INGENIERIA INDUSTRIAL

Combinatorial Challenges in Forest Management Modelling

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Classic Forest Problems

- Linear Programming
- MIP's
  - Harvesting + Road Construction
  - Adjacency
  - Machine Location
  - Uncertainty problems

Linear Programming

- Traditional models appeared during 70's (US Forest Service)
- Forest represented by basic units (stands), sharing homogeneous forest areas.
- Maximize Net return
- Main Decisions/Constraints:
  - # of Ha of stands to Harvest each period

Linear Programming

Maximize $$\sum_t \sum_t c_i^t x_i^t$$

Subject to $$\sum_t v_i^t x_i^t = H^t$$

$$\sum_t x_i^t \leq a_i$$

$$\beta^L H^{t-1} \leq H^t \leq \beta^H H^{t-1}$$ **Flow control**

$$x_i^t \geq 0$$

where: $$H^t \geq 0$$

- x_i^t **N° of Has. Stand i, harvested period t**
- c_i^t **is Net return of harvesting 1 ha. of stand i, period t**
- H^t **is total volume harvested in period t**
- v_i^t **is the volume per ha. Obtained in stand i, period t**

Linear Programming

- All variables are continuos.
- Easy to solve in reasonable time by any LP commercial solver.
- Does not consider spatial relationships.
- Widely used.

MIP: Road Construction

- Spatial relationships introduced during 70’s and 80’s.
- Road Building 0-1 decisions, to access areas to be harvested, with an associated cost.

MIP: Road Construction

- Main Decisions
  - Road Building
  - Amount of Timber Flow per road
  - Harvest
- Main Constraints
  - Flow Capacity
  - Relation flows roads
  - Flow Conservation at different nodes (production, intersection and demand)
  - Demand bounds

MIP: Road Construction

- **Applied in**

  US Forest Service 1980's

  Weintraub, Kirby et al Operation Research (1994)

  Solution algorithm: LP and Heuristics

MIP: Road Construction

- Chile
  - Forestal Millalemu 1990’s
  - Andalaft et al, Operation Research (1999)

Model

- Main Decisions
  - Harvest stands per period (Three products, 17 independent forests), potential roads (two types), road upgrade posibility.
  - Stocking yards.
- Main Constraints
  - Flow conservation within different nodes (Origin, Intersection, stocking and exit).
  - Flow needs road building.
  - Road and stocking capacities.
  - Global Demand constraints.

Model

- **Variables:**

$$x_{s,t} = Has. of\ stand\ s\ harvested\ in\ period\ t$$

$$y_{i,t}^{k} = Timber\ volume\ of\ type\ in\ period\ t, origin\ i$$

$$F_{ij,r}^{k,t} = Flow\ of\ timber\ on\ arc\ (i,j)\ type\ r, period\ t$$

$$Z_{k,m}^{t} = Amount\ of\ timber\ delivered\ in\ market\ m, period\ t$$

$$I_{c}^{t,k} = Inventory\ of\ timber, period\ t, stocking\ yard\ c$$

$$W_{ij,r}^{t} = \begin{cases} 1 & if\ road\ (ij)\ is\ built\ at\ standard\ r\ in\ period\ t \\ 0 & \sim \end{cases}$$

$$V_{ij}^{t} = \begin{cases} 1 & if\ road\ (ij)\ is\ upgraded\ in\ period\ t \\ 0 & \sim \end{cases}$$

$$E_{s,t} = \begin{cases} 1 & if\ stand\ s\ is\ harvested\ in\ period\ t \\ 0 & \sim \end{cases}$$

Model

- Objective Function: Max net present profit
  - Sales income
  - Harvesting Cost
  - Production Cost
  - Transportation costs
  - Road building and upgrading costs
  - Stocking cost

Model

- Main Constraints
  - Flow Conservation
  - Flow and road construction/upgrading relation
  - Demands

Solution Approaches

- Strengthenings:
  - Adjustment of Capacities: flow capacities, tight bound using max production per arcs.
  - Inequalities
    - Road-to-Road triggers: This constraint states that no isolated road should be built.

$$W_{ij,r}^{t} \leq \sum_{q \leq t} \sum_{I \in N(ij)} \sum_{r} W_{I,r'}^{q} \qquad \forall r, t, ij \text{ a potential road}$$

N(ij) := set of potential roads connecting to (ij)

Solution Approaches

- **Inequalities**
  - **Project-to-Road triggers: This constraint states that no isolated stand can be entered.**

$$E_{s,t} \leq \sum_{q \leq t} \sum_{ij \in N(s)} \sum_{r} W_{ij,r'}^{q} \qquad \forall t, \forall s \text{ not connected to an existing r}$$

N(s) := set of potential roads accessing stand s

- **Liftings**
  - **Road building and upgrading constraints can be lifted with respect to time.**

Solution Approaches

- Liftings
  - Road building:

$$\sum_k \sum_{\theta \in \Psi(t)} F_{ij,r1}^{k,\vartheta} \leqslant U_{ij,r1}^{t} \cdot \sum_{\theta \in \Psi(t)} W_{ij,r1}^{\theta},$$

$$\forall ij \quad \text{potential road, } \forall t = \text{summer } (r1 = \text{dirt}).$$

  - Road upgrading:

$$\sum_k \sum_{g \leqslant t} F_{ij,r2}^{k,g} \leqslant U_{ij,r2}^{t} \left(\sum_{\theta \in \Psi(t)} W_{ij,r2}^{\theta} + \sum_{\theta \in \Psi(t)} V_{ij}^{\theta} \right),$$

$$\forall ij \text{ potential road,} \quad \forall t \ (r2 = \text{gravel}),$$

$$\sum_k \sum_{g \leqslant t} F_{ij,r2}^{k,g} \leqslant U_{ij,r2}^{t} \cdot \sum_{\theta \in \Psi(t)} V_{ij}^{\theta},$$

$$\forall ij \text{ existing dirt road,} \quad \forall t \ (r2 = \text{gravel}).$$

Lagrangean Relaxation

- Areas are linked through the demand constraints, each period.
- Forest company are geographically independent, possible decomposition of the problem once demand constraints are dualized.
- The problem splits into separate sub-problems, one per area, plus one problem for the timber sales.
- These problems have a much simpler structure and thus are easier to solve.

Lagrangean Heuristic

- The solutions obtained through the Lagrangean relaxation may not satisfy the demand constraints.
- Two ways:
  - Not enough roads built to carry timber to cover demand
  - Harvest excessive timber in some periods and not enough in others
- The heuristic procedure builds a minimum number of additional roads to carry enough timber to satisfy demand, and readjusts production among periods

Test Data

Instances

	No. of Potential Roads	No. of Pre-existing Dirt Roads	No. of Pre-existing Gravel Roads	Density of Potential Arcs
MO	39	106	60	low
MR	145	0	60	low
MC	193	0	60	high

Computational Results

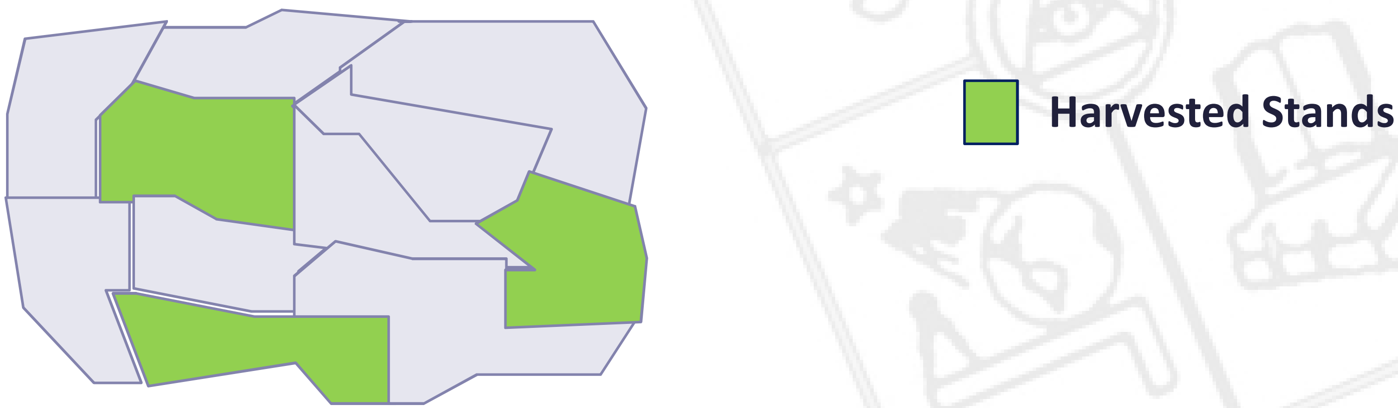
Comparison between solution approaches.

Instance	Using B&B Original Formulation		Adjusting Capacities		Adding Triggers		Lifting Constraints		Lagrangean Relaxation	
	Gap	Time	Gap	Time	Gap	Time	Gap	Time	Gap	Time
MO	17.1	3,605	2.9	3,604	0.9	188	0.4	151	0.3	160
MR	51.5	3,606	5.0	3,605	3.5	3,605	1.8	249	1.1	244
MR_LP	161.8	3,605	71.5	3,606	16.2	3,607	1.9	1,530	1.6	2,416
MC	33.3	3,609	13.1	3,607	9.3	3,609	1.4	1,288	1.9	1,237
MC_LP	123.1	3,606	24.8	3,606	20.3	3,612	16.8	3,608	2.6	4,080
MC_LD	42.5	3,608	11.2	3,610	15.7	3,609	12.8	3,608	1.7	2,107
MC_LP_LD	42.3	3,605	6.0	3,605	2.0	3,606	6.0	3,607	1.5	829

Results on real planning problems show that even as these problems become more complex, the proposed solution strategies lead to very good solutions, reducing the residual gap for the most difficult data set from 162% to 1.6%, and for all data sets to 2.6% or less.

Adjacency Constraints

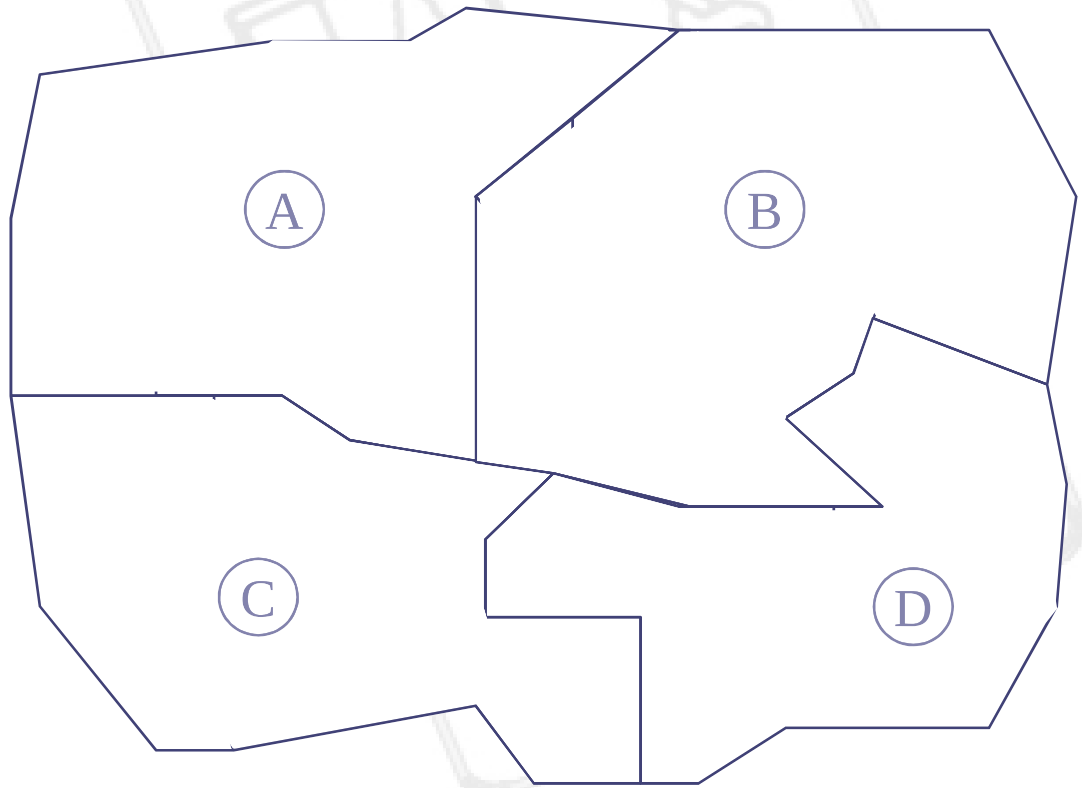
- Harvesting with environmental constraints.
- Main form of constraints
- Harvest with maximum opening size (adjacency)
- Blocks no larger than 40 Has.



First Approach: URM

- Forest planner forms cutting units by blocking basic cells together a priori using GIS (Barrett 1997).

- Max Area of 40 ha implies no adjacent blocks can be harvested at the same time
- For example if A is harvested B,C cannot



URM Formulation

$X_i^t = 1$ if *block* i harvested in period t.

$H^t =$ Volume harvested in period t.

