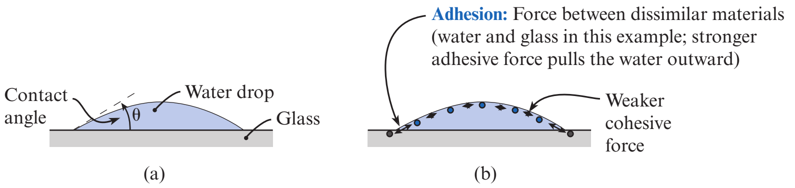


FIGURE 2.19
Water wets glass because adhesion is greater than cohesion. Wetting is associated with a contact angle less than 90°.

On some surfaces, such as Teflon and wax paper, a drop of water will bead up (Fig 2.20) because *adhesion between the water and Teflon is less than cohesion of the water*. A surface on which water beads up is called *hydrophobic* (water hating). Surfaces such as glass on which water drops spread out are called *hydrophilic* (water loving).

Capillary action describes the tendency of a liquid to rise in narrow tubes or to be drawn into small openings. Capillary action is responsible for water being drawn into the narrow openings in soil or into the narrow openings between the fibers of a dry paper towel.

When a capillary tube is placed into a container of water, the water rises up the tube (Fig. 2.21) because the adhesive force between the water and the glass pulls the water up the tube. This is called capillary rise. Notice how the contact angle for the water is the same in Figs. 2.19 and 2.21. Alternatively, when a fluid is nonwetting (such as mercury on glass), then the liquid will display capillary repulsion.

To derive an equation for capillary rise (see Fig. 2.22), define a system comprised of the water inside the capillary tube. Then, draw an FBD. As shown, the pull of surface tension lifts the column of water. Applying force equilibrium gives

$$\text{weight} = \text{surface tension force}$$

$$\gamma\left(\frac{\pi d^2}{4}\right)\Delta h = \sigma\,\pi\, d\cos\theta \tag{2.22}$$

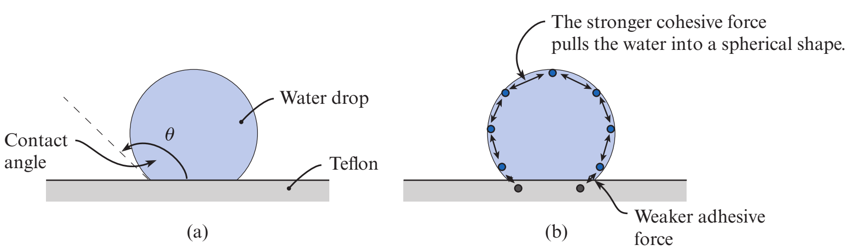


FIGURE 2.20
Water beads up a hydrophobic material such as Teflon because adhesion is less than cohesion. A nonwetting surface is associated with a contact angle greater than 90°.

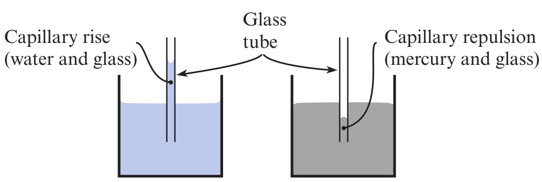


FIGURE 2.21
Water will rise up a glass tube (capillary rise), whereas mercury will move downward (capillary repulsion).

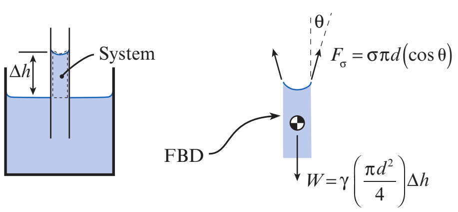


FIGURE 2.22
Sketches used for deriving an equation for capillary rise.

Assume the contact angle is nearly zero so that $\cos\theta \approx 1.0$. Note that this is a good assumption for a water/glass interface. Eq. (2.22) simplifies to

$$\Delta h = \frac{4\sigma}{\gamma d} \tag{2.23}$$

EXAMPLE. Calculate the capillary rise for water (20°C) in a glass tube of diameter $d = 1.6$ mm.

