Practico 5

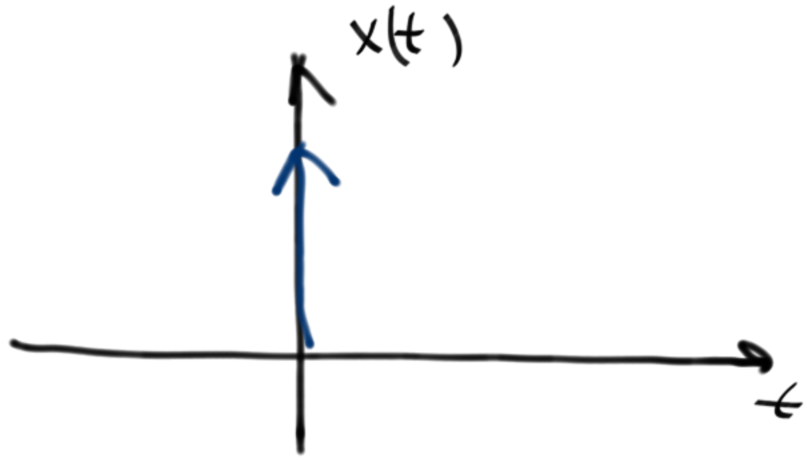
Series de Fourier

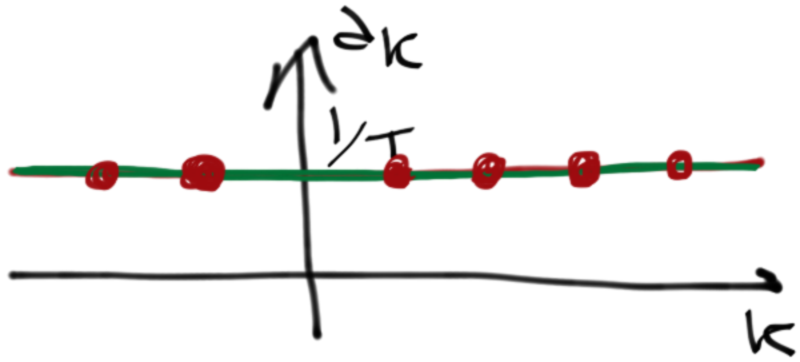
Propiedades y ejemplos

Ejemplo 3.8

a) $x(t) = \delta(t)$

$$a_k = \frac{1}{T}\int_{\langle T\rangle} \delta(t)\,\overbrace{e^{-j\omega_0 k t}}^{1} = \frac{1}{T}$$

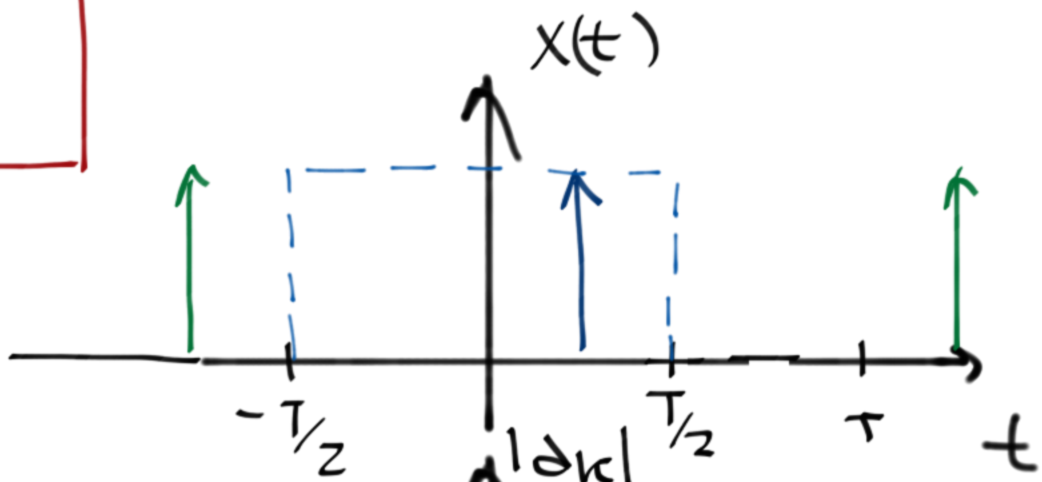




b) $x(t) = \delta(t-\tau)$

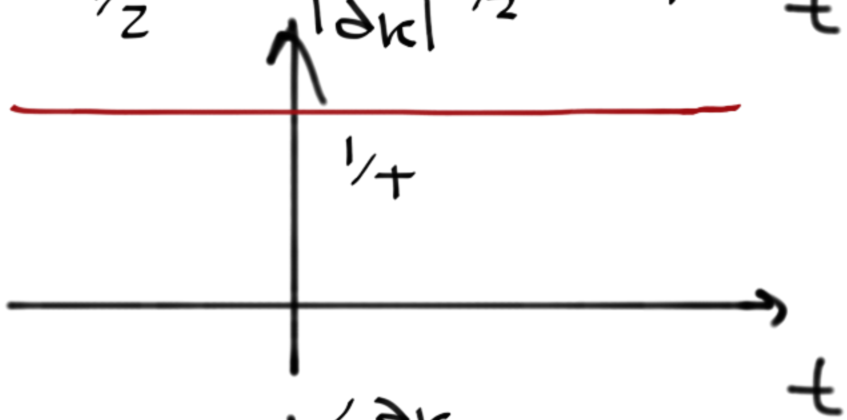
$$a_k = \frac{1}{T}\int_{\langle T\rangle} \delta(t-\tau)\, e^{-j\omega_0 k t}\, dt$$