URM model

$$\text{Max} \sum_i \sum_t C_{it} X_i^t$$

s.t. 1) $H^t = \sum_i a_{it} X_i^t$

2) $0.85 \; H^{t-1} \leq H^t \leq 1.15 \, H^{t-1}$

3) $X_i^t + X_j^t \leq 1$ if i, j adjacent

4) $X_i^t = 0,1$ $H^t \geq 0$

This is a weak formulation

Solving URM like Problems

- Heuristics:
  - Tabu search (Murray and Church 1995)
  - Simulated annealing (Murray and Church 1995)
  - Monte Carlo simulations (O'Hare et al. 1989, Nelson and Brodie 1990)
- Exact techniques:
  - Dynamic programming (Hoganson and Borges 1998)
  - Column generation (Barahona et al. 1992). Sub problem is set packing.
  - Formulation strengthening (Murray and Church 1996)

    Use cliques instead of pair wise relations

URM: Column Generation

- Column generation (Barahona et al. 1992).
- 3 Periods problem.
- Sub-problem is set packing.
- The generation of columns is done by solving a stable set problem.
- To preserve the adjacency properties, fractional solutions in the master problem are rounded off to integrality through a heuristic procedure.

URM: Column Generation - MP

$$Max\ Z = \sum_i \sum_j C_{ij} X_{ij}$$

$$\sum_j X_{ij} = 1 \qquad \forall i$$

$$\sum_i \sum_j A_{ijt} X_{ij} \geq F_t \qquad \forall t \quad Minimum\ required\ Timber\ production$$

$$\sum_{i \in Hr} \sum_j L_{ijt} X_{ij} \geq D_{rt} \qquad \forall t, r \quad Hr\ set\ of\ areas\ in\ the\ zone\ r$$

$$0 \leq X_{ij} \leq 1 \qquad \forall i, j$$

where:

- X_{ij} is 1 if area i is managed with alternative j
- A_{ijt} is total timber production of area I, period t ,under choice j
- D_{rt} Minimum number of acres of mature standing timber required for zone r in period t

URM: Column Generation - SP

- Sub-problem consist in a stable set problem (NP-HARD)
- Three stages to solve it:
  1. Greedy Heuristic
  2. If it does not produce a candidate to enter the basis of MP, we solve a LP that represents the stable set problem.
  3. If these 2 phases are still not successful in finding new candidates, use B&B or B&C algorithm to make sure we do not miss any candidate.

URM: Column Generation

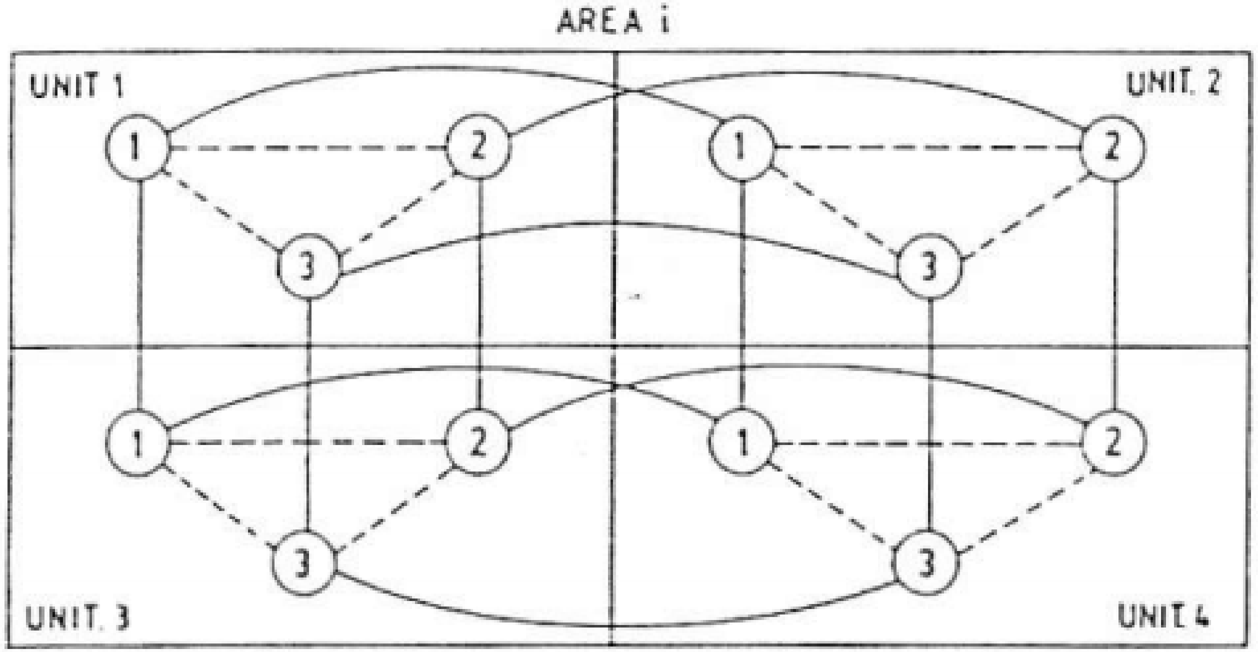
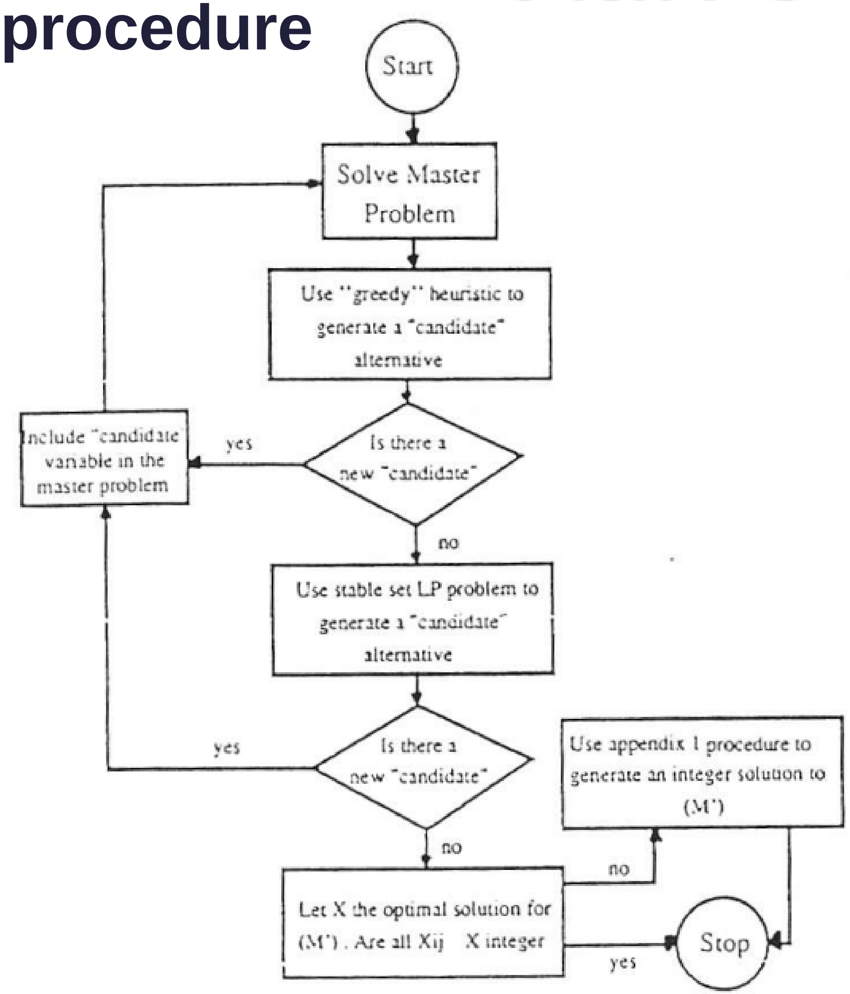


FIGURE 3. The graph reflecting neighboring nodes. The solid lines correspond to adjacency restrictions. The dashed lines correspond to constraining one harvest timing option for each unit.

URM: Column Generation

- **Flowchart of procedure**



Computational Results

Configuration of the three problems.

Problem	Zone	Area	Problem	Zone	Areas
Type 1	1	I, III, V, VII	Type 3	1	II, IX, XI, XI
	2	II, IV, VI, VIII		2	II, II, IV, X
Type 2	1	II, VII, VIII, IX		3	V, VI, VI, IX
	2	II, IV, V, X		4	VI, VI, VII, VII
	3	IV, VI, VI, XI		5	IV, IV, VII, VIII

Computational results obtained for three types of forest configurations.

Problem	P1A	P1B	P2A	P2B	P3A	P3B
Average infeasib (%)	1.1	1.1	0.4	0.1	0.0	0.2
Max infeasib (%)	3.8	5.8	3.8	1.2	0.0	1.1
Reduction in objective value (%)	1.0	−0.2	0.6	0.1	−0.2	0.5

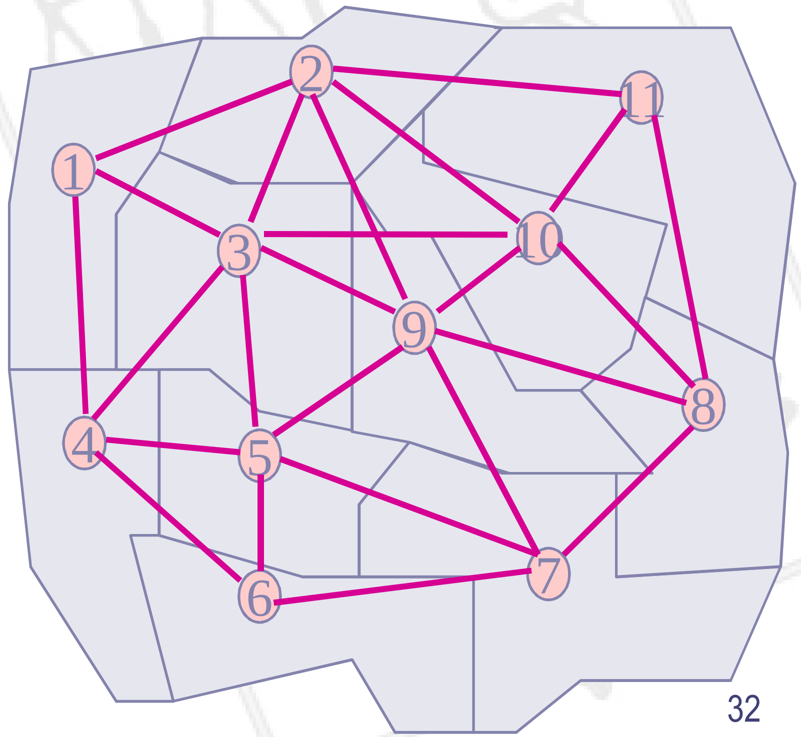
Second Approach: ARM (Murray 1999)

- Incorporate block construction to model
- Basic cells as small as one Ha.
- Considerable profit gains compared to URM (Murray and Weintraub (2002)).
- Far more complex combinatorially
- Solving the ARM
- Mostly Heuristics: (Hokans 1983, Lockwood and Moore 1993, Barrett et al. 1998, Clark et al. 1999, Richards and Gunn 2000, Boston and Bettinger 2001).
- Few exact approaches (McDill and Braze (2000) and Martins et al. (2001), Goycoolea et. al. (2003))

Modeling ARM: Forest Map

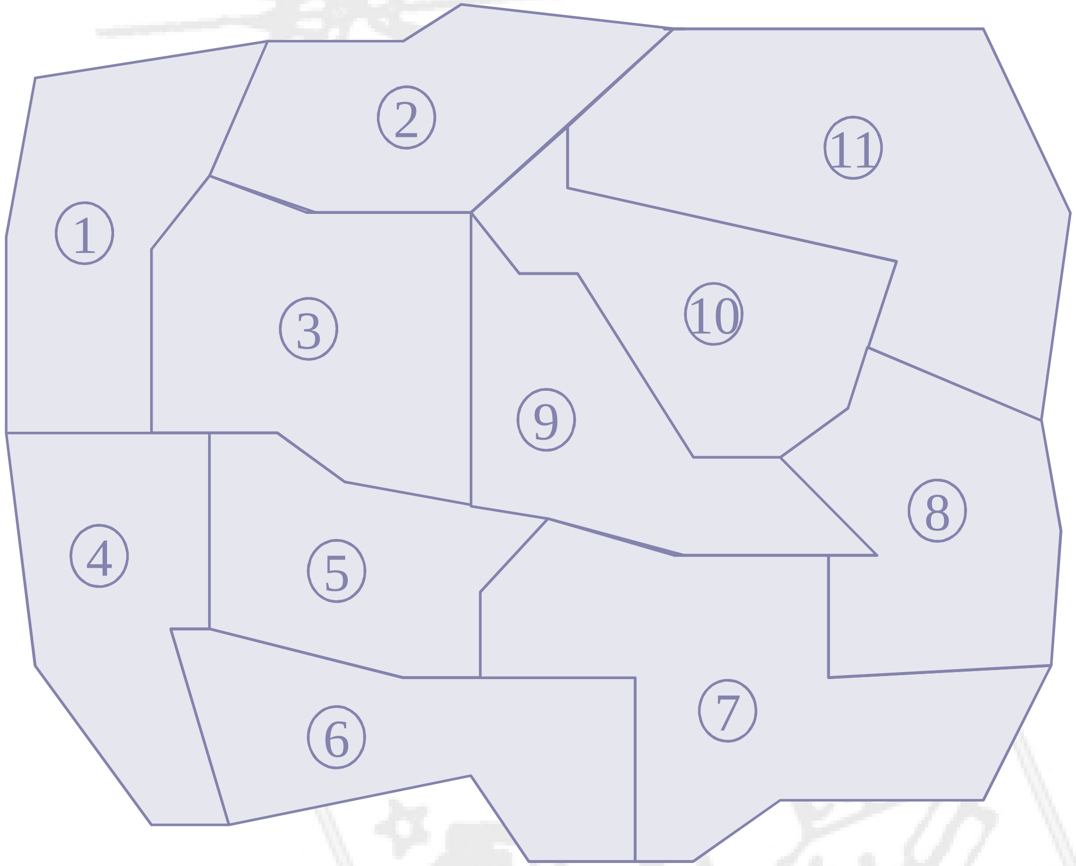
- Forest partitioned into basic cells
- Basic Cells:
  - Known: Area, Volume per Period, Net Profit per Period

- Graph G(V,E):
  - V = {Basic Cells}
  - (u,v) $\in$ E if cells u and v are adjacent



Feasible Clusters

- Feasible Cluster:
  - Any set of contiguous collection of cells
  - Area does not exceed the given maximum area restriction



- Compatible Clusters
  - Are not adjacent
  - Do not share a common cell

Cluster Packing Problem

Maximize $$\sum_{S} c_S x_S$$

subject to $$x_S + x_{S'} \leq 1$$ **for each pair S, S' of incompatible clusters**

$$x_S \in \{0,1\}$$ **for each cluster S $\in$ Λ.**

where:

- c_S is Net Profit of cluster S
- x_S = 1 if cluster s is harvested
- This is a weak formulation:
  - Many constraints
  - LP many fractions

First Strengthened Formulation

- (Martins et al. 2001) more compact formulation
  - For each pair of incompatible cluster S,S' there must exist an arc (u,v) in G such that u Є S and v Є S'

Maximize $$\sum_{S} c_S x_S$$

subject to $$\sum_{S \in \lambda(u,v)} x_S \leq 1$$ for each arc (u,v) in G(V,E)

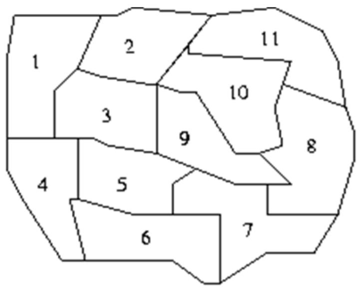
$$x_S \in \{0,1\}$$ for each cluster S ∈ Λ.