Solution. From Table A.5, $\gamma = 9790\ \text{N/m}^3$. From Fig. 2.18, $\sigma = 0.0728$ N/m. Now, calculate capillary rise using Eq. (2.23):

$$\Delta h = \frac{4(0.0728\ \text{N/m})}{(9790\ \text{N/m}^3)(1.6 \times 10^{-3}\ \text{m})} = \boxed{18.6\ \text{mm}}$$

Example 2.4 shows a case involving a nonwetting surface.

EXAMPLE 2.4

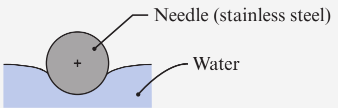
Applying Force Equilibrium to Determine the Size of a Sewing Needle That Can Be Supported by Surface Tension

Problem Statement

The Internet shows examples of sewing needles that appear to be "floating" on top of water. This effect is due to surface tension supporting the needle. Determine the largest diameter of sewing needle that can be supported by water. Assume that the needle material is stainless steel with $SG_{ss} = 7.7$.

Define the Situation

A sewing needle is supported by the surface tension of a water surface.



Assumptions

- The sewing needle is a cylinder.
- Neglect end effects.

Properties

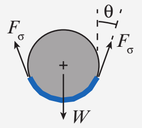
- Water (20°C, 1 atm, Fig. 2.18): $\sigma = 0.0728$ N/m
- Water (4°C, 1 atm, Table F.6): $\gamma_{H_2O} = 9810\ \text{N/m}^3$
- SS: $\gamma_{ss} = (7.7)(9810\ \text{N/m}^3) = 75.5\ \text{kN/m}^3$

State the Goal

d(mm) ⬅ diameter of the largest needle that can be supported by the water

Generate Ideas and Make a Plan

Because the weight of the needle is supported by the surface tension force, draw an FBD. Select a system comprised of the needle plus the surface layer of the water. The FBD is



Apply force equilibrium:

Force due to surface tension = Weight of the needle

$$F_\sigma = W \tag{a}$$

From Eq. (2.21)

$$F_\sigma = \sigma 2L\cos\theta \tag{b}$$

where L is the length of the needle. The weight of the needle is

$$W = \left(\frac{\text{weight}}{\text{volume}}\right)[\text{volume}] = \gamma_{ss}\left[\left(\frac{\pi d^2}{4}\right)L\right] \tag{c}$$

Combine Eqs. (a), (b), and (c). Also, assume the angle θ is zero because this gives the maximum possible diameter:

$$\sigma 2L = \gamma_{ss}\left(\frac{\pi d^2}{4}\right)L \tag{d}$$

Plan. Solve Eq. (d) for d and then plug numbers in.

Take Action (Execute the Plan)

$$d = \sqrt{\frac{8\sigma}{\pi\gamma_{ss}}} = \sqrt{\frac{8(0.0728\ \text{N/m})}{\pi(75.5 \times 10^3\ \text{N/m}^3)}} = \boxed{1.57\ \text{mm}}$$

Review the Solution and the Process

Notice. When applying specific gravity, look up water properties at the reference temperature of 4°C.

2.7 Vapor Pressure, Boiling, and Cavitation

A liquid, even at a low temperature, can boil as it flows through a system. This boiling can reduce performance and damage equipment. Thus, engineers need to be able to predict when boiling will occur. This prediction is based on vapor pressure.

Vapor pressure, p_v (kPa), is the pressure at which the liquid phase and the vapor phase of a material will be in thermal equilibrium. Vapor pressure is also called *saturation pressure,* and the corresponding temperature is called *saturation temperature*.

Vapor pressure can be visualized on a *phase diagram*. A phase diagram for water is shown in Fig. 2.23. As shown, water will exist in the liquid phase for any combination of temperature and pressure that lies above the line. Similarly, the water will exist in the vapor phase for points below the line. Along the line, the liquid and vapor phases are in thermal equilibrium. When boiling occurs, the pressure and temperature of the water will be given by one of the points on the line. In addition to Fig. 2.23, data for vapor pressure of water are tabulated in Table A.5.

