f cont en $[a,b]$ y derv. (a,b)

1) $\forall x \in (a,b),\ f'(x) \geq 0 \Leftrightarrow f$ creciente

2) $\forall x \in (a,b),\ f'(x) \leq 0 \Leftrightarrow f$ decreciente

3) $\forall x \in (a,b),\ f'(x) = 0 \Leftrightarrow f$ constante

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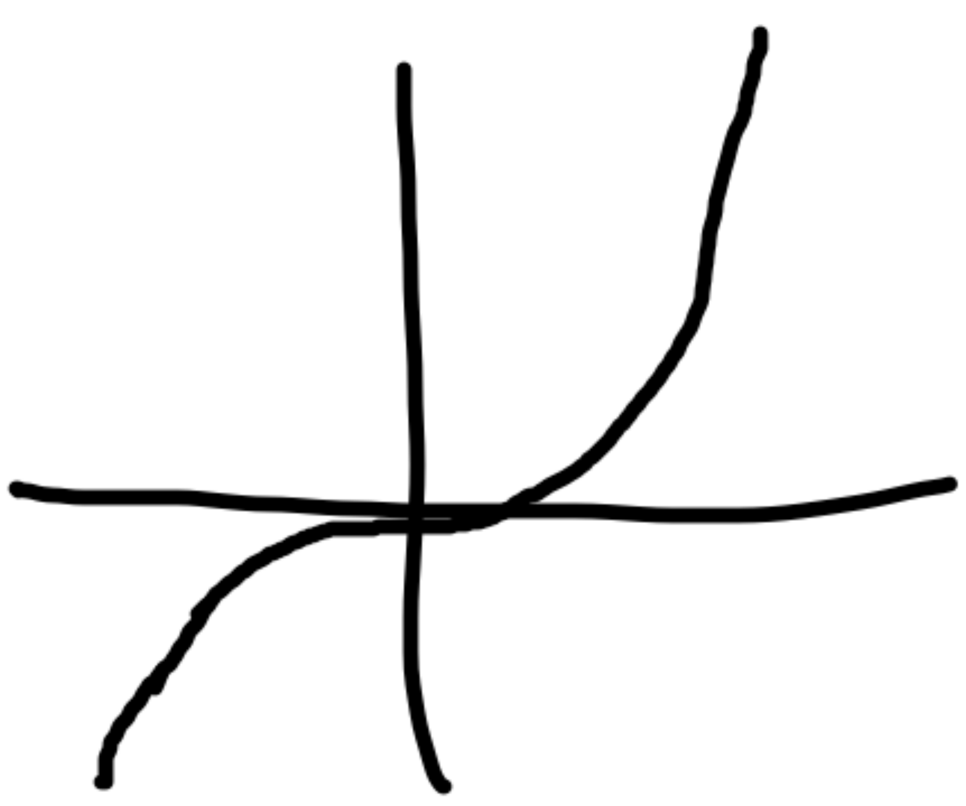
5) $\forall x \in (a,b),\ f'(x) > 0 \Rightarrow f$ est. creciente

6) $\forall x \in (a,b),\ f'(x) < 0 \Rightarrow f$ est. decreciente

$f(x) = x^3$ $\qquad f'(x) = 3x^2$

$f'(0) = 0$



Teorema Cauchy

(H) f y g continuas $[a,b]$

f y g derivables (a,b)

(T) $\exists\, c \in (a,b)$, $f'(c)(g(b)-g(a)) =$

$= g'(c)(f(b)-f(a))$

Obs.) En el caso particular. $g'(x) \neq 0$

la expresión anterior es equivalente

$$\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}$$

Regla de L'Hôpital

Cuando $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$

No se puede decir nada sobre f/g $\quad \frac{f(x)}{g(x)} \xrightarrow{?} ?$

La regla permite resolver la ind.