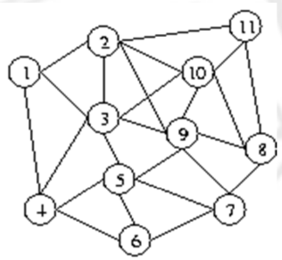
where:

- c_S is Net Profit of cluster S
- $\lambda(u,v)$ is the set of all clusters S such that u Є S or v Є S

Second Strengthened Formulation : Cluster Graph

Define a graph of clusters
G (Λ, Γ) each node in Λ is a feasible cluster, Γ: arcs joining incompatible clusters.





Cluster (1,2,3) is node i ∈ Λ
Cluster (9, 10, 11) is node j ∈ Λ

Arc (i, j) ∈ Γ.

Leads to model node packing in graph G (Λ, Γ)
One approach: define maximal cliques in graph G (Λ, Γ) .

Cliques Cluster Packing Formulation

Maximize $$\sum_{S} c_S x_S$$

subject to $$\sum_{S \in K} x_S \leq 1$$ for each maximal clique K in G(^,Γ)

$$x_S \in \{0,1\}$$ for each cluster S $\in \Lambda$.

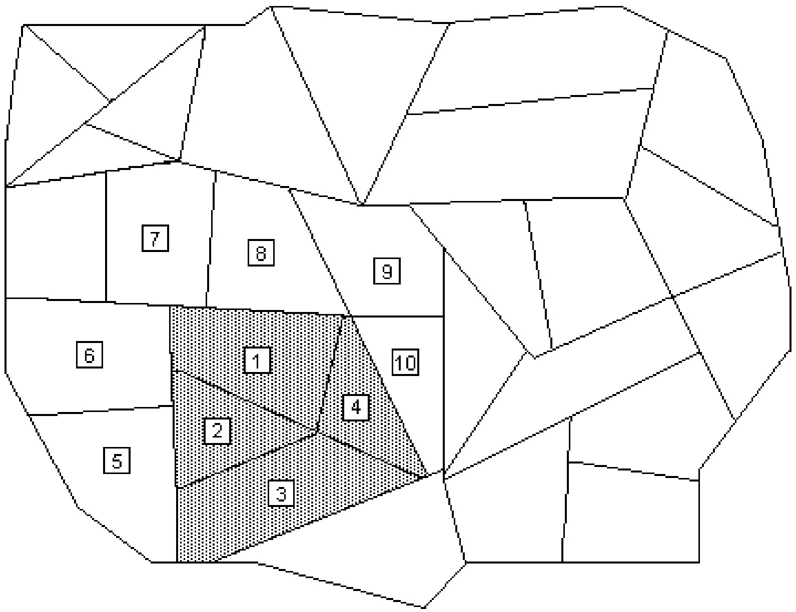
where:

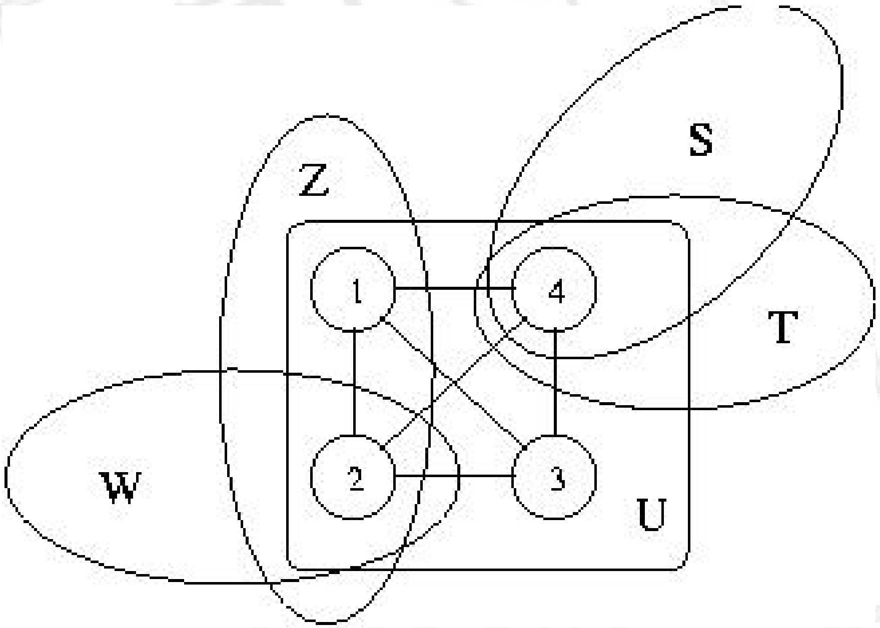
- c_S is Net Profit of cluster S

- These are stronger constraints.
- Note that each pair of incompatible Clusters (S, S') defines an arc in G (^,Γ) and is contained in some maximal clique.
- Problem: Number of max cliques K in G (^,Γ) is too large

Third Strengthened Formulation

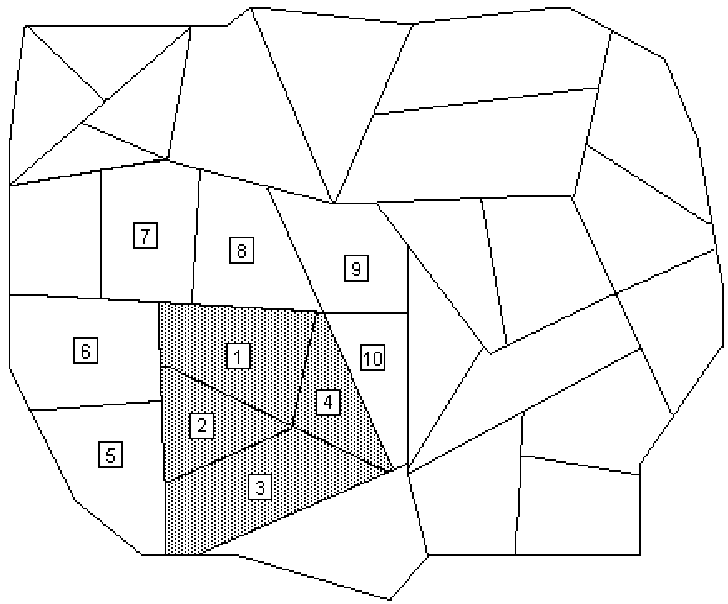
- Use constraint projection to generate strong inequalities valid for the cluster packing problem.

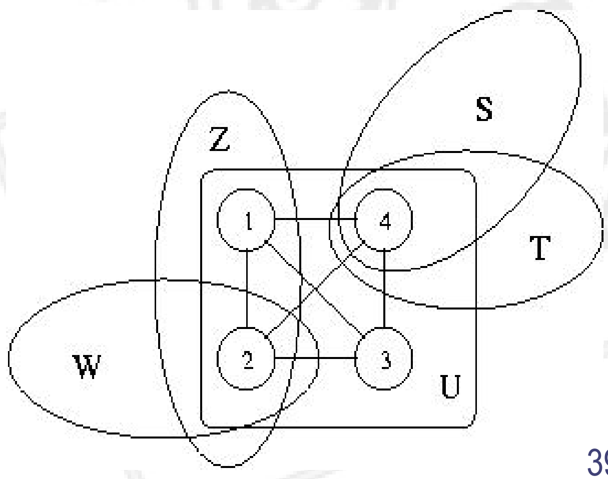




Projected Clique's in G(V,E)

- For each clique in G(V,E) generate a large set of incompatible clusters in G(^,Γ)
- Thus form a clique in G(^,Γ)
- Even if clique (1,2,3,4) in G(V,E). may be maximal not necessarily the case for projected clique.
- Example cluster R defined by nodes (5,6,7,8,9,10) does not intersect clique {1,2,3,4} but is incompatible with S,T,U,V, W.
- Thus $(XR)+Xs+XT+Xu+Xw+Xw \leq 1$
- In this form we can obtain facets of projected clique constraints associated with clique k.





Projected Cliques Cluster Packing Formulation (Goycoolea et. al. (2003))

Maximize $$\sum_{S} c_S x_S$$

subject to $$\sum_{S \in \Lambda(K)} x_S \leq 1$$ for each maximal clique K in G(V, E)

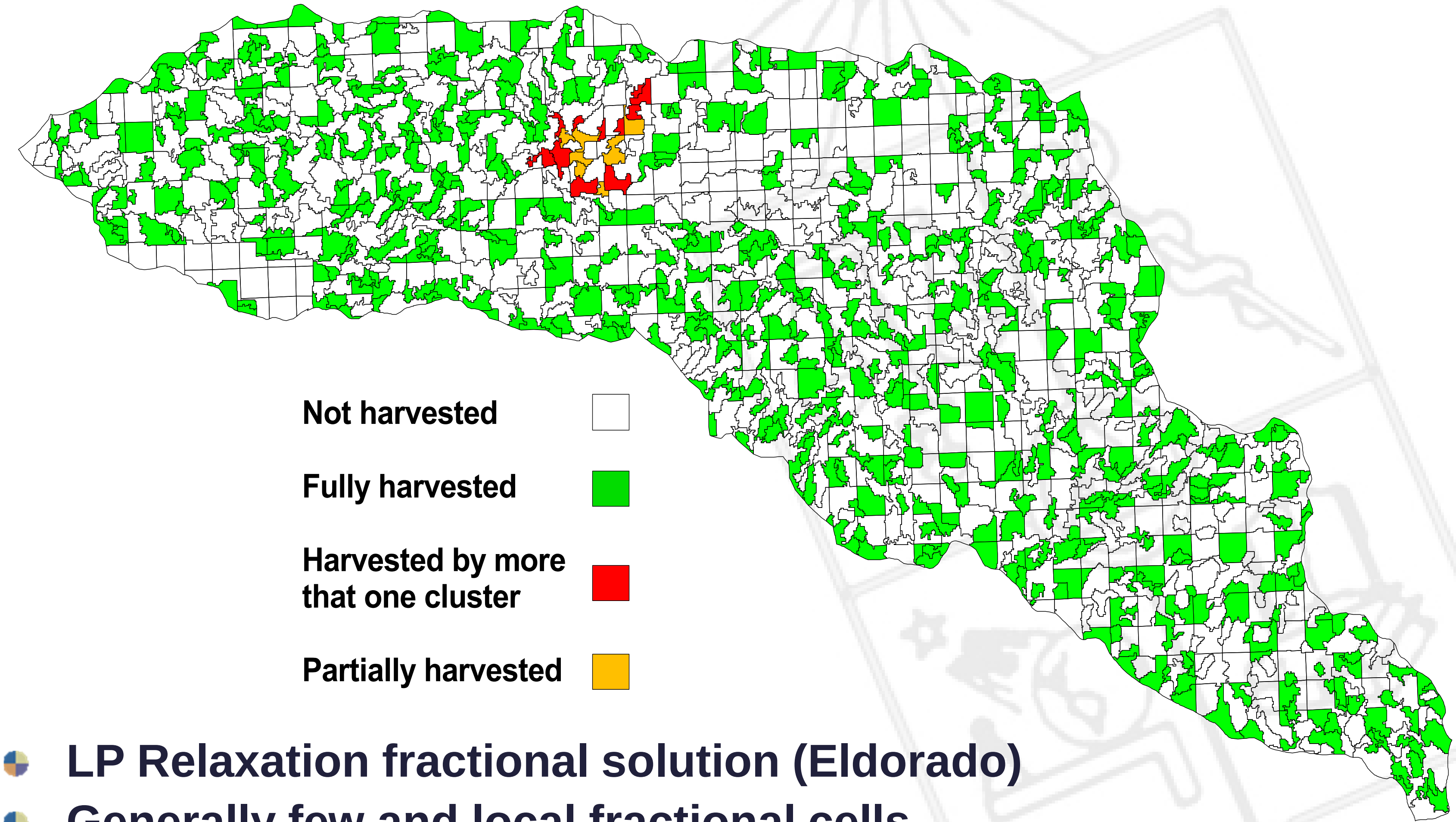
$$x_S \in \{0,1\}$$ for each cluster S $\in \Lambda$.

where:

- c_S is Net Profit of cluster S
- $\Lambda(K)$ is the set of all clusters that intersect maximal clique

- This set packing formulation is solved to integrality at the root node by CPLEX 8.1

Fractional Properties of the LP Formulation



- LP Relaxation fractional solution (Eldorado)
- Generally few and local fractional cells
- Generally solved in Node 0

Computational Results

- Butter Creek 351 units
- El Dorado 1363 units
- Random square problems
- Formulations
  - ARM-ARC: Martins et al.'s arc based formulation
  - ARM-PC: Goycoolea et. al.'s projected clique formulation
  - Another approach
    - ARM-MB: add explicitly all constraints of minimal infeasible clusters.