EXAMPLE. Water at 20°C flows through a venturi nozzle and boils. Explain why. Also, give the value of pressure in the nozzle.

Solution. The water is boiling because the pressure has dropped to the vapor pressure. Table 2.1 indicates that p_v can be looked up in Table A.5. Thus, the vapor pressure at 20°C (Table A.5) is $p_v = 2.34$ kPa absolute. This value can be validated by using Fig. 2.23.

Review. Vapor pressure is commonly expressed using *absolute pressure*. Absolute pressure is the value of pressure as measured relative to a pressure of absolute zero.

Cavitation is a phenomena associated with boiling in which vapor bubbles form and then collapse. Cavitation occurs when the fluid pressure at a location in a flowing fluid drops to the vapor pressure. One of the main reasons that cavitation is important is it damages propellers, pump impellers, flow passageways on dams, and so on. More information on cavitation is presented in §5.5.

2.8 Characterizing Thermal Energy in Flowing Gases

Engineers characterize thermal energy changes using properties introduced in this section. **Thermal energy** is the energy associated with molecules in motion. This means that thermal energy is associated with temperature change (sensible energy change) and phase change (latent energy change). For most fluid problems, thermal properties are not important. However, thermal properties are used for compressible flow of gases (Chapter 12).

Specific Heat, *c*

Specific heat characterizes the amount of thermal energy that must be transferred to a unit mass of substance to raise its temperature by one degree. The dimensions of specific heat are energy per unit mass per degree temperature change, and the corresponding units are J/kg·K.

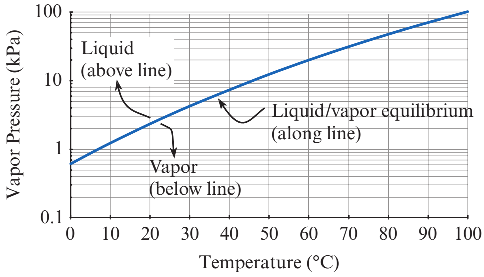


FIGURE 2.23
A phase diagram for water.

The magnitude of c depends on the process. For example, if a gas is heated at *constant volume*, less energy is required than if the gas is heated at *constant pressure*. This is because a gas that is heated at constant pressure must do work as it expands against its surroundings.

The *constant volume specific heat*, c_v, applies to a process carried out at constant volume. The *constant pressure specific heat*, c_p, applies to a process carried out at constant pressure. The ratio c_p/c_v is called the **specific heat ratio** and is given the symbol k. Values for c_p and k for various gases are given in Table A.2.

Internal Energy

Internal energy includes all the energy in matter except for kinetic energy and potential energy. Thus, internal energy includes multiple forms of energy, such as chemical energy, electrical energy, and thermal energy. Specific internal energy, u, has dimensions of energy per unit mass. The units are J/kg.

Enthalpy

When a material is heated at constant pressure, the energy balance is

$$(\text{Energy added}) = \begin{pmatrix}\text{Energy to increase}\\ \text{thermal energy}\end{pmatrix} + \begin{pmatrix}\text{Energy to do work}\\ \text{as the material expands}\end{pmatrix}$$

The work term is needed because the material is exerting a force over a distance as it pushes its surroundings away during the process of thermal expansion.

Enthalpy is a property that characterizes the amount of energy associated with a heating or cooling process. Enthalpy per unit mass is defined mathematically by

$$(\text{enthalpy}) = (\text{internal energy}) + (\text{pressure}/\text{density})$$

$$h = u + p/\rho$$

Ideal Gas Behavior

For an ideal gas, the properties h, u, c_p, *and* c_v depend only on temperature, not on pressure.

2.9 Summarizing Key Knowledge

Describing Your System

To describe what you are analyzing, apply three ideas:

- The *system* is the matter that you select for analysis.
- The *surroundings* are everything else that is not part of the system.
- The *boundary* is the surface that separates the system from its surroundings.