$$\boxed{a_k = \frac{1}{T}\cdot e^{-j\omega_0 \tau k}}$$



obs: tren de impulsos

$$x(t) = \sum_{\ell=-\infty}^{\infty} \delta(t-\ell\tau)$$

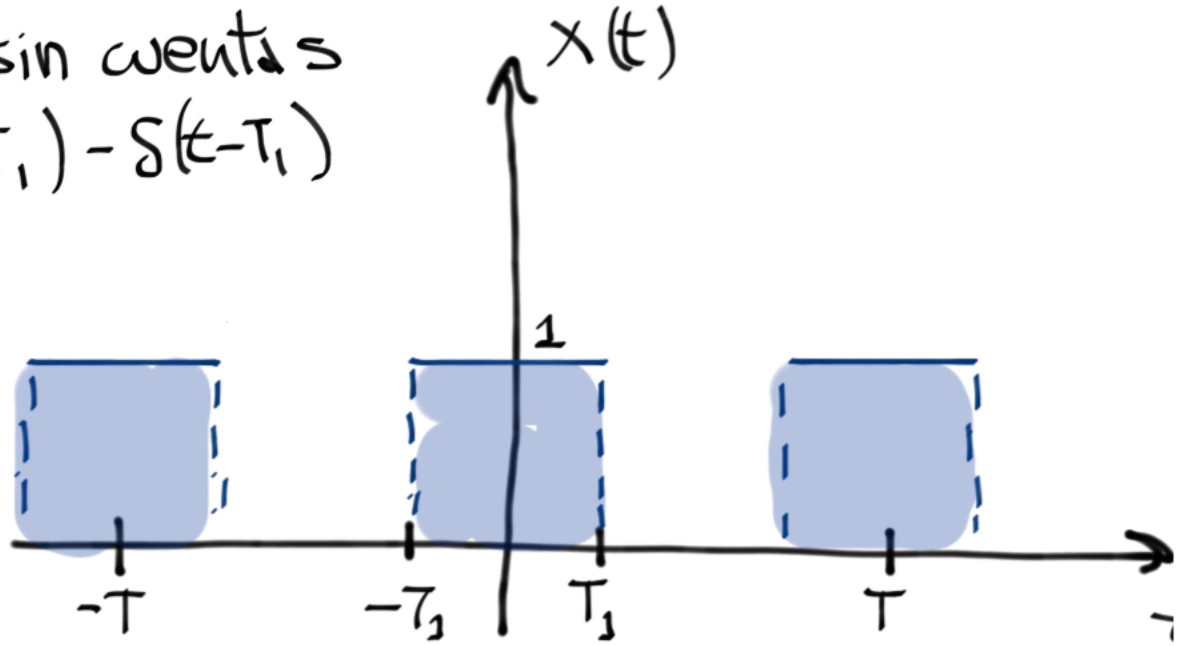


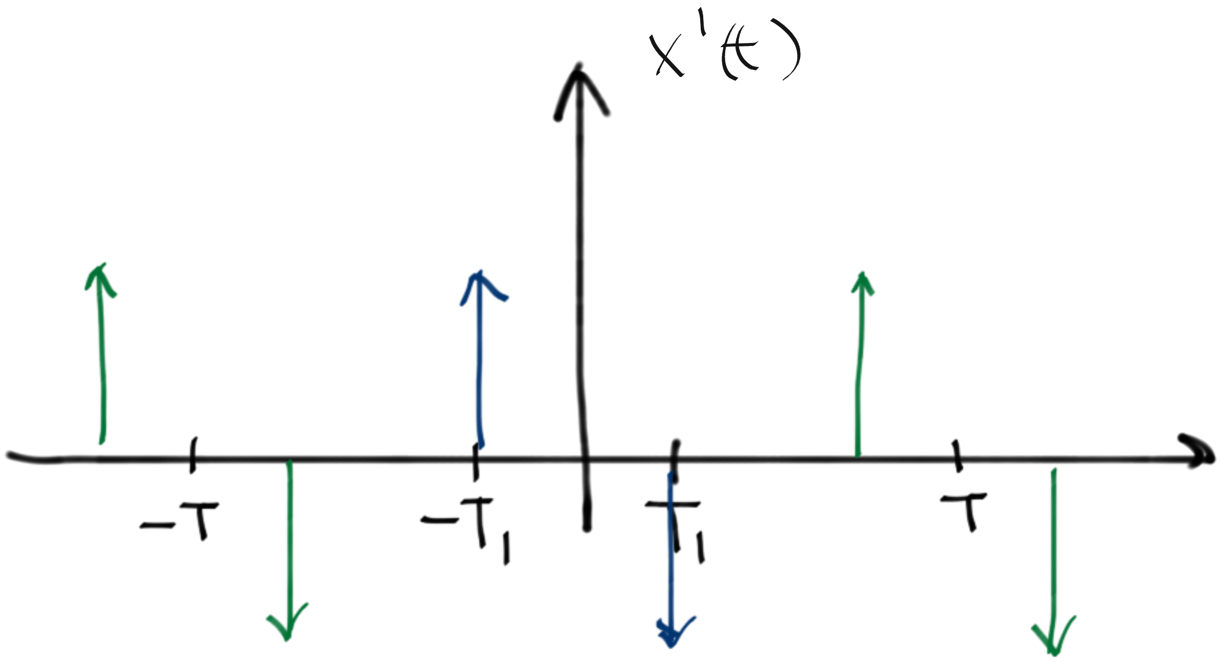
c) idem 3.5 pero sin cuentas

$$g(t) = x'(t) = \delta(t+T_1) - \delta(t-T_1)$$

$$g(t) \overset{\mathcal{F}}{\longleftrightarrow} a_k$$

$$x(t) \overset{\mathcal{F}}{\longleftrightarrow} b_k$$





$$a_k = \frac{1}{T}\left(e^{j\omega_0 T_1 k} - e^{-j\omega_0 T_1 k}\right)$$

$$b_k = \frac{1}{T}\,\frac{e^{j\omega_0 T_1 k} - e^{-j\omega_0 T_1 k}}{j\omega_0 k} = \boxed{\frac{T_1}{T}\,\mathrm{senc}(\omega_0 T_1 k)}$$

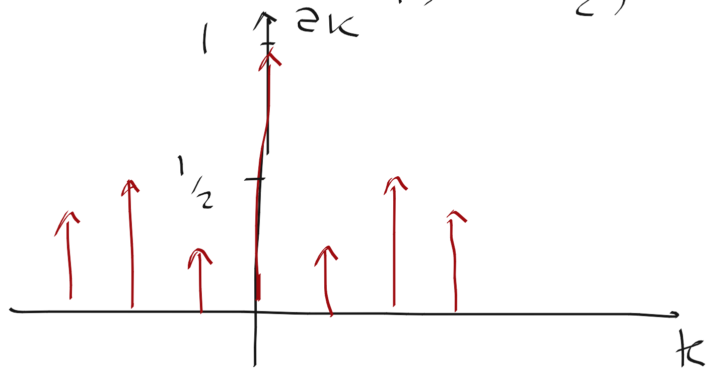
Practico 6

Series de Fourier

Propiedades y ejemplos

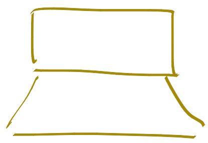
Ejemplo 3.2 (p187)

$$x(t)=\sum_{k=-\infty}^{\infty} a_k \cdot e^{jk\omega_0 t} \qquad a_0=1,\ a_{\pm 1}=\frac{1}{4},\ a_{\pm 2}=\frac{1}{2},\ a_{\pm 3}=\frac{1}{3}$$



$$x(t)=1+\underbrace{\underbrace{\frac{1}{4}\cdot e^{j\omega_0 t}}_{a_1}+\underbrace{\frac{1}{4}\cdot e^{-j\omega_0 t}}_{a_{1}}}_{\text{Conjugados}}+\underbrace{\frac{1}{2}e^{j2\omega_0 t}}_{a_2}+\underbrace{\frac{1}{2}e^{-j2\omega_0 t}}_{a_{-2}}+\underbrace{\frac{1}{3}\cdot e^{j3\omega_0 t}}_{a_3}+\underbrace{\frac{1}{3}\cdot e^{-j3\omega_0 t}}_{a_{-3}}$$

$$\boxed{x(t)=1+\frac{1}{2}\cos(\omega_0 t)+\cos(2\omega_0 t)+\frac{2}{3}\cdot\cos(3\omega_0 t)}$$