Computational Results

Instance	ARM-MB Obj. Value	ARM-MB Sol. Time	ARM-ARC Obj. Value	ARM-ARC Sol. Time	ARM-PC Obj. Value	ARM-PC Sol. Time
8x8	1,335,635.36 (26.58% gap)	14,400.00	1,352,200.90 (16.27% gap)	14,400.00	1,426,754.85	5.79
12x12	2,392,595.65 (45.02% gap)	14,400.00	2,671,223.69 (22.41% gap)	14,400.00	2,883,748.66	134.58
16x16	*	*	3,305,557.42 (44.58% gap)	14,400.00	4,887,757.03	2,557.07
Butter Creek	9,419.27 (14.65% gap)	14,400.00	9,928.73 (3.48% gap)	14,400.00	10,110.88	2.88
El Dorado	1,672,065 (3.39% gap)	14,400.00	1,696,935 (0.08% gap)	14,400.00	1,697,695	6.12

Multi-period Formulation with Volume Restrictions

Maximize $$\sum_{S,t} c_{S,t} x_{S,t}$$

subject to

$$\sum_{S \in \Lambda(K)} x_{S,t} \leq 1$$ **for each maximal clique K in G(V, E) and for each period t**

$$\sum_{S,t \;\; u \in S} x_{S,t} \leq 1$$ **for unit u in V**

$$(1-\Delta)\sum_{S} v_{S,t-1} x_{S,t-1} - \sum_{S} v_{S,t} x_{S,t} \leq 0$$ **for each period t>1**

$$\sum_{S} v_{S,t} x_{S,t} - (1+\Delta)\sum_{S} v_{S,t-1} x_{S,t-1} \leq 0$$ **for each period t>1**

$$x_{S,t} \in \{0,1\}$$ **for each cluster S $\in$ Λ and for each period t**

Numerical Results for Multi-period Model

- **Instances:**
  - Using Δ=0.1, 0.15 (±10%, ±15%)
- **Difficult to solve with Volume Constraints**

Map	Time Periods	Δ	IP Time	B&B Nodes	Best Solution Time [s]	GAP [%]	1st sol under 1% GAP [s]	1st Feasible Time [s]	1st Feasible GAP
eldorado15	12	0.10	28800	2133	18606	1.47	-	18606	1.47
eldorado15	15	0.10	28800	1575	18315	0.83	10839	10839	1.00
ran12by12	12	0.10	28800	388	-	-	-	-	-
ran12by12	15	0.10	28800	394	-	-	-	-	-
eldorado15	12	0.15	28800	2087	11211	0.50	10719	2323	1.51
eldorado15	15	0.15	28800	2067	20733	0.59	20274	20274	0.77
ran12by12	12	0.15	28800	634	-	-	-	-	-
ran12by12	15	0.15	28800	342	-	-	-	-	-

Elastic Constraints

- What if we consider the volume requirements as more of a guide rather than hard constraints
  If they are violated by a small amount, the solutions would likely be acceptable to forest managers

- Elastic Constraints
  Permit small violations, but penalize violations in the objective

- Effects
  The volume constraints are “inactive” and do not generate new extreme points (good integrality properties)

Multi-period Model with Elastic Volume Restrictions

Maximize $$\sum_{S,t} c_{S,t} x_{S,t} - \sum_{t>1} \underline{p}_t l_t - \sum_{t>1} \overline{p}_t u_t$$

Subject to $$\sum_{S \in \Lambda(K)} x_{S,t} \le 1$$ for each maximal clique K in G(V, E) and for each period t

$$\sum_{S,t \; u \in S} x_{S,t} \le 1$$ for unit u in V

$$(1 - \Delta_E) \sum_S v_{S,t-1} x_{S,t-1} - \sum_S v_{S,t} x_{S,t} \le l_t$$ for each period t>1

$$\sum_S v_{S,t} x_{S,t} - (1 + \Delta_E) \sum_S v_{S,t-1} x_{S,t-1} \le u_t$$ for each period t>1

$$x_{S,t} \in \{0,1\} \qquad l_t, u_t \ge 0$$ for each cluster S $\in$ Λ and for each period t

Multi-period Formulation with Volume Restrictions

- Use of:
  - Elastic constraints
  - Constraint Branching
  - Integer Allocation Heuristic

Solving the Elastic Constraint Model

- Choosing independent elastic penalties still difficult.
- Branch & bound method
  - Elastic Constraints help Integer allocation
  - Constraint branching to resolve fractions
    - Diversifies greedy nature of heuristic
  - Integer allocation heuristic at each B&B node
    - Volume violation corrections carried out in integer allocation

Numerical Results for Elastic Model B&B Method

- Instances:
  - Using Δ= 0.1, 0.15 (±10%, ±15%), ΔE=0.09, 0.14 (±9%, ±14%)
- GAP's comparable to strict volume constraint table:
  - GAP's calculated with respect to strict volume constraint LP
  - Solutions are feasible for strict volume constraint model with the respective Δ

Map	Time Periods	Δ	IP Time	B&B Nodes	Best Solution Time [s]	GAP [%]	1st sol under 1% GAP [s]	1st Feasible Time [s]	1st Feasible GAP
eldorado15	12	0.10	14400	23	5555	0.41	1706	1706	0.43
eldorado15	15	0.10	14400	13	12541	0.44	4307	4307	0.45
ran12by12	12	0.10	14400	75	5059	3.43	-	663	8.70
ran12by12	15	0.10	14400	25	13856	4.52	-	614	14.42
eldorado15	12	0.15	14400	18	12216	0.30	1160	1160	0.33
eldorado15	15	0.15	14400	13	9916	0.29	2387	2387	0.34
ran12by12	12	0.15	14400	199	9684	2.29	-	312	5.07
ran12by12	15	0.15	14400	20	9124	4.97	-	504	7.99

Conclusions for Elastic Model B&B Method

- Initial tests show elastic constraint method generates good integer feasible solutions quickly.
- First integer feasible solutions are obtained between 10 to 150 times faster than CPLEX and are of higher quality.
- Other improvements in computacional capabilities appear possible.

Old growth: Tabu Search

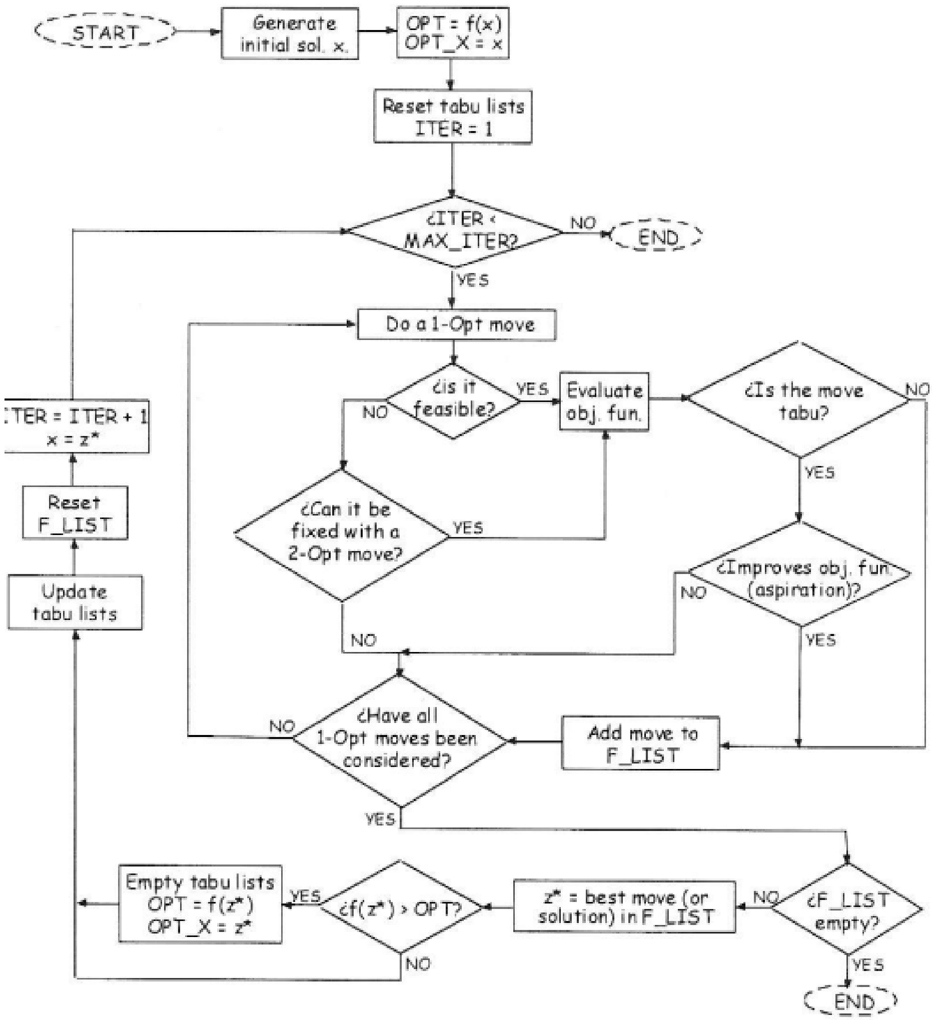
- Caro et al 2003
- Multiperiod harvest-scheduling ARM model with adjacency constraints + old growth patch size and total old growth area restrictions.

- Tabu search procedure with 2-Opt moves (exchanging at most 2 units) was developed.

Algorithm

- Neighbors:
  - OPT-1: change the harvesting period of one node (or cutting unit) from period t1 to t2, or no harvest to harvest in period t3, or from harvesting in period t4 to no harvest.
  - OPT-2: involves simultaneously changing the harvesting period of two nodes (including not harvesting)

Algorithm Flowchart



Computational Results

Table 1. Cases with exact solution.

Case	Min. Vol. (m^3)	CPLEXP 6.6 Obj. Func.	CPLEXP 6.6 CPU (sec)	Basic Tabu procedure Initial	Best	Ave.	C. Var. (%)	CPU (sec)
4x5x2_a1	15	57.8	6,958.4	43.3	57.8	56.8	9	0.60
4x5x2_a2	20	57.8	6,436.2	46.6	57.8	54.3	9	0.08
4x5x2_a3	25	57.3	8,675.3	56.5	57.2	52.2	5	0.55
4x5x2_b1	30	92.6	30,809.3	82.3	92.5	85.5	7	0.08
4x5x2_b2	35	92.5	23,766.0	82.3	92.5	84.8	6	0.07
4x5x2_b3	40	89.8*	156,665.4	82.7	91.0	83.5	4	0.31

* Best integer solution (GAP 3%) after 44 execution hr.

Computational Results

Table 2. Comparison for the Iberian instance tested under different implementations of the Tabu procedure.

	Best O.F.	Average O.F	CPU (sec)
	($)		
1-Opt heuristic	791,560.10	756,101.59	63
Basic 2-Opt procedure	980,271.45	955,911.10	156,318
Efficient implementation	978,447.86	959,116.79	784
Neighborhood reduction	979,739.72	951,088.15	59
Intensification	980,639.30	949,216.28	70
Diversification	991,174.04	974,071.43	193
Random Tabu tenure	984,804.84	952,684.45	71
Alternative Tabu criteria	983,107.28	951,182.94	62
Probabilistic move selection	981,055.27	953,878.23	90
Best combination	1,004,853.01	973,437.78	200

Table 3. Computer results for large-scale instances.

Instance	Obj. Fun. ($)	CPU (sec)	Trivial bound	Trivial gap (%)
64x100x7	2,996,839.3	7,396	3,099,654.4	3.32
144x100x7	1,288,845.9	31,012	1,340,063.0	3.82
180x150x10	2,191,304.7	89,292.1	2,511,582.2	12.75

Old growth exact formulation

- Carvajal et al 2011
- Harvest scheduling problem with both maximum clear-cut constraint and old growth conservation requirements.

- Objective: Maximize profit while preventing large clear-cut areas, maintaining a minimum average ending age of the forest and a connected (contiguous) region of old growth forest.

Old growth exact formulation

Extension of ARM model that considers old growth patches, with enough area to be a wildlife habitat.

z_v: **Old-growth variable. Takes the value 1 if stand v belongs to the old-growth forest and 0 otherwise.**

Main Constraints:

- **Stand selected at old growth can not be harvested**
- **Old growth forest has minimum area.**

Old growth exact formulation

- Problem is connectivity
- Two non-adjacent nodes, u and v belong to a connected set if there is a path of nodes connecting them.
- In any cut set separating u and v, there must be at least one node connected to u and to v: “there exists a path U between u and v, such that for every node cut set S separating u and v, the intersection of S and U is not empty.