Sea f y g dos funciones tales que f, g definidas y derivables $(a; a+\delta)$ $(a \in \mathbb{R}, \delta > 0)$

- $\lim_{x \to a^+} f(x) = \lim_{x \to a^+} g(x) = 0$
- $g'(x) \neq 0 \quad \forall x \in (a, a+\delta)$
- Existe $\lim_{x \to a^+} \frac{f'(x)}{g'(x)} = l \in \mathbb{R}$

Entonces $\lim_{x \to a^+} \frac{f(x)}{g(x)} = \lim_{x \to a^+} \frac{f'(x)}{g'(x)} = l$

Ejemplo

① $$\lim_{x\to 0} \frac{\overbrace{e^x - 1 - x}^{\to 0}}{\underbrace{x^2}_{\to 0}} = \lim_{x\to 0} \frac{e^x - 1}{2x} =$$

$$\lim_{x\to 0} \frac{e^x}{2} = \frac{1}{2}$$

② $$\lim_{x\to 0} \frac{\overbrace{\cos x - 1}^{\to 0}}{\underbrace{x^2}_{\to 0}} = \lim_{x\to 0} \overbrace{\frac{-\operatorname{sen} x}{2x}}^{\to 1} = -1/2$$

(3) $\lim\limits_{x \to 0} \dfrac{L(1+x) - x}{x^2} = \lim\limits_{x \to 0} \dfrac{\frac{1}{1+x} - 1}{2x} =$

$= \lim\limits_{x \to 0} \dfrac{\frac{1 - 1 - x}{1+x}}{2x} = \lim\limits_{x \to 0} \dfrac{-\cancel{x}}{2\cancel{x}(1+x)} = -1/2$

1. $A \subseteq (\mathbb{R}-\mathbb{Q})$, $\sqrt{2} \in A$ y $\sup(A) = \pi$

1. $B = \{x \in \mathbb{R},\ x > \pi\}$ entonces $b \in \mathbb{R}$ es cota superior de A si y solo si $b \in B$ F