Obs: ver simulación en PC 

Obs:
* puedo descomponer cualquier señal como una serie de cosenos
* puedo hacer lo mismo con senos

* no lo vamos a ver pero está en las notas

Representación como cosenos

obs: * ej. anterior $a_k = a_{-k}^*$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jw_0kt} = a_0 + \sum_{k=1}^{\infty} a_k e^{jw_0kt} + \sum_{k=-\infty}^{-1} a_k e^{jw_0kt}$$

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k e^{jw_0kt} + \sum_{k=1}^{\infty} a_{-k} e^{-jw_0kt}$$

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k e^{jw_0kt} + \sum_{k=1}^{\infty} a_k^* \cdot e^{-jw_0kt}$$

$$a_k = A_k e^{j\theta_k}$$

$$x(t) = a_0 + \sum_{k=1}^{\infty} \mathrm{Re}\{a_k \cdot e^{jw_0kt}\}$$

$$\boxed{x(t) = \sum_{k=0}^{\infty} A_k \cdot (\cos w_0t + \theta_k)}$$

obs: * puedo descomponer cualquier señal como suma de cosenos

* idem seno

Practico 7

Series de Fourier en t. discreto

Propiedades y ejemplos

PROPIEDADES

* no las vamos a ver (se pueden consultar en el libro, tabla 3.2, p. 221)
* Parseval.

Parseval

$$\underbrace{\frac{1}{N} \sum_{n=\langle N \rangle} |x[n]|^2}_{\text{reparto de energía en el tiempo.}} = \underbrace{\sum_{k=\langle N \rangle} |a_k|^2}_{\text{reparto en frecuencia.}}$$

Ejemplo 3.12 (cont.)

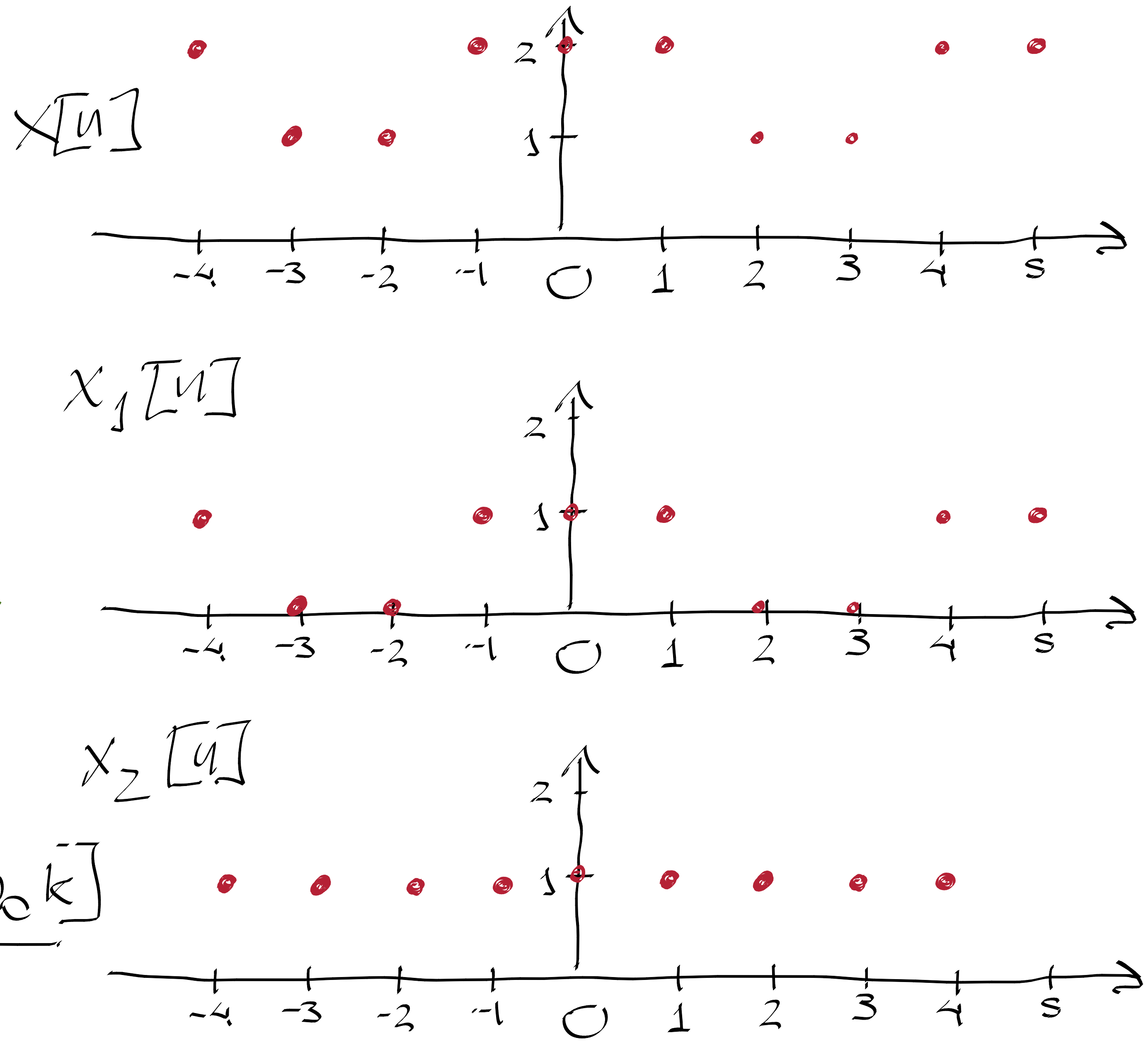
c) simulación

* $E = \frac{1}{N} \sum_{k=-N_1}^{N_1} 1 = \boxed{\frac{2N_1+1}{N}}$ caso particular $E = \frac{6}{30} = \boxed{\frac{1}{5}}$