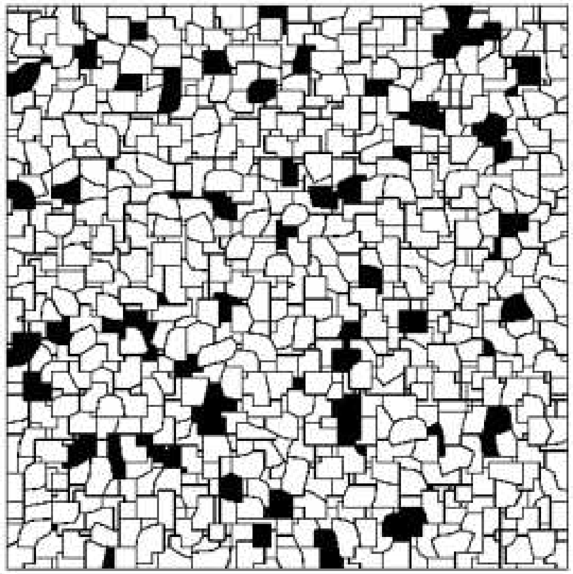
Old growth exact formulation

- This is represented by cut inequalities.

- Too many cut inequalities: Constraint generation.

Instances and Results

Old-growth area for FLG9A in time period three when not imposing connectivity.



Characteristics of the FMOS instances used in our computational study and parameters used in the optimization model.

Name	Stands	Area (ha)	A_{max} (ha)
Rebain-McDill	50	1000	40
Gavin	352	6310	40
Hardwicke	423	6948	40
FLG9A	850	9999	48.6
Shulkell	1039	4498.7	16
El Dorado	1363	21147	48.5

Parameters used in our computational study for solcing the different forest planning problems.

Name	Value
L	0.15
U	0.15
H	40 Years
O_{age}	60 Years
A_{min}	20% of total area

Number of connected patches obtained in FLG9A when not imposing connectivity.

	Patches	Largest (%)
El Dorado	70	17.6
FLG9A	57	5.8
Shulkell	10	25

Instances and Results

Table 7 NPV and gaps for best solutions obtained.

Instance	Model	1 period NPV (%)	1 period Gap (%)	3 periods NPV (%)	3 periods Gap (%)	5 periods NPV (%)	5 periods Gap (%)
ElDorado	ARM	100	0.01	100	0.31	100	0.03
	OGARM	100	0.01	99.81	0.19	99.46	0.02
	OGPARM[a]	99.9	0.57	97.1	2.5	96.84	2.11
FLG9A	ARM	100	0.01	100	0.01	100	0.01
	OGARM	100	0.01	98.92	0.01	97.38	0.01
	OGPARM[a]	100	0.09	95.99	1.87	95.41	0.9
NBCL5A	ARM	100	0	100	0.01	100	0.01
	OGARM	100	0	100	0.01	100	0.01
	OGPARM[a]	99.74	0.03	96.78	0.08	96.67	0.01
Shulkell	ARM	100	0.01	100	0.01	100	0.01
	OGARM	100	0.01	100	0.01	100	0.01
	OGPARM[a]	100	0.01	98.57	0.05	98.42	0.07

NPV is expressed as a porcentage of the ARM NPV value, for example the OGPARM NPV entry is $NPV_{OGPARM}/NPV_{ARM} \cdot 100\%$. The "Gap" column contains the gap between the best upper bound and the best feasible solution found, for the referred instance

Instances and Results

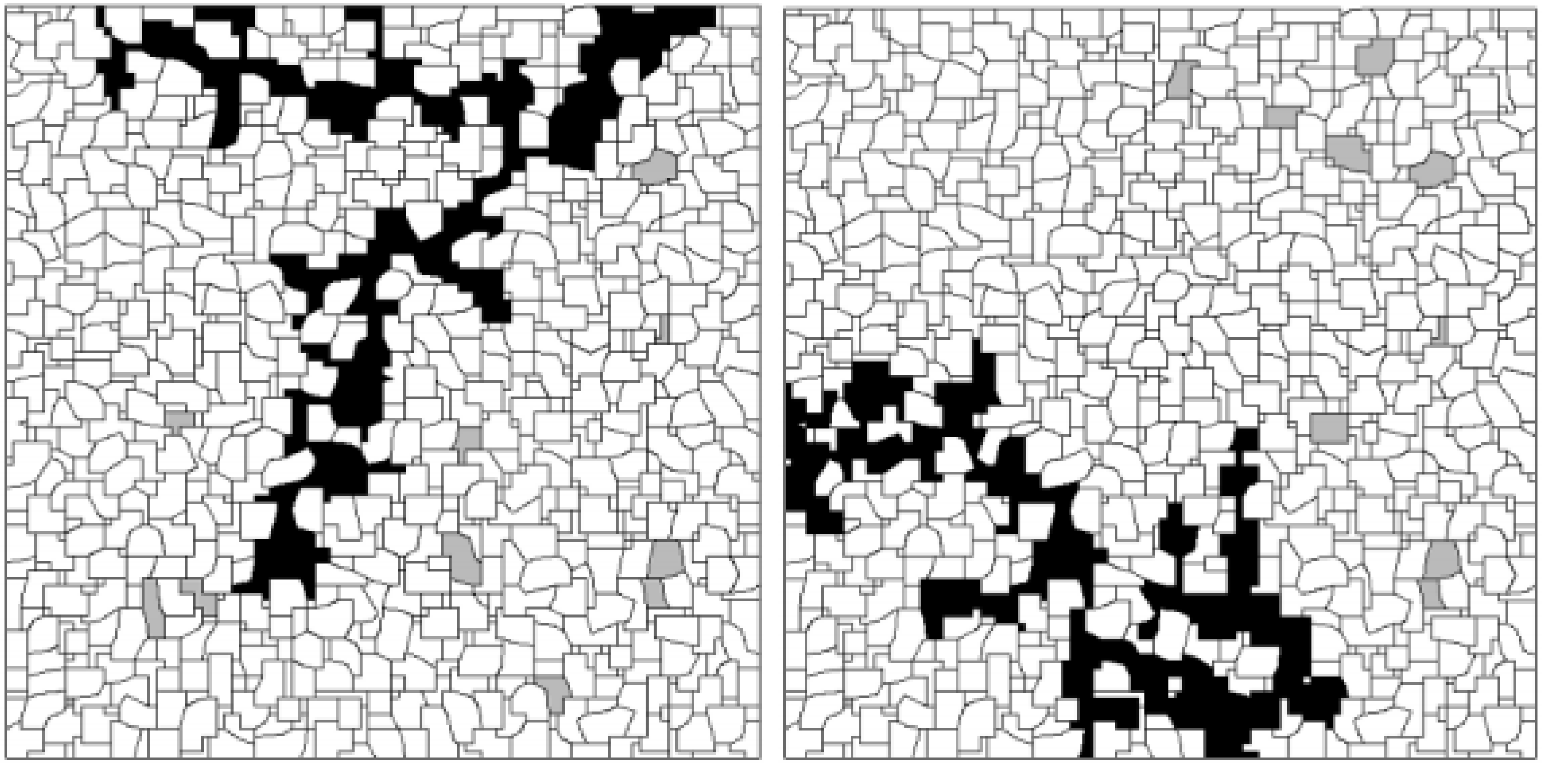


Figure 5 Two solutions for FLG9A. Stands in black are the ones selected for the old-growth forest. For simplicity stands that are harvested in some period are showed in white and nonharvested stands in gray.

Machine Location Problem

Main Decisions:

- **Where to locate the machinery, Skidders and Towers to harvest the Forest.**
- **Road building**

PLANEX: Harvesting Machinery Allocation

- Need to harvest 300 to 1,000 ha in next 4 months

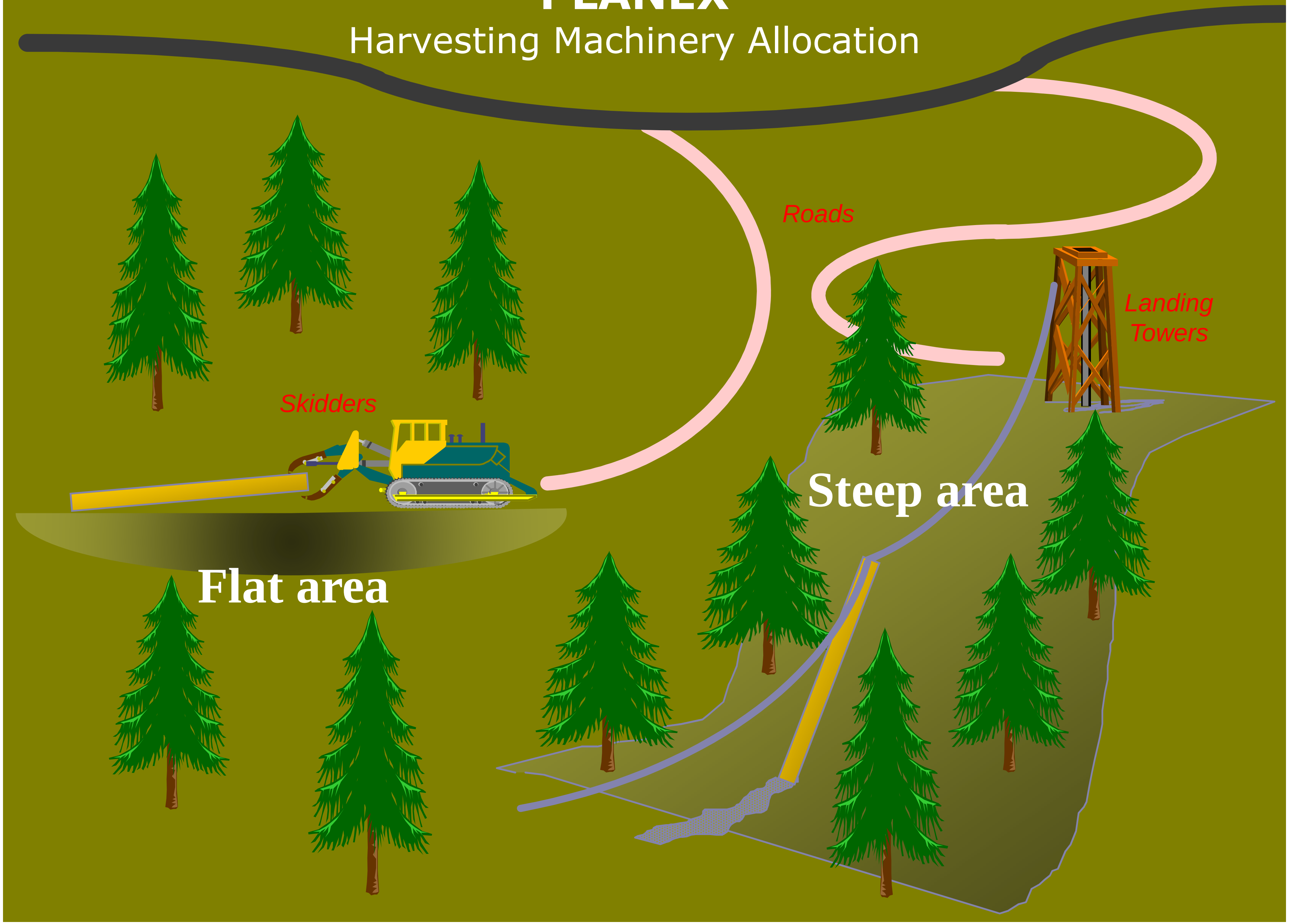
- Process:
  - Fell trees
  - Bring trees to roadside:
    - Skidders for flat area
    - Cable logging (towers) in steeper slopes
  - Load on trucks

PLANEX- Main Decisions

- Where to allocate tractors and towers
- Which areas to assign to each machine
- The road network

PLANEX

Harvesting Machinery Allocation



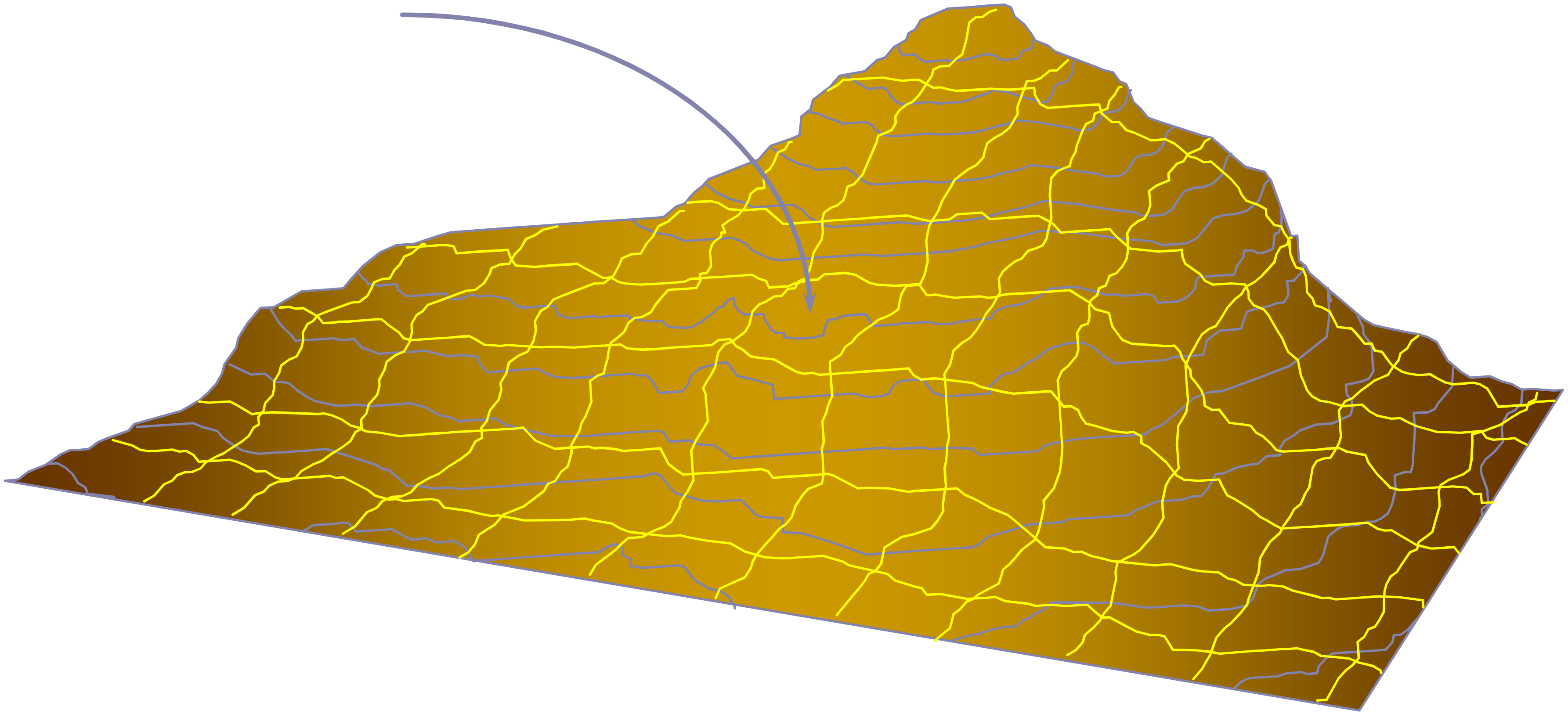
Manual Approach: Engineer w/topographic maps