$\pi = \sup(A) \longrightarrow \pi$ cota sup. de A

$\pi \notin B$

2. $3{,}141516 \in A$ F

$3{,}141516 \in \mathbb{Q}$ $\qquad A \cap \mathbb{Q} = \emptyset$

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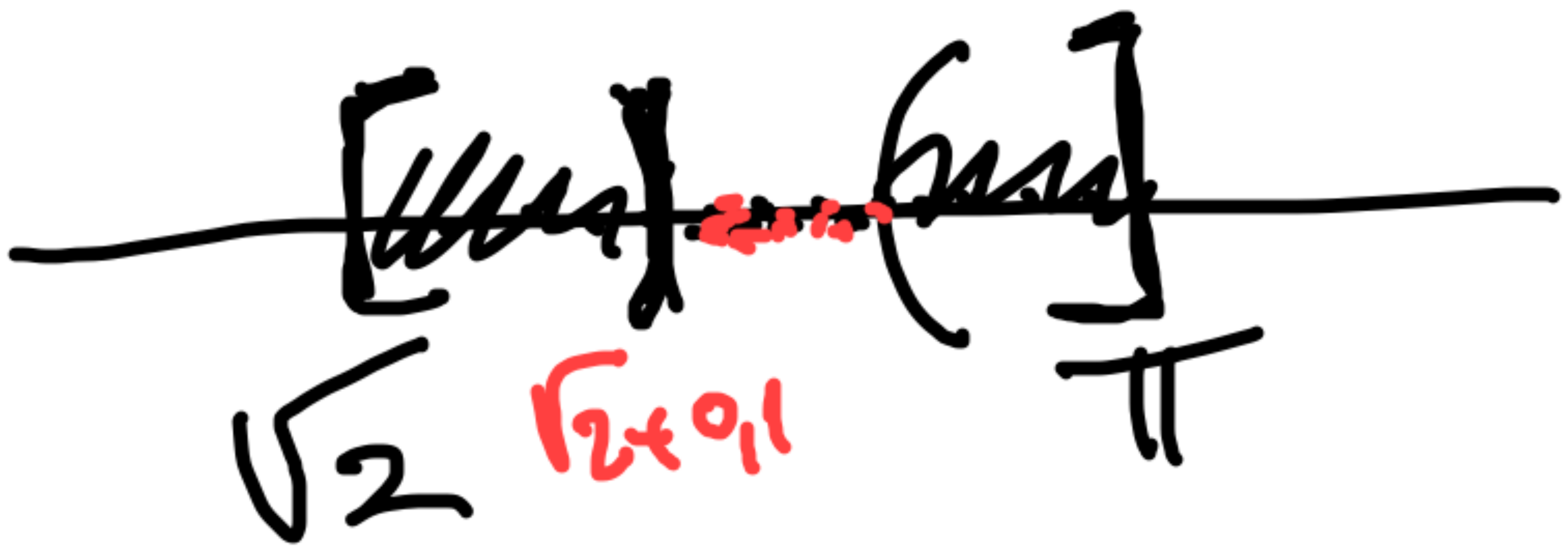
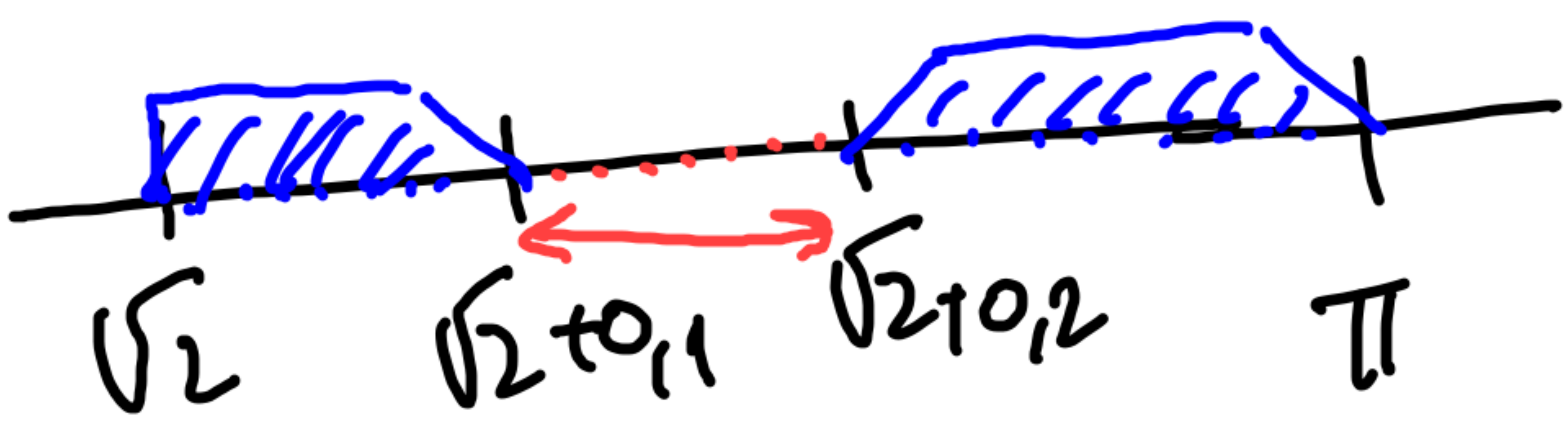
3) $\forall x \in (\mathbb{R} - \mathbb{Q})$, $\boxed{\sqrt{2} < x < \pi}$ entonces $x \in A$



$\underbrace{[\sqrt{2}, \sqrt{2}+0{,}1)} \cup (\sqrt{2}+0{,}2 ; \pi)$

No hay dato importante

$A = \left\{ x \in \mathbb{R}-\mathbb{Q} \;/\; \begin{array}{l} \sqrt{2} \leq x < \sqrt{2}+0{,}1 \text{ o } \\ \sqrt{2}+0{,}2 < x < \pi \end{array} \right\}$



4) A acot. inferiormente

$A \subseteq (\mathbb{R}-\mathbb{Q})$ $\quad \sqrt{2} \in A$ y $\sup(A) = \pi$

No hay datos suficientes de A

por ej $\quad A = \{x \in (\mathbb{R}-\mathbb{Q});\ x < \pi\}$

A no está acot. inferiormente.