* Simulación de la reconstrucción en PC:

  → reconstrucción exacta
  → reconstrucción parcial: – uso sólo $N_r < N$ coefs. (ver que pasa con N chico)
  – recalcular con N grande

Ejemplo 3.13 (p. 224)



$$x[n] = x_1[n] + x_2[n]$$

$N_1 = 1$, $N = 5$

$x_1[n] = P_{N_1}[n]$ $\qquad \omega_0 = \dfrac{2\pi}{N}$

$x_2[n] = 1$

$x_1[n] \xleftrightarrow{\mathcal{F}} a_k$

$x_2[n] \xleftrightarrow{\mathcal{F}} b_k$

$\boxed{k \neq rN}$

$$a_k = \begin{cases} \dfrac{1}{N} \cdot \dfrac{\operatorname{sen}\left[\left(N_1 + \frac{1}{2}\right)\omega_0 k\right]}{\operatorname{sen}\left[\frac{\omega_0}{2} k\right]} \\[2ex] \dfrac{2N_1 + 1}{N} \end{cases}$$

$$a_k = \begin{cases} \dfrac{1}{5} \cdot \dfrac{\operatorname{sen}\left[\frac{3}{\cancel{2}} \frac{\cancel{2}\pi}{5} \cdot k\right]}{\operatorname{sen}\left[\frac{\cancel{2}\pi}{5} \cdot \frac{k}{\cancel{2}}\right]} = \dfrac{1}{5} \dfrac{\operatorname{sen}\left[\frac{3\pi}{5} \cdot k\right]}{\operatorname{sen}\left[\frac{\pi}{5} \cdot k\right]} \\[2ex] \dfrac{3}{5} + 1 \end{cases}$$

$$b_k = \delta[k]$$

Practico 9

Series de Fourier en t. discreto

Sistemas basados en ecuaciones en diferencias

FILTROS DISCRETOS BASADOS EN EC. DIF.

Ejemplo IIR (p 244)

$$x[n] = e^{j\omega t}$$

$$y[n] - a\,y[n-1] = x[n]$$

$$y[n] = H(e^{j\omega}) \cdot e^{j\omega t}$$

$$H(e^{j\omega})\,\cancel{e^{j\omega t}} - a\,H(e^{j\omega}) \cdot \cancel{e^{j\omega t}} \cdot e^{-j\omega} = \cancel{e^{j\omega t}}$$

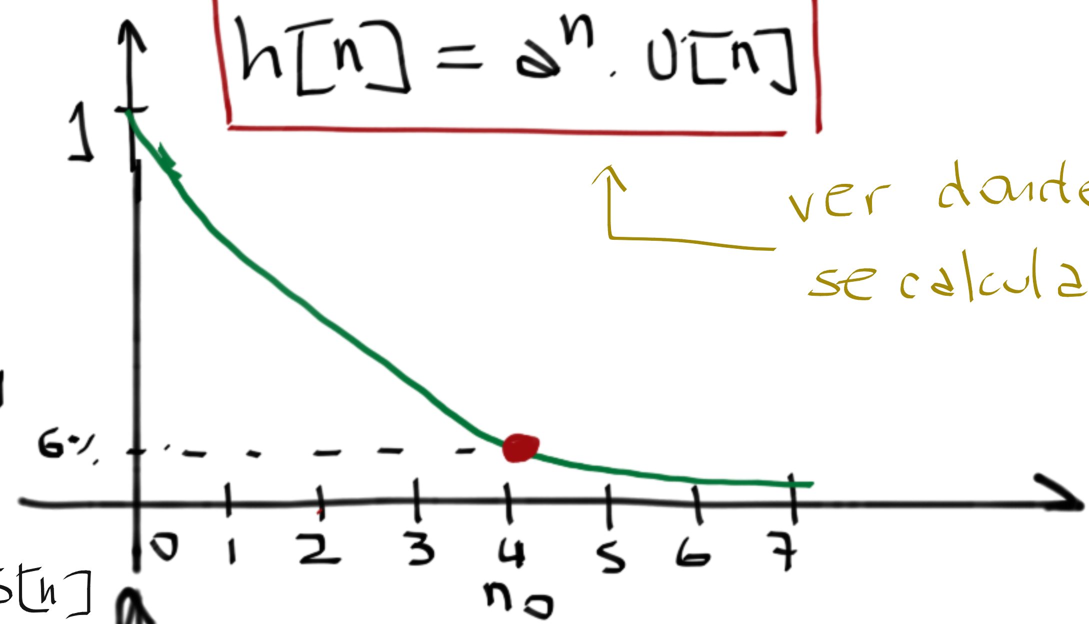
$$(1 - a e^{-j\omega})\,H(e^{j\omega}) = 1$$