- **Long, tedious work**
- **Can only analyze one scenario**
- **GIS for information on:**
  - **Topography, standing timber, existing roads**

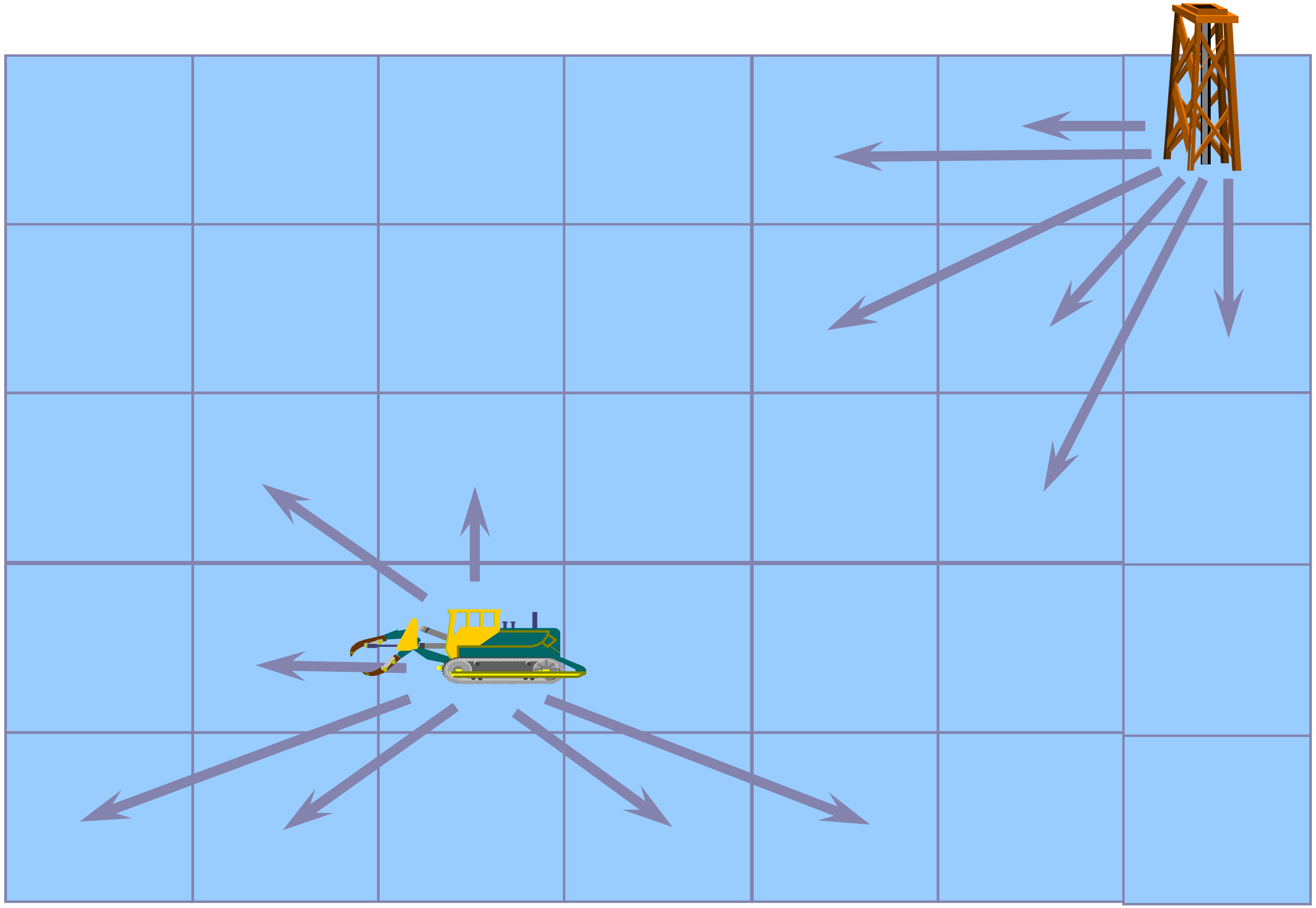
- **Raster form 10x10m2 cells**
- **Friendly graphic interface**
- **Heuristic algorithm**
- **Runs take about 15 minutes for large areas**
- **Ability to test several scenarios**