5) $\sqrt{3} \notin A$ $\quad A = \{x \in (\mathbb{R}-\mathbb{Q});\ 0 < x < \sqrt{3}$ ó $\sqrt{3} < x < \pi\}$

$$\sum_{i=1}^{i=n} i = \frac{n(n+1)}{2}$$

<u>Paso Base</u> $n = 1$

$$\sum_{i=1}^{i=1} i = 1$$

$$\frac{1(1+1)}{2} = 1 \checkmark$$

Paso Inductivo

HI $n = h$

$$\sum_{i=1}^{i=h} i = \frac{h(h+1)}{2}$$

TI $n = h+1$

$$\sum_{i=1}^{i=h+1} i = \frac{(h+1)(h+2)}{2}$$

$$\sum_{i=1}^{i=h+1} i = \sum_{i=1}^{i=h} i + \sum_{i=h+1}^{i=h+1} i$$

$$= \frac{h(h+1)}{2} + h+1 =$$

$$= \frac{h(h+1) + 2(h+1)}{2}$$

$$= \frac{(h+1)(h+2)}{2}$$

② $\lim\limits_{x \to 0^+} e^{1/x} \cdot x^2$

$e^{1/x} \to +\infty$, $1/x \to +\infty$, $x^2 \to 0^+$

$t = 1/x$ $\quad x \to 0^+ \quad t \longrightarrow +\infty$

$x^2 = 1/t^2$

$$\lim_{t \to +\infty} e^t \cdot 1/t^2 = \lim_{t \to +\infty} \frac{e^t}{t^2} = +\infty$$

(por órdenes de infinito)

$f: [0,+\infty) \to \mathbb{R}$

$$f(x) = \begin{cases} \dfrac{x^2+x+1}{x^2}\, e^{-1/x} & \text{si } x \in (0,+\infty) \\ 0 & \text{si } x = 0 \end{cases}$$

f es cont en $(0,+\infty)$ por tratarse de (cociente, producto) de funciones cont. en $(0,+\infty)$

$$\lim_{x\to 0^+} \underbrace{\frac{x^2+x+1}{x^2}}_{\to +\infty}\; \underbrace{e^{\overbrace{-1/x}^{\to -\infty}}}_{\to 0} =$$

c.v. $t = \frac{1}{x}$ $\quad x \to 0^+ \Rightarrow t \to +\infty$

$$= \lim_{x\to 0^+} \left(1 + \frac{1}{x} + \frac{1}{x^2}\right) \cdot e^{-1/x} = \lim_{t\to +\infty} (1+t+t^2)\, e^{-t} =$$

$$= \lim_{t\to +\infty} \frac{1+t+t^2}{e^t} = 0 \quad \left(\text{por órdenes de infinitos}\right)$$

$f(0)=0$

$\lim_{x\to 0^+} f(x)=0$ $\Longrightarrow$ f es cont. en 0^+

f cont en 0^+

f cont. en $(0,+\infty)$ $\Longrightarrow$ f es cont. $[0,+\infty)$

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(b) $f(x) = \dfrac{x^2+x+1}{x^2} \cdot e^{-1/x}$

$$f'(x) = \frac{(2x+1)x^2 - (x^2+x+1)2x}{x^4} \cdot e^{-1/x} + \frac{x^2+x+1}{x^2} \cdot e^{-1/x} \cdot \left(\frac{1}{x^2}\right)$$