$$\boxed{H(e^{j\omega}) = \frac{1}{1 - a e^{j\omega}}}$$

$$\boxed{s[n] = \frac{1 - a^{n+1}}{1 - a} \cdot u[n]}$$

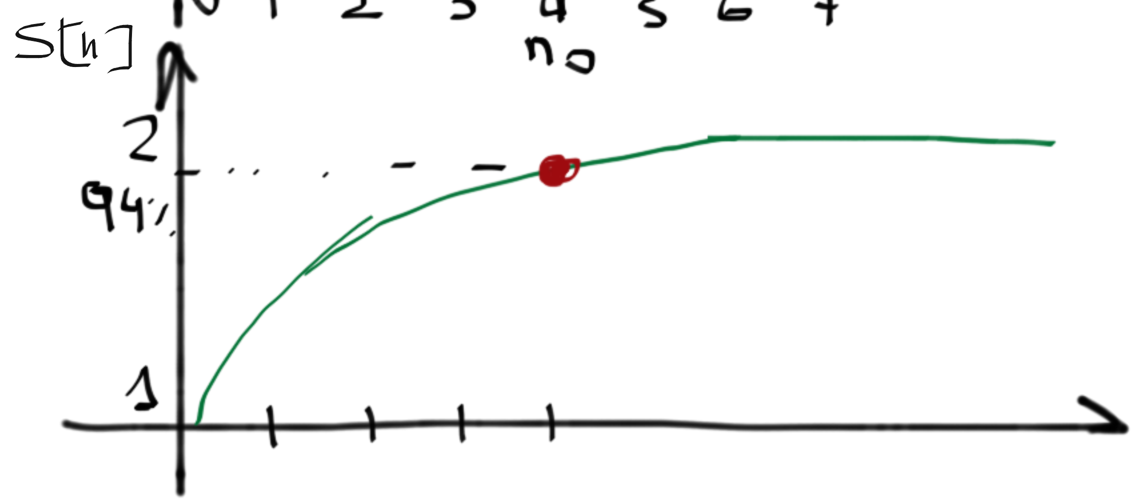
$$\boxed{h[n] = a^{n} \cdot u[n]}$$

ver donde se calcula



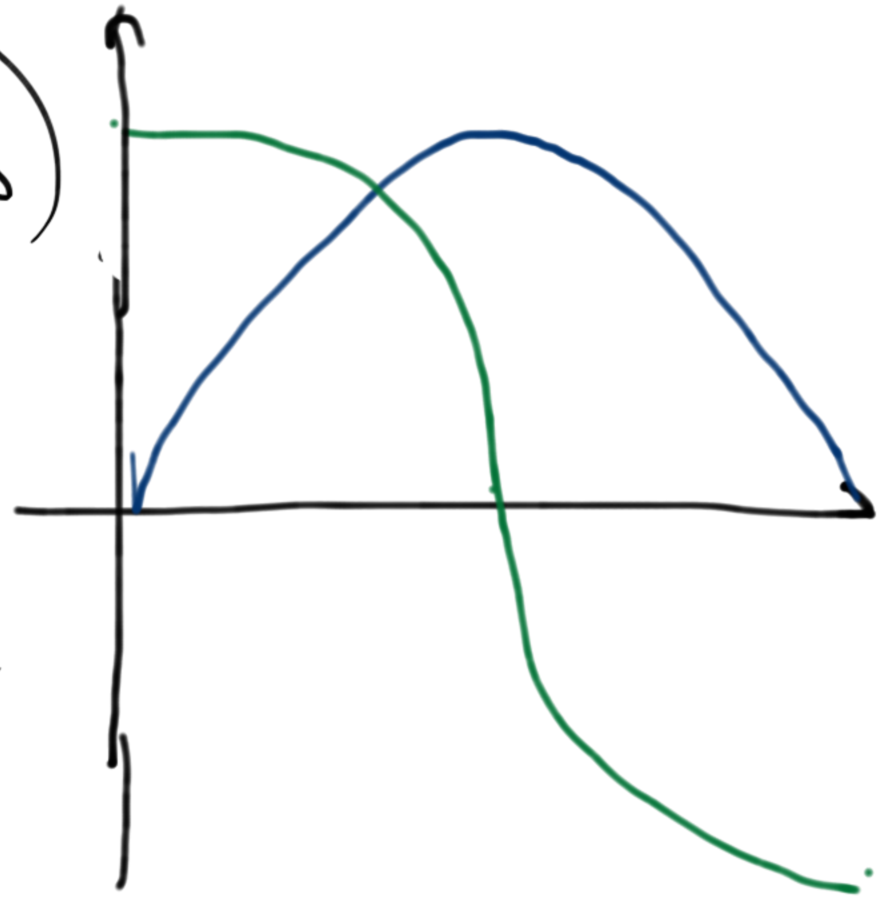
$$n_0 = \left\lceil \frac{\log(0.1)}{\log(a)} \right\rceil = \lceil 3.3 \rceil = 4$$

$$a^4 = 0.0625$$



$$\left|H(e^{j\omega})\right| = \frac{1}{\sqrt{(1-a\cos w)^2 + a^2 \operatorname{sen}^2 w}}$$

$$\angle H(e^{jw}) = -\operatorname{atan}\left(\frac{a \operatorname{sen} w}{1 - a\cos w}\right)$$

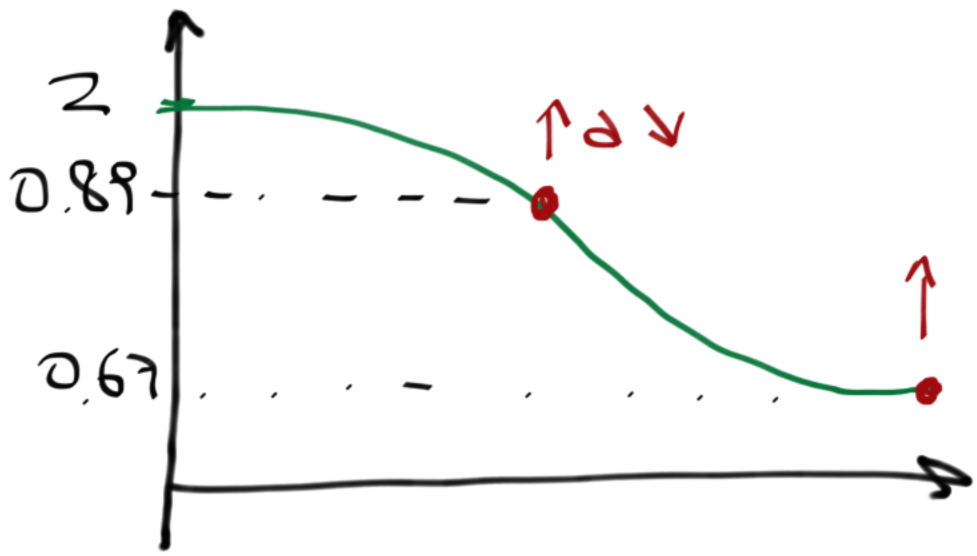


$$\left|H(e^{j0})\right| = 2$$

$$\left|H e^{(j\frac{\pi}{2})}\right| = \frac{1}{\sqrt{1+a}} = 0.89$$

$$\left|H(e^{j\pi})\right| = \frac{1}{\sqrt{(1+a)^2}} = 0.67$$

obs:
* pasabajos: $a>0$
* pasaaltos: $a<0$
* ver 3.17 matlab



$$S[n] = u[n] * h[n]$$

$$S[n] = \sum_{k=-\infty}^{\infty} u[k] \cdot \alpha^k u[n-k] = \sum_{k=0}^{n} \alpha^k = \frac{1-\alpha^{n+1}}{1-\alpha}$$

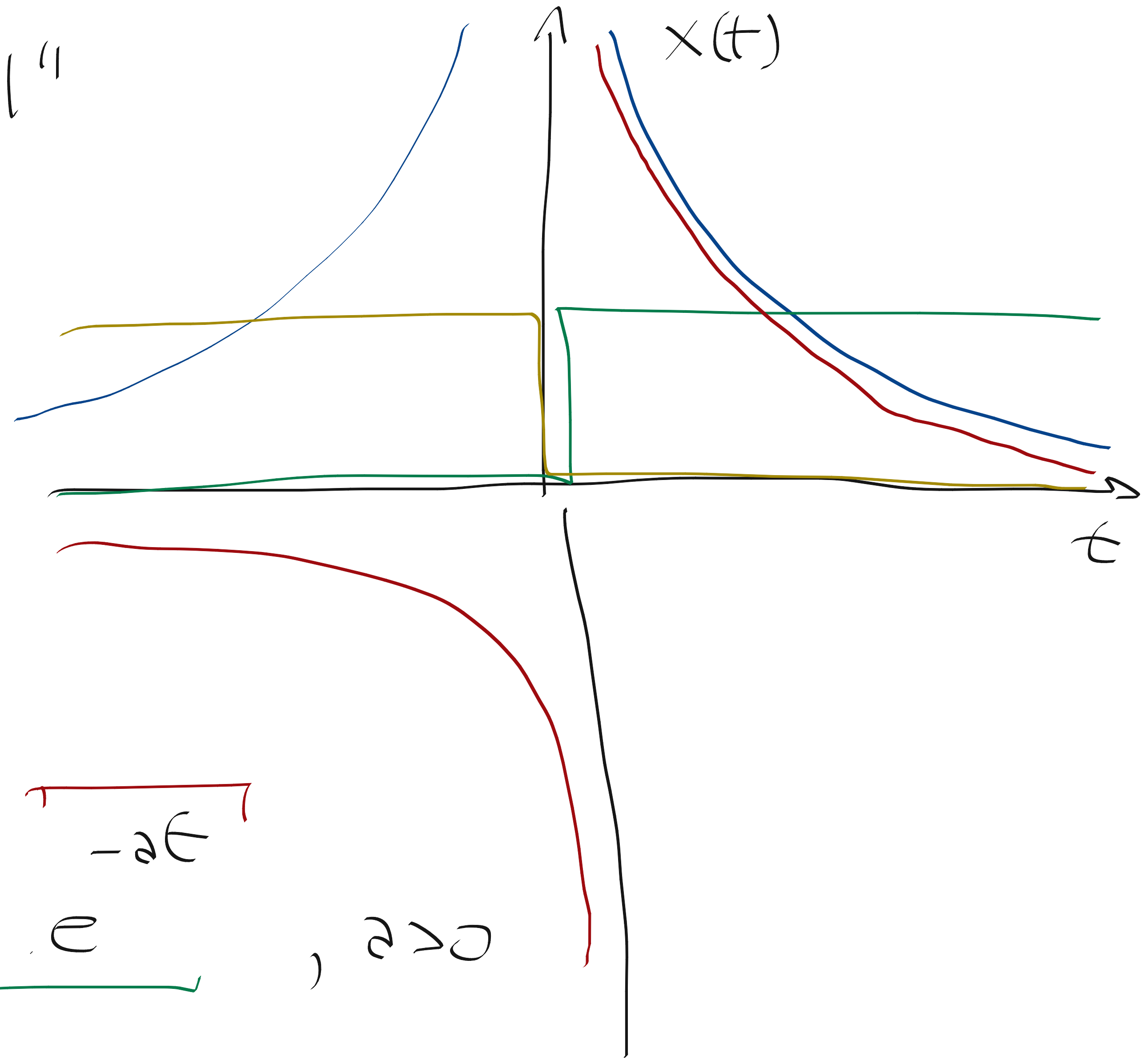
$$\boxed{S[n] = \frac{1-\alpha^{n+1}}{1-\alpha} u[n]}$$

Practico 10

Transformada de Fourier

Ejemplos de cálculo

Ejemplo 4.2: "exponencial bilateral"



1) es lenta o rápida?
2) cómo se compara con la anterior
3) escribir la función

$$x(t) = u(-t)\,e^{at} + u(t)\,e^{-at} \quad , \; a>0$$

$$x(t) = \begin{cases} e^{-at} & t>0 \\ e^{at} & t<0 \end{cases}$$

$$x(t) = e^{-a|t|}$$

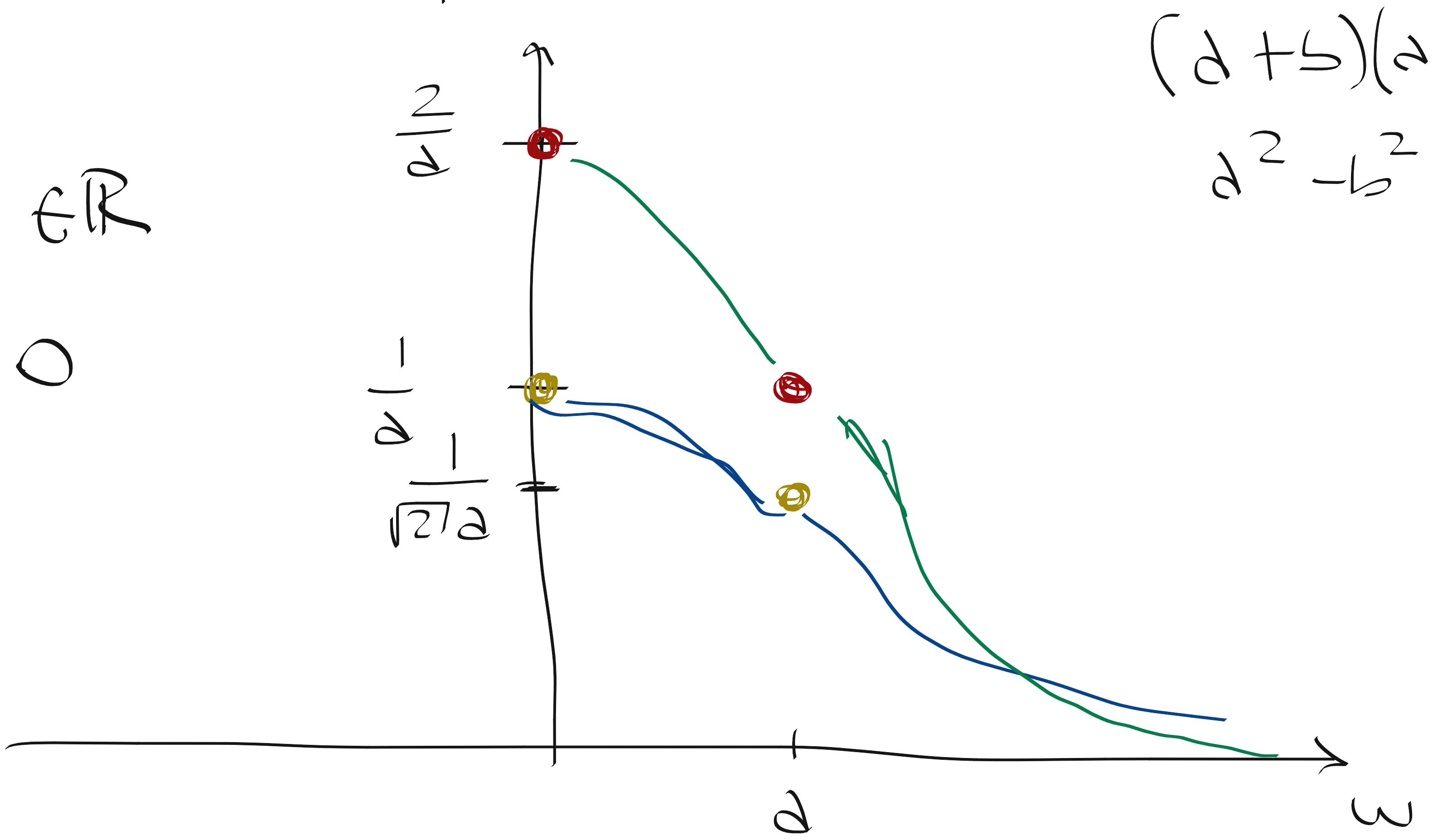
4) calcular $X(j\omega)$

$$e^{at}\,e^{-j\omega t} \qquad e^{-at}\,e^{-j\omega t}$$

$$\frac{1}{a-j\omega} + \frac{1}{a+j\omega} = \frac{2a}{a^2+\omega^2}$$

$$(a+b)(a-b)$$
$$a^2 - b^2$$

Obs:
* $X(j\omega) \in \mathbb{R}$
* fase $= 0$



Practico 11

Transformada de Fourier

Ejemplos integradores

Ejemplo integrador

* limitaciones del análisis: oculograma vs batido

Practico 12

Transformada de Fourier

Ejemplos tiempo discreto

Ejemplo 5.3 (p. 365)

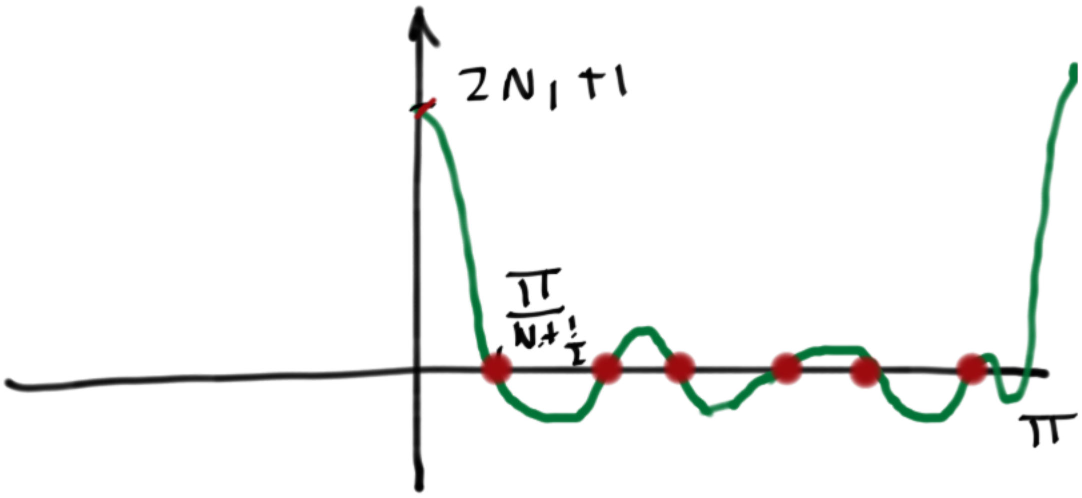
$$x[n] = \begin{cases} 1 & |n| \le N_1 \\ 0 & |n| > N_1 \end{cases}$$

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-jnw} = \sum_{n=-N_1}^{N_1} e^{-jnw} \underset{\substack{\text{C.V.} \\ m=n+N_1}}{=} \sum_{n=0}^{2N_1} e^{-j(m-N_1)w}$$

$$= e^{+jN_1w} \sum_{n=0}^{2N_1} e^{-jmw} = e^{+jN_1w} \cdot \frac{1-e^{-j(2N_1+1)w}}{1-e^{-jw}} =$$

$$= \frac{e^{+jN_1w} - e^{-j(N_1+1)w}}{1-e^{-jw}} \cdot \frac{e^{j\frac{w}{2}}}{e^{j\frac{w}{2}}} = \frac{e^{j(N_1+\frac{1}{2})w} - e^{-j(N_1+\frac{1}{2})w}}{e^{j\frac{w}{2}} - e^{-j\frac{w}{2}}}$$

$$\boxed{X(e^{jw}) = \frac{\operatorname{sen}[(N_1+\frac{1}{2})w]}{\operatorname{sen}(\frac{w}{2})}}$$



obs: * senoc periódico

$$\lim_{w \to 0} X(e^{jw}) = \frac{w(N_1+\frac{1}{2})}{w/2} = \boxed{2N_1+1}$$

ceros: $\boxed{w_k = \frac{\pi}{N_1+\frac{1}{2}}}$

Para pasar

Convergencia

$$\hat{x}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} 2 \cdot \frac{\operatorname{sen}(wT_1)}{w} e^{jwt} dw$$

$$|\hat{x}(t)|^2 \leq \frac{1}{2\pi}\int_{-\infty}^{\infty} 2 \left|\frac{\operatorname{sen}(wT_1)}{w}\right|^{2 \leq 1} \cdot \left|e^{jwt}\right|^{=1} dw \leq \frac{2}{\pi}\int_{\sigma \; B}^{\infty \; A} \frac{\operatorname{sen}^2(wT_1)}{w^2} dw$$

A) $\frac{2}{\pi}\int_{1}^{\infty} \frac{\operatorname{sen}^2(wT_1)}{w^2} dw \leq \int_1^{\infty} \frac{1}{w^2} dw < \infty$

B) $\frac{2}{\pi}\int_{0}^{1} \frac{\operatorname{sen}^2(wT_1)}{w^2} dw$ y finito!!