PLANEX - Information

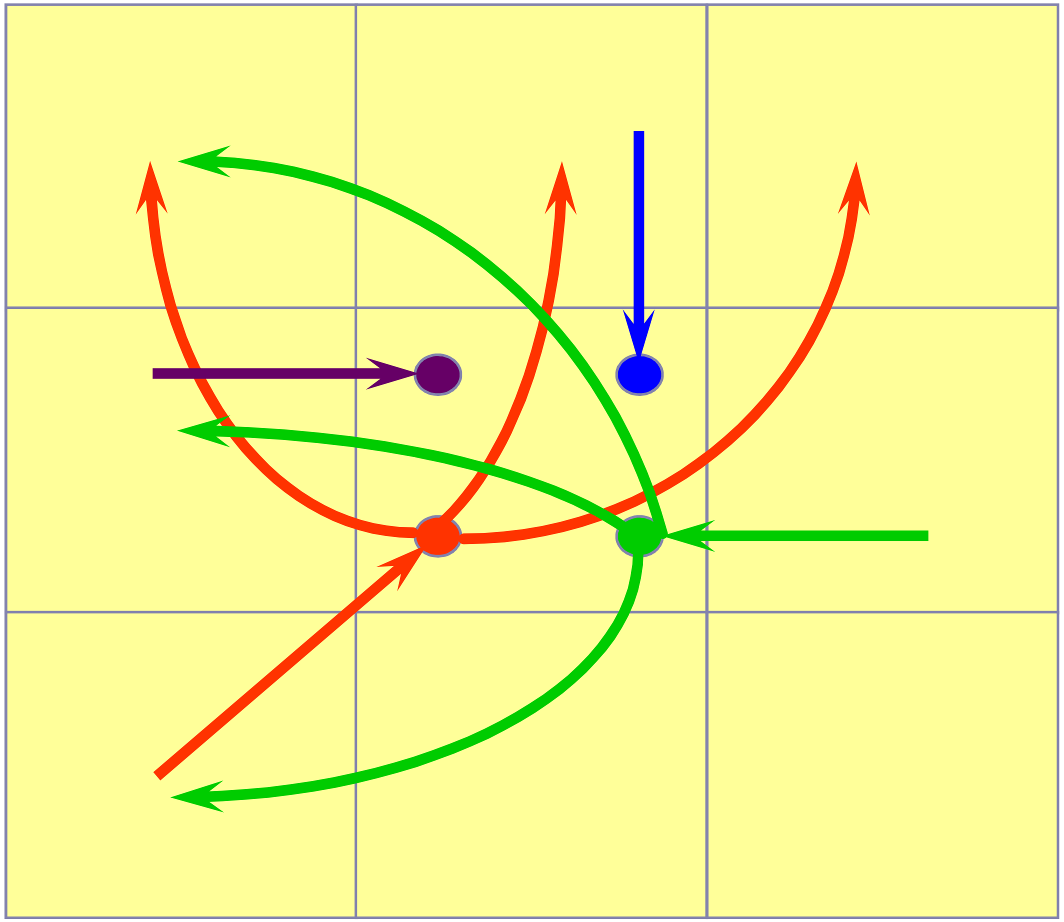
10 x 10 meter cells

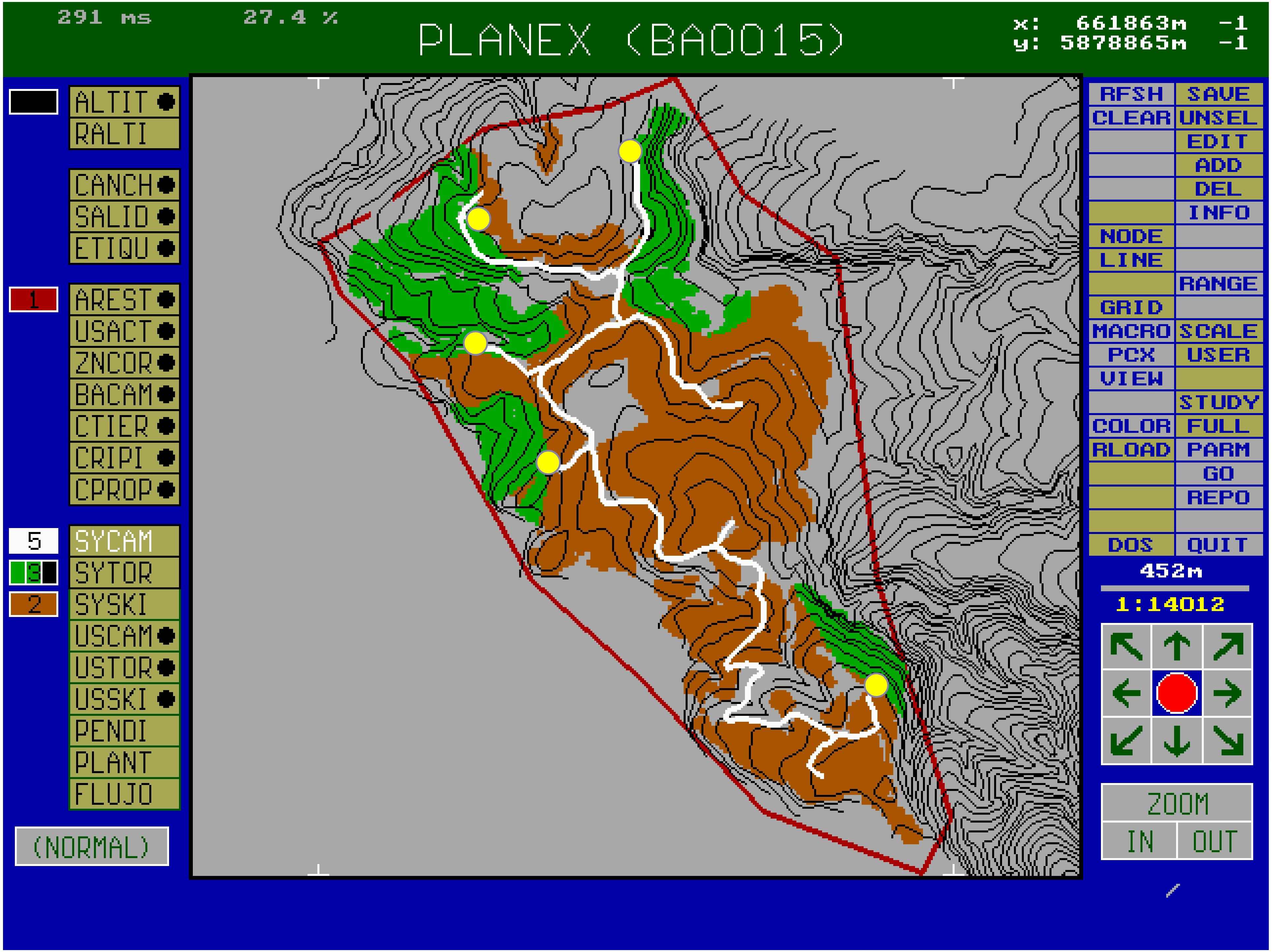


PLANEX - Reach of Harvesting Equipment



PLANEX - Feasible Turns for Harvesting Equipment

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SYCAM
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USCAM
USTOR
USSKI
PENDI
PLANT
FLUJO
(NORMAL)
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SAVE
CLEAR
UNSEL
EDIT
ADD
DEL
INFO
NODE
LINE
RANGE
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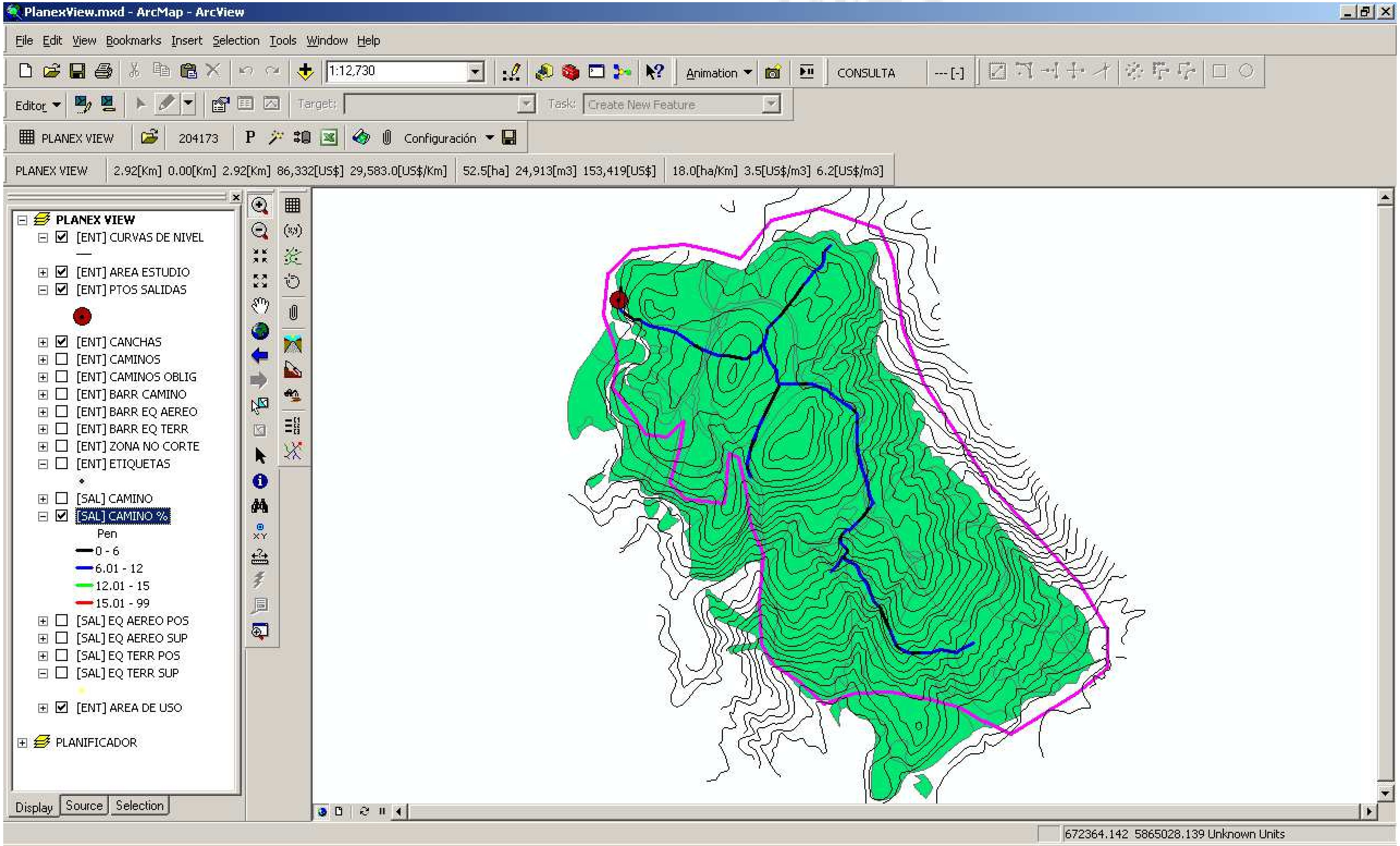
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PLANEX VIEW
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[ENT] AREA ESTUDIO
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[ENT] CANCHAS
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Display Source Selection
672364.142 5865028.139 Unknown Units

A Mathematical Model

- **Installation decision variables**

$$x_i^k = \begin{cases} 1, & \text{if machinery of type k is located in cell } i \in T^k \\ 0, & \text{otherwise} \end{cases}$$

- **Road construction decision variables**

$$z_{qr} = \begin{cases} 1, & \text{if road section } (q,r) \in A \text{ is built (i.e., } (q,r) \text{ has to be constructed if it does not already exist)} \\ 0, & \text{otherwise} \end{cases}$$

Model

- **Variables associated with timber volume harvested.**

w_{ij}^{k} : timber volume harvested in cell using machinery type *k* in cell *i* $\in T^k$

Y_i : timber volume harvested through cell

f_{qr} : timber volume flowing through road section (q,r)

g_s : timber volume flowing through exit

Savings with PLANEX

- Roads: 10% - 60% of the original network.
- Almost US$ 500,000 per year of operational NPV, integrated with the Tactic System (1997)
- Up to 40% of the planification time in “hard” problems.
- 15% of the cost l
- n US$/m3

Lagrangian Relaxation Approach (Vera et al, 2002)

- **Separates the problem into a Machine location + Road Construction**
- **Two classes of machines: towers and skidders.**
- **Roads Construction**
- **Flow Conservation between different nodes (origins, intersection and exits)**

Model

Location Subproblem:

$$\max \sum_{i\in T}\sum_{j\in M} P_{ij}^{k}(\mu_i + \alpha_{ij}^{2k})w_{ij} - \sum_{i\in T}\sum_{k} \alpha_{ik}^{1} x_i^{k}$$

$s.a.$

$$\sum_{k}\sum_{i\in T^k} w_{ij}^{k} P_{ij}^{k} \leq \Omega_j, \quad \forall j \in M$$

$$w_{ij}^{k} \leq x_i^{k}\Omega_j, \quad \forall i, j, k\, P_{ij}^{k} = 1$$

$$w_{ij}^{k} = x_i^{k}\Omega_j, \quad \forall j \in M\, P_{ij}^{k} = 1$$

$$\sum_{k} x_i^{k} \leq 1, \quad \forall i \in T$$

$$x \in \{0,1\}, \quad w \geq 0,$$

Model

Network Design Subproblem:

$$\max \sum_{i\in T}(\beta+\mu_i)y_i - \sum_{(i,j)\in A}\alpha_{ij}^3 z_{ij} - \sum_{(i,j)\in A}\alpha_{ij}^4 f_{ij}$$

s.a.

$$f_{rq} + f_{qr} \le z_{qr}K_{qr}\ , \quad \forall (q,r)\in A^{\mathrm{P}}$$

$$\sum_{(q,r)\in A} f_{qr} - \sum_{(r,t)\in A} f_{rt} = \begin{cases} -y_r & r\in T \\ 0 & r\in N-(T\cup S) \\ g_r & r\in S \end{cases}$$

$$\sum_{i\in T} y_i = \sum_{s\in S} g_s$$

$$z_{qr} \le \sum_{(q,t)\in A^{\mathrm{P}}} z_{qt} + \sum_{(t,q)\in A^{\mathrm{P}}} z_{tq} + \sum_{(r,t)\in A^{\mathrm{P}}} z_{rt} + \sum_{(t,r)\in A^{\mathrm{P}}} z_{tr}\ , \quad \forall (q,r)\in \bar{A}$$

$$\frac{y_i}{\sum_{j:P_{ij}^k\ge 1}\Omega_j} \le \sum_{(r,i)\in A^{\mathrm{P}}} z_{ri} + \sum_{(i,t)\in A^{\mathrm{P}}} z_{it}, \quad \forall i\in T$$

$$\sum_{i\in T} y_i \le \sum_{j\in M}\Omega_j$$

$$y_i \le \sum_{j:P_{ij}^k=1}\Omega_j, \quad \forall i\in T$$

$$z\in\{0,1\},\ f\ge 0,\ y\ge 0,\ g\ge 0$$

Solving Strategy

1. Defining additional constraints, in order to make the original formulation stronger.
2. Partition of the problem using LR approach.
3. Strengthen the partitioned subproblems, if possible.
4. Solve the LR using a pure subgradient algorithm, or a combined hybrid approach, using subgradient iterations followed by a Dantzig-Wolfe method or by bundle method.
5. Obtain primal feasible solutions using Lagrangian heuristic.

Strengthenings

- **Location to road trigger**

$$\sum_{k=1}^{K} x_i^k \leq \sum_{(i,q)\in A^p} z_{iq} + \sum_{(r,i)\in A^p} z_{ri}$$

- **Road to road triggers**

$$z_{q,r} \leq \sum_{(r,t)\in A^p} z_{rt} + \sum_{(t,r)\in A^p} z_{tr} + \sum_{(q,t)\in A^p} z_{qt} + \sum_{(t,q)\in A^p} z_{tq}$$

Where:

$z_{i,j}$ **is 1 if we build the road (i,j).**

x_i^k **is 1 if we locate machine of type k in cell i.**

Obtaining feasible solutions

1. If the solution is feasible, keep it.
2. If not, road network is not compatible with the locations defined by the subproblem, so machine locations are not connected to the exit.
3. Auxiliary problem consisting of all machine locations and an auxiliary road network consisting of minimum spanning tree connecting all possible machine locations to the exit.
4. Then solve the auxiliary linear problema to take out all timber to the exits.
5. Delete all roads which are not taking any flow of timber.

Computational Results

Test Instances

DIMENSIONS	SET 1	SET 2 (simple)	SET 3 (complex)
Area (hs.)	10	40	40
Number of cells	1.000	4.071	4.071
Tower loc. points	4	17	17
Skidders loc. points	6	41	41
Constraints	1.620	16.046	16.046
Continuous variables	955	12.688	12.688
Potential roads	16	65	109
Binary variables	26	123	167

Computational Results

INSTANCE	Linear Relaxation		Branch & Bound		Lagrangian relaxation		
	normal	strengthened	normal	strengthened	Subgradient	Hybrid	Bundle
SET 1							
Feas. sol	84,629	85,452	85,992	85,992	85,992	85,992	85,992
Bound	89,588	87,056	85,992	85,992	86,874	86,486	86,486
Gap (%)	5.5	1.8	0.0	0.0	1.0	0.6	0.6
Time (min)	0.05	0.03	0.42	0.15	4.70	5.43	4.02
SET 2							
Feas. sol	410,300	421,992	410,258	415,248	414,259	415,248	415,248
Bound	433,885	421,670	431,581	415,248	421,345	418,123	417,253
Gap (%)	5.4	2.5	4.9	0.0	1.7	0.7	0.5
Time (min)	5.60	7.49	425.21	17.45	82.41	87.45	78.49
SET 3							
Feas. sol	400,063	407,038	381,427	415,248	415,547	*	*
Bound	434,177	428,382	420,156	426,174	425,782		
Gap (%)	7.9	5.0	9.2	2.6	2.4		
Time (min)	5.70	8.78	453.57	342.47	165.58		

Conclusions

1. **Hard Problem: B&B algorithm was not able to solve the basic formulation in reasonable time.**
2. **B&B leads to significantly lower gaps, but at the cost of higher CPU times compared to LR approach.**
3. **Significant improvemen is obtained by strengthening the formulation of the model.**
4. **The LR approach appears worse for easier problems.**

Tabu Search (Andrés Diaz et al, 2004)

- **Objective: Selecting the locations for the machines and design the access road network connecting the existing network with the points where machinery is installed.**
- **Formulated as 2 problems: Plant location and fixed charge network flow problems.**
- **Two types of machines (towers and skidders)**

Algorithm

- $x = [x_1, x_2, \dots, x_{|T|}]$ **feasible selection of locations, with T potential location set.**

2. **Neighbor solution x':**
   - **1-OPT: set of modifications where some location $i \in T$ is opened or closed.**
   - **2-OPT: set of modifications involving a pair of locations, in which one is opened and the other is closed.**
3. **Run the Tabu Search**

Algorithm

- At the end of Tabu Search, set of solutions is available for the machine location sub problem.
- During the resolution of this sub-problem, we use the road sections of the minimum spanning tree covering the potential locations and the exits of the forest to estimate the road network construction cost and the transportation cost.
- To evaluate more exactly the cost of each solution in the set of solutions, obtain the best Steiner tree covering the opened locations.

Numerical Results

- **Instances**

Problem data

Problem	1	2	3	4
Area (ha)	10	40	210	500
Number of cells	1000	4071	21,000	50,000
Number of potential tower locations	4	17	90	216
Number of potential skidder locations	6	41	150	398
Number of exit cells	1	1	5	11
Number of potential road sections	16	109	330	978
Number of existing road sections	0	45	36	102

Numerical Results

		CPLEX 8.1		Tabu
$\delta = 50\ US\$/m^3$				
Problem 1	Objective function (US$)	85,992.56		85,992.56
	Upper bound (US$)		85,992.56	
	Gap		0%	
	Time (minutes)	0.020		0.001
Problem 2	Objective function (US$)	414,502.59		416,858.19
	Upper bound (US$)		427,966.33	
	Gap		3.25%	
	Time (minutes)	600.00		0.12
Problem 3	Objective function (US$)	2,040,319.79		2,041,777.38
	Upper bound (US$)		2,102,811.93	
	Gap		3.06%	
	Time (minutes)	600.00		1.04
Problem 4	Objective function (US$)	6,222,050.04		6,259,090.23
	Upper bound (US$)		6,420,475.28	
	Gap		3.19%	
	Time (minutes)	520.07		3.81

Conclusions

- **Numerical results indicate that the heuristic approach is very attractive and leads to better solutions than those provided by "state-of-the-art" integer programming codes in limited computation times**