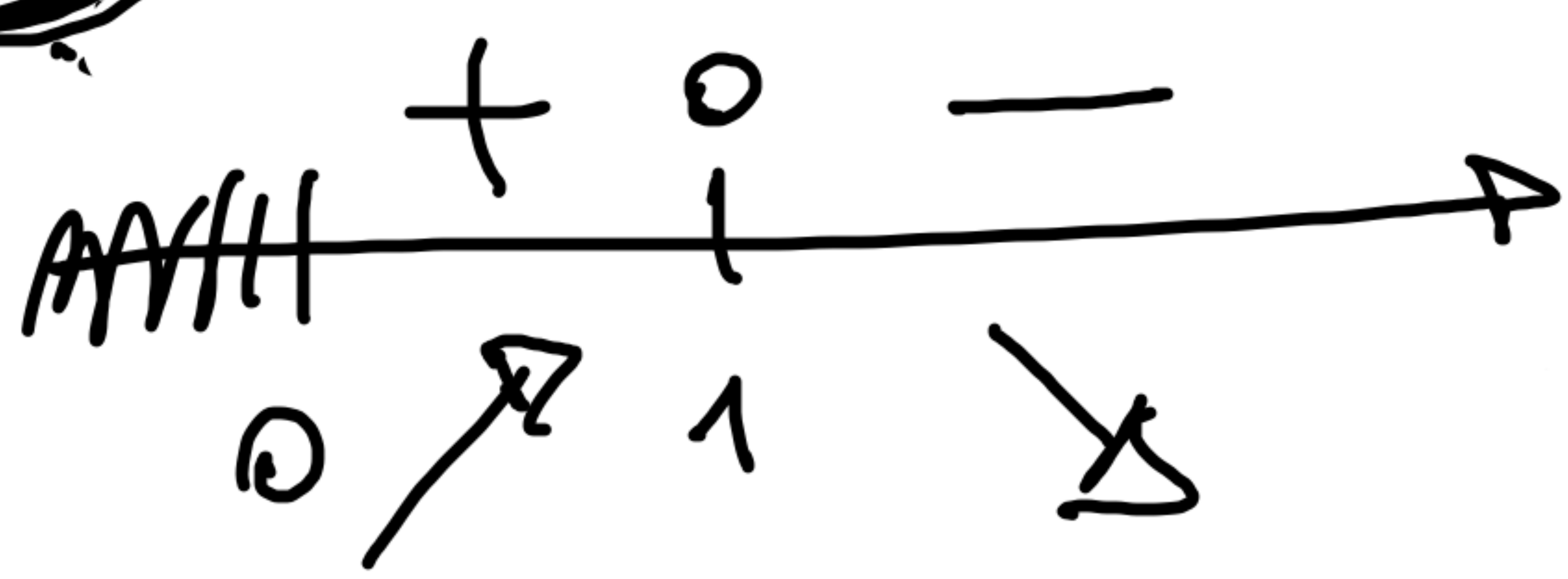
$$f'(x) = \frac{e^{-1/x}}{x^4}\left(\cancel{2x^3} + \cancel{x^2} - \cancel{2x^3} - \cancel{2x^2} - 2x + \cancel{x^2} + x + 1\right) =$$