To describe the *conditions* of your system, apply four ideas:

- The *state* of a system is the condition of the system as specified by values of the properties of the system.
- A *property* is a characteristic of a system that depends only on the present state.
- *Steady state* means that all properties of the system are constant with time.
- A *process* is a change of a system from one state to another state.

Finding Fluid Properties

- To characterize the weight or mass of a fluid, use ρ, γ, or SG. If you know one of these properties, then you can calculate the other two using these equations: $\gamma = \rho g$ and $SG = \rho/\rho_{H_2O,\,(4°C)} = \gamma/\gamma_{H_2O,\,(4°C)}$.
- To characterize viscous effects (i.e., frictional effects), you can use *viscosity* μ, which is also called *dynamic viscosity* or *absolute viscosity*. You also will often use a different property, called *kinematic viscosity*, which is defined by $\nu = \mu/\rho$.
- When looking up properties, make sure that you account for the variation in the value of the property as a function of temperature and pressure.

- Quality in documentation involves listing the name of the property, the source of the property data, the units, the temperature and pressure, and any assumptions that you make.

Density Topics

- Modeling a fluid as *constant density* means that you assume the density is constant with position and time. *Variable density* means the density can change with position, time, or both.
- A gas in steady flow can be idealized as a constant density if the Mach number is less than 0.3. Liquids for most flow situations can be idealized as constant density. Two notable exceptions are water hammer problems and acoustics problems.
- All fluids, including liquids, will compress (i.e., decrease in volume) if the pressure is increased. The amount of volume change can be calculated by using the bulk modulus of elasticity.
- Specific gravity (S or SG) gives the ratio of the density of a material to the density of liquid water at 4°C.

Stress

In mechanics, **stress** is an entity that expresses the internal forces that material particles exert on each other. Stress is the ratio of force to area at a point and is resolved into two components:

- Pressure (normal stress) is the ratio of normal force to area.
- Shear stress is the ratio of shear force to area.

To relate force to stress, integrate the stress over area.

- The general equation for the *pressure force* is $\mathbf{F}_p = \int_A -p\mathbf{n}\, dA$. For the special case of a uniform pressure acting on a flat surface, this equation simplifies to $F_p = pA$.
- The general equation for the *shear force* is $\mathbf{F}_s = \int_A \tau\, \mathbf{t}\, dA$. For the special case of a uniform shear stress acting on a flat surface, this equation simplifies to $F_s = \tau A$.
- When a force acts between Body #1 (a fluid body) and Body #2 (any other body), the force can usually be identified as one of seven forces: (1) the pressure force, (2) the shear force, (3) the buoyant force, (4) the drag force, (5) the lift force, (6) the surface tension force, or (7) the thrust force. Except for the surface tension force, each of these forces is associated with a pressure distribution, a shear stress distribution, or both.

The Viscosity Equation

- The viscosity equation is useful for calculating the shear stress in a flowing fluid. The equation is $\tau = \mu\,(dV/dy)$. If μ is constant, then τ is linearly related to dV/dy.
- For many flows, the velocity gradient is the first derivative of velocity with respect to distance (dV/dy).
- The no-slip condition means that the velocity of fluid in contact with a solid surface will equal the velocity of the surface.
- A *Newtonian fluid* is one in which a plot of τ versus dV/dy is a straight line. A *non-Newtonian* fluid is one in which a plot of τ versus dV/dy is not a straight line. In general, non-Newtonian fluids have more complex molecular structures than Newtonian fluids. Examples of non-Newtonian fluids include paint, toothpaste, and molten plastics. Equations developed for Newtonian fluids (i.e., many textbook equations) often do not apply to non-Newtonian fluids.
- Couette flow involves a flow through a narrow channel with the top plate moving at a speed of V_o. In Couette flow, the shear stress is constant at every point and is given by $\tau_o = (\mu V_o)/H$.

Surface Tension and Vapor Pressure

- A liquid flowing in a system will boil when the pressure drops to the vapor pressure. This boiling often is detrimental to a design.
- Surface tension problems are usually solved by drawing an FBD and summing forces.
- The formula for capillary rise of water in a round glass tube is $\Delta h = (4\sigma)/(\gamma d)$.

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