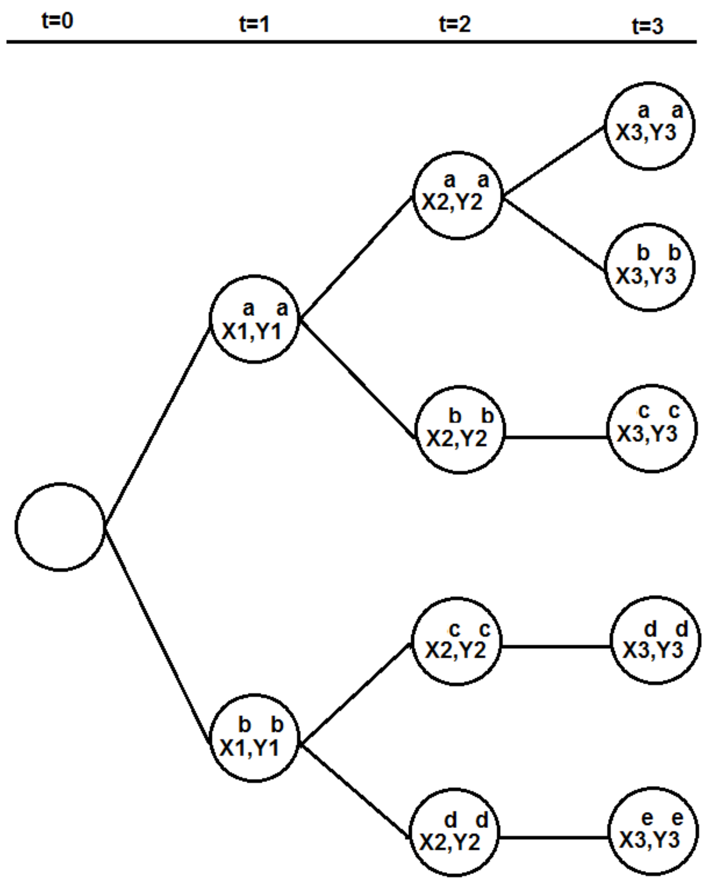
- **Solution times significantly smaller.**

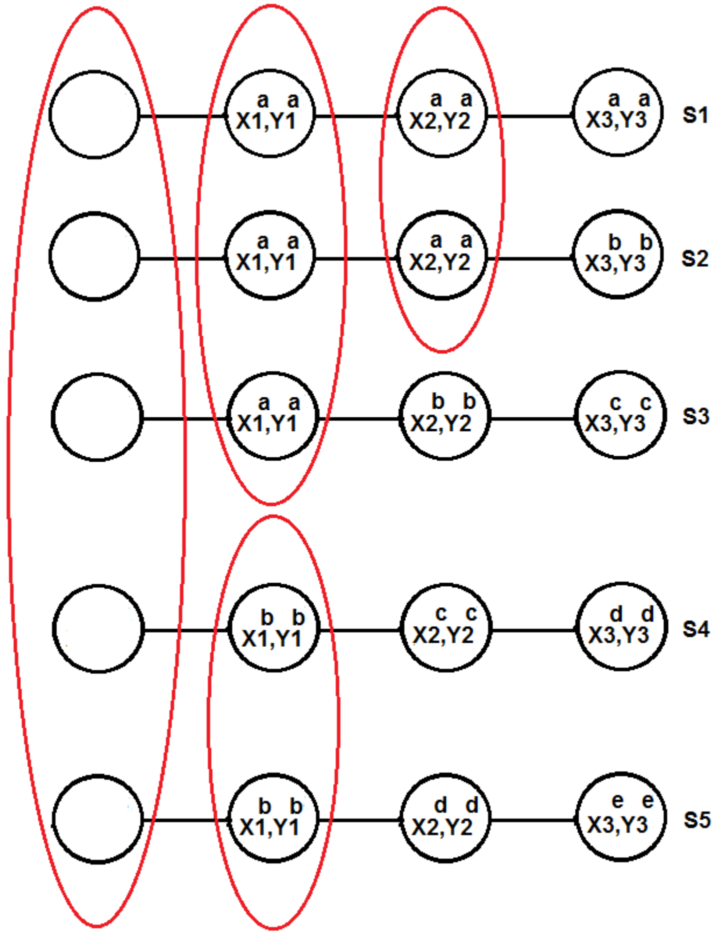
- **The numerical results do not vary too much when typical parameters such as the tabu tenure are modified, except for the dimension of neighborhood**

Stochastic Problems

- Scenario trees representing different uncertainty sources.
- Future Prices and Timber Volumes.
- Non-Anticipativity Constraints.
- Starting point: Andalaft (2003) problem from Millalemu instance, considering only one forest (instead of the original 17, linked by demand.

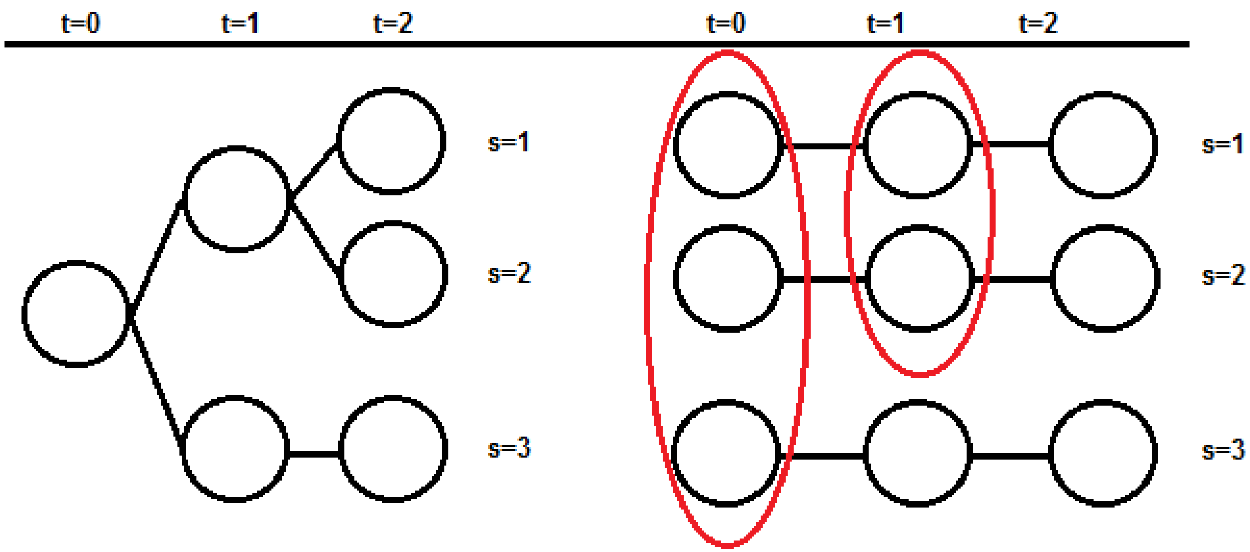
Scenario Trees





Non-Anticipativity

"If two scenarios are indistinguishable up to some stage, then the decisions in those scenarios, until that stage, must be identical"



Scenario Trees

- Scenario w consists in a realization of a random parameter during an horizon planning.
- Represent reality
  - Black swans
- Scenario tree generation: very hard to develop a general way to create them
  - Expert judgement
  - Random Walks converging long range average

Uncertainty in Forest production planning

- Escudero et al 2010
- Planning forest harvest and access to road construction under uncertainty problem.
- Uncertainty is represented by scenario trees, containing prices of timber and demand bounds.
- 18 Scenarios from los Copihues (Chile) real forest.
- MIP: Flow, Harvest, Road Build, 4 periods.
- Difficult to solve: Too many constraints, Non-Anticipativity constraints do not allow to Split the problem.

Uncertainty in Forest production planning: solving approach

- Branching: BFC approach due to the large scale of the problem.
- Average Scenario Solution (AVSC) is solved by simulating what happens in a given scenario (w) when applying the average scenario solution.
- BFC approach led to better solutions than the deterministic approach under most scenarios.
- Deterministic couldn’t find feasible solutions in multiple scenarios, for all cases.

Uncertainty in Forest production planning: Results

Table 1. Comparison of the Results

	Z_{AVSC}	Z_{BFC}	GAP %
SC 1	7860376.2	8141684.5	3.6
SC 2	74986706.4	7832291.9	4.5
SC 3	7272765.9	7681257.3	5.6
SC 4	6876035.1	7248863.7	5.4
SC 5	6751277.3	7288986.5	8.0
SC 6	6420668.3	6913420.5	7.7
SC 7	6440966.1	6739744.0	4.6
SC 8	6077296.2	6359584.0	4.6
SC 9	6003190.1	6111671.1	1.8
SC 10	Infeasible	5604078.0	—
SC 11	Infeasible	4945591.0	—
SC 12	Infeasible	4541990.9	—
SC 13	Infeasible	4324647.4	—
SC 14	Infeasible	4149814.2	—
SC 15	Infeasible	3335188.5	—
SC 16	Infeasible	3067968.2	—
SC 17	Infeasible	2866035.9	—
SC 18	Infeasible	2593300.9	—
tt	—	11942	—
Z_{IP}	—	5541451.0	—

Stochastic Forest Planning: A Progressive Hedging Approach

- Badilla et al 2010
- Medium term (4 stages) forest planning with an integrated approach considering both harvesting and road construction decisions in the presence of uncertainty.
- Price and growth uncertainties.
- Use of Strengthenings (Andalaft et al 1999)
- Many more scenarios than previously reported in the literature.
- Scenario-based decomposition method- Progressive Hedging

Stochastic Forest Planning: A Progressive Hedging Approach

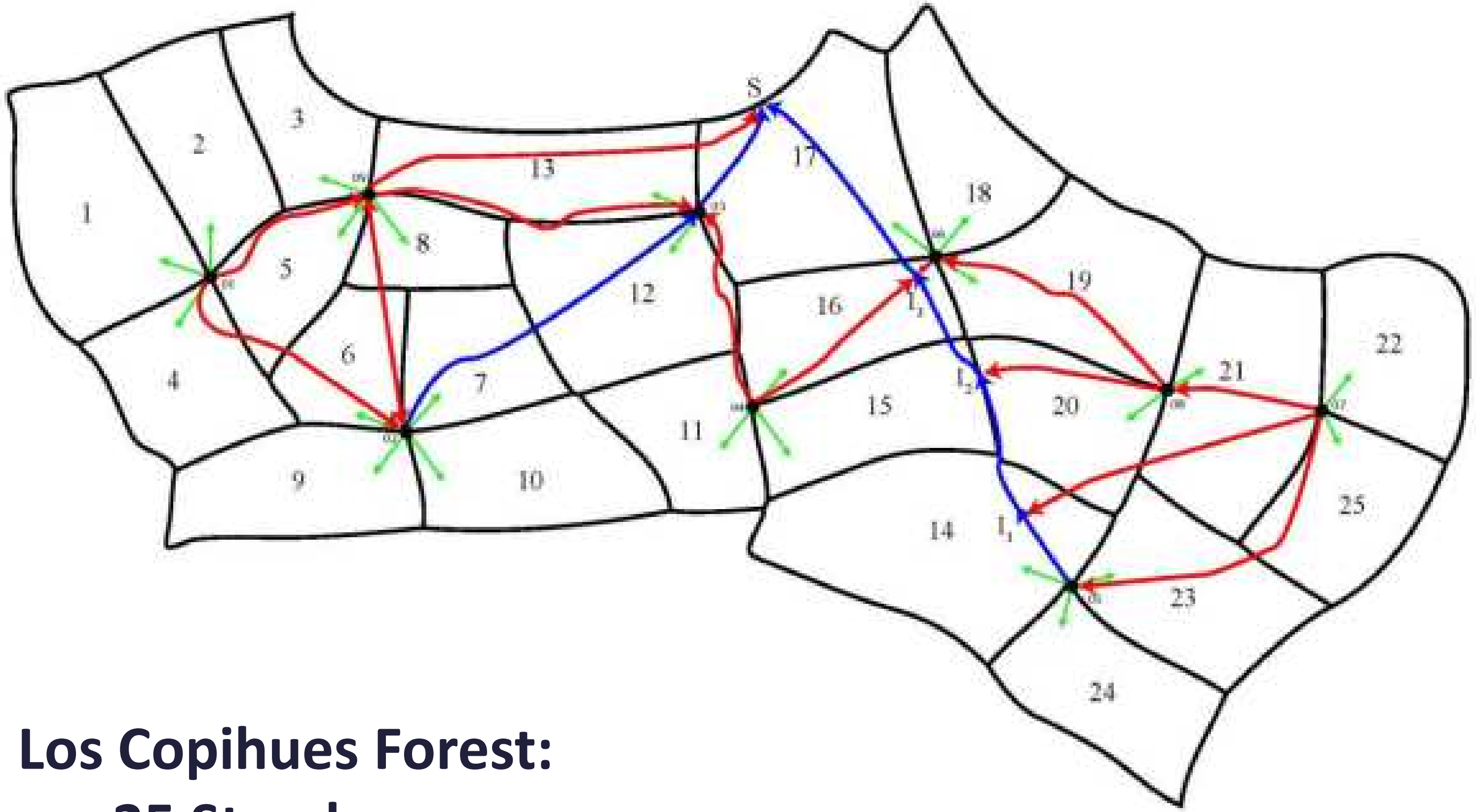
- **Progressive Hedging Algorithm**

1. Solve each scenario under $min_{x_s \in Q_s} f_s(x_s)$
2. Compute the solution in each node, $\overline{x} = \sum_{N_t : s \in N_t} p_s \, x_s;$
3. If solutions are similar $\|x - \overline{x}\| < \varepsilon$ then stop
4. Update penalty factor $w_i = \rho(x - \overline{x}) + w_{i-1}$
5. Solve each penalized scenario: $min_{x_s \in Q_s} f_s(x_s) + w_s x_s + \frac{\rho}{2} \|x_s - \overline{x}\|^2$
6. Return to 2

Stochastic Forest Planning: A Progressive Hedging Approach

- Progressive Hedging:
  - Separates problem per scenario
  - Implicit non-anticipativity constraints.
  - Natural parallel implementation
  - Different Techniques to improve its performance (hot starts, fixing variables, computing penalty term, etc.)

Instance



- **Los Copihues Forest:**
  - **25 Stands**
  - **9 Origin nodes, 3 Intersection nodes and 1 Exit node**
  - **15 Existing and 11 Potencial Roads**

Computacional Results

Instances