$$f'(x) = \frac{e^{-1/x}}{x^4}(-x+1)$$

$f'(x) = 0 \rightarrow x = 1$

si $x \neq 0$ $\quad sg(f'(x)) = sg(-x+1)$

$sg(f'(x))$

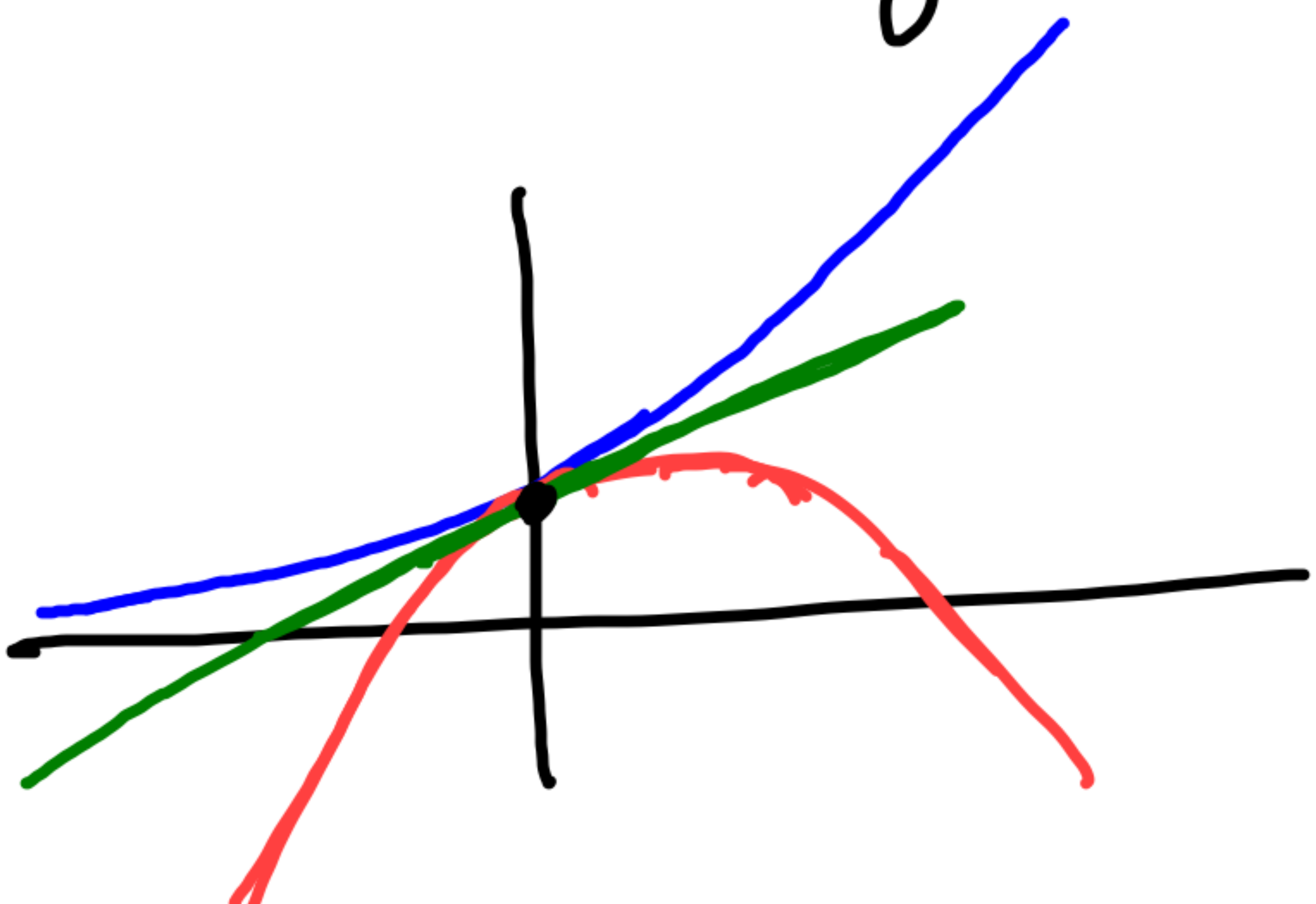


En $x=1$ presenta un máx relativo

$$f(1) = \frac{1+1+1}{1} e^{-1} = \frac{3}{e}$$

④ $f(x) = e^x$

$g(x) = -x^2 + ax + b$



$y = f'(a)(x-a) + f(a)$

ec. de la recta tg en $(a, f(a))$

$f(0) = g(0)$ $\qquad$ $f'(0) = g'(0)$

$\left.\begin{array}{l} f(0) = e^0 = 1 \\ g(0) = b \end{array}\right\} \rightarrow b = 1$