* ¿a cuanto converge?

$$\hat{x}(T_1) = \frac{1}{2\pi}\int_{-\infty}^{\infty} 2\frac{\operatorname{sen}(wT_1)}{w} \cdot e^{jwT_1} \cdot dw = \frac{1}{\pi}\int_{-\infty} \frac{1}{2jw}\left(e^{j2wT_1} - 1\right)$$

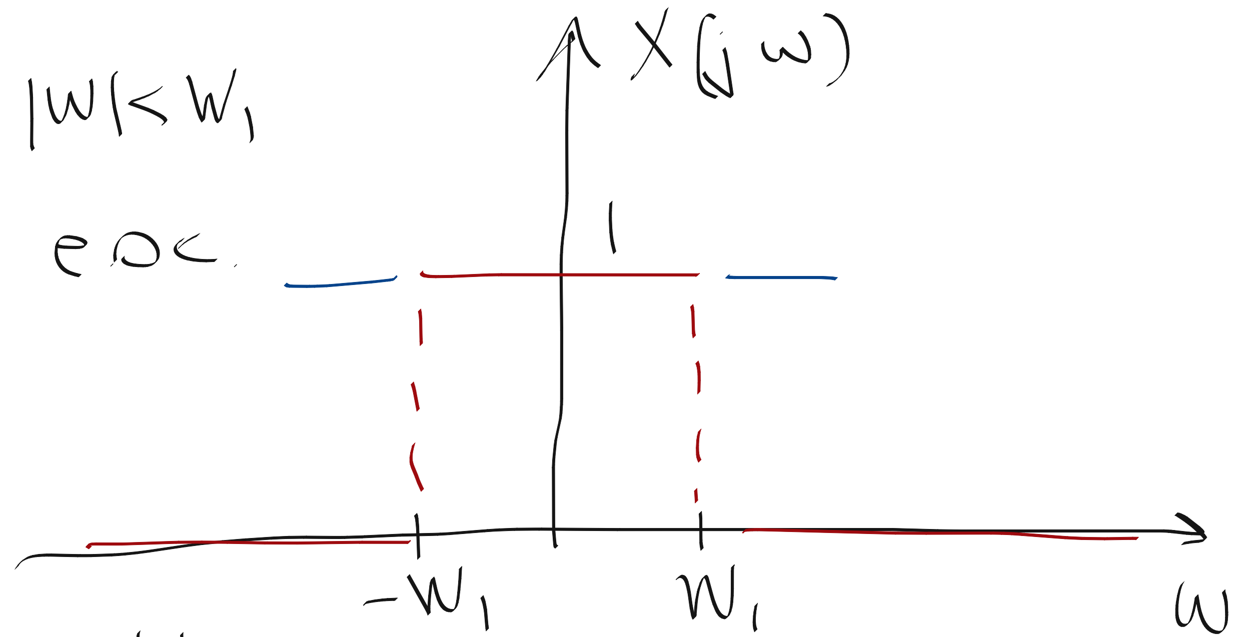
$$\hat{x}(T_1) = \frac{1}{2j\pi} \cdot \left(\int_{-\infty}^{\infty} \frac{e^{j2wT_1}}{w} - \int_{-\infty}^{\infty}\frac{1}{w}\right)$$

$$\int_{-\infty}^{\infty} \frac{1}{w} \cdot e^{j2wT_1} dw$$

$$\boxed{\text{sin terminar}}$$

Ejemplo 4.5

$$X(j\omega) = \begin{cases} 1 & |\omega| < W_1 \\ 0 & e.o.c. \end{cases}$$

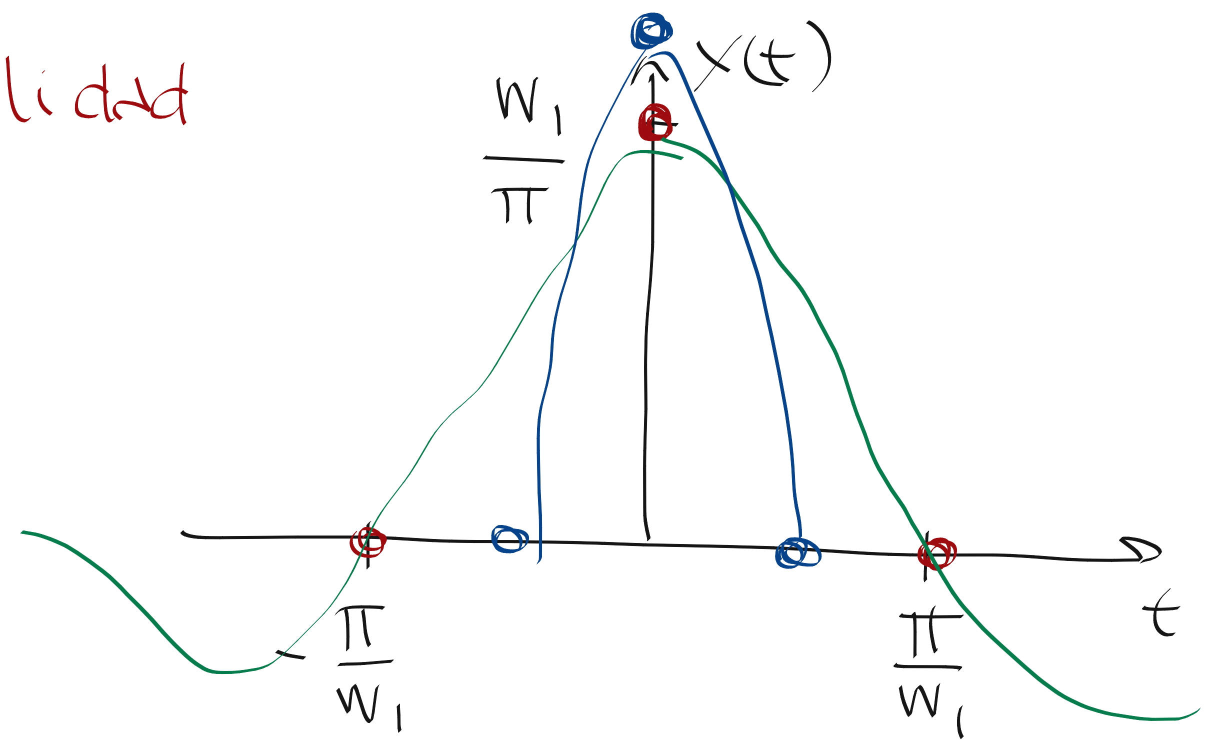


$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

$$x(t) = \frac{1}{2\pi}\int_{-W_1}^{W_1} 1 \cdot e^{j\omega t}\, d\omega = \frac{1}{2\pi} \cdot \frac{e^{j\omega t}}{jt}\Bigg|_{-W_1}^{W_1} = \frac{1}{2\pi} \cdot \frac{e^{jW_1 t} - e^{-jW_1 t}}{jt} = \frac{W_1}{W_1} \cdot \frac{\operatorname{sen}(W_1 t)}{\pi t}$$

$$\boxed{x(t) = \frac{W_1}{\pi} \cdot \operatorname{senc}(W_1 t)}$$

Dualidad



obs: * dualidad (con el 4.4)

Ejemplo 4.4 (rec. parcial)