$f'(x) = e^x$ $\qquad$ $g'(x) = -2x + a$

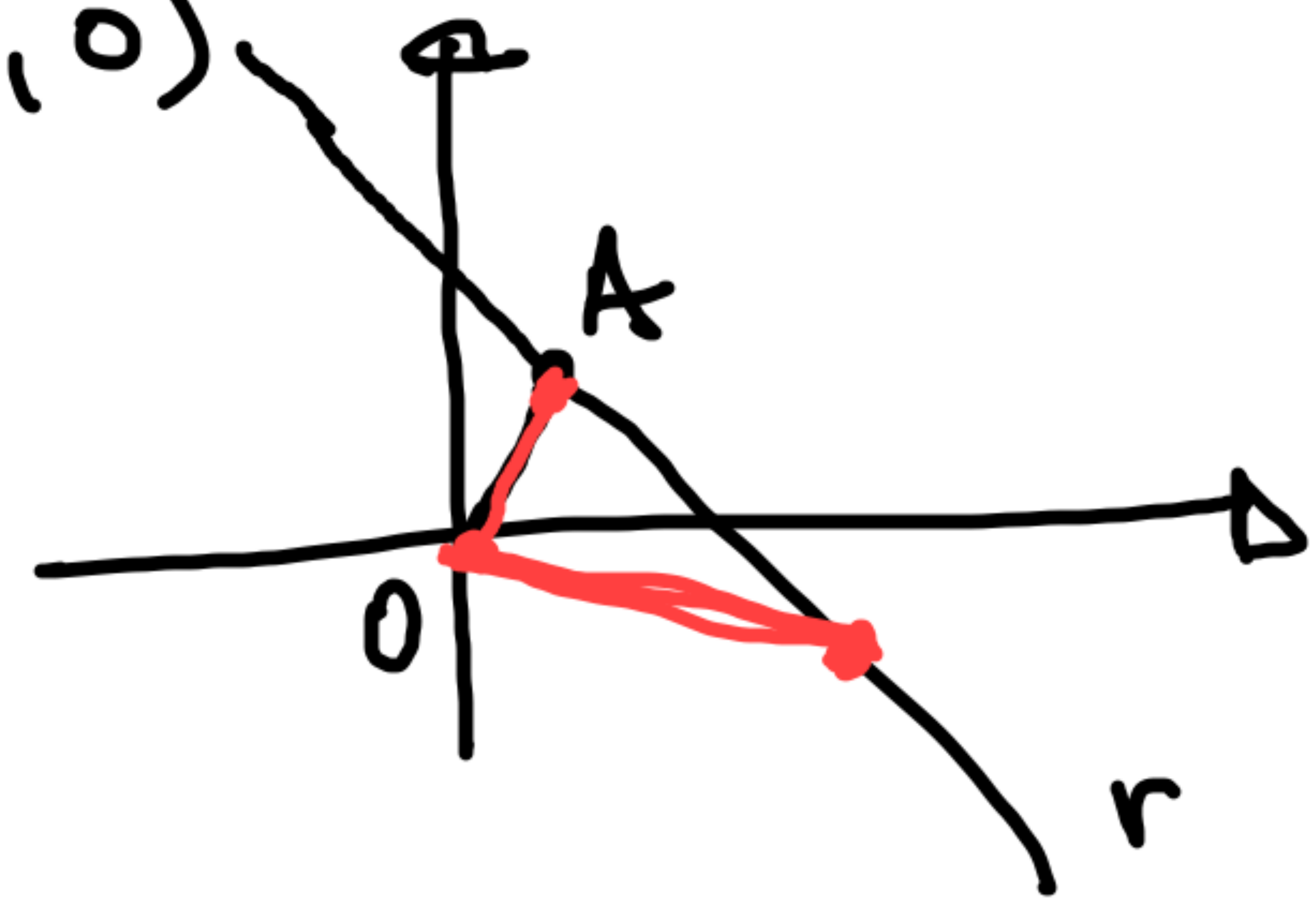
$g'(0) = a$

$f'(0) = 1 \longrightarrow a = 1$

$\boxed{r: 3x+4y-7=0}$

$A \in r$, $d(A,O)$ sea mínima.

$O(0,0)$



$A(x_A, y_A)$

$B(x_B, y_B)$

$$d(A,B) = \sqrt{(x_A-x_B)^2+(y_A-y_B)^2}$$

$A(x,y)$

$$d(A,O) = \sqrt{x^2+y^2}$$

$$3x+4y-7=0 \rightarrow y = -\frac{3}{4}x+\frac{7}{4}$$

$$d(A,O) = \sqrt{x^2+\left(-\frac{3}{4}x+\frac{7}{4}\right)^2}$$

$$d(x) = \sqrt{x^2 + \left(-\frac{3}{4}x + \frac{7}{4}\right)^2}$$

$$d'(x) = \frac{1}{2\sqrt{x^2 + \left(-\frac{3}{4}x + \frac{7}{4}\right)^2}} \cdot \left(2x + \cancel{2}\left(-\frac{3}{4}x + \frac{7}{4}\right) \cdot \left(-\frac{3}{4}\right)\right) =$$

$$\frac{2x + \frac{9}{8}x - \frac{21}{8}}{2\sqrt{x^2 + \left(-\frac{3}{4}x + \frac{7}{4}\right)^2}} = \frac{25x - 21}{16\sqrt{x^2 + \left(-\frac{3}{4}x + \frac{7}{4}\right)^2}}$$

> 0

$$25x - 21 = 0 \rightarrow x = \frac{21}{25}$$

$sg(d'(x))$: $-$ 0 $+$ at $21/25$, $\searrow$ $\nearrow$

$$d(21/25)$$

$$\left(21/25 \, ; \, d(21/25)\right)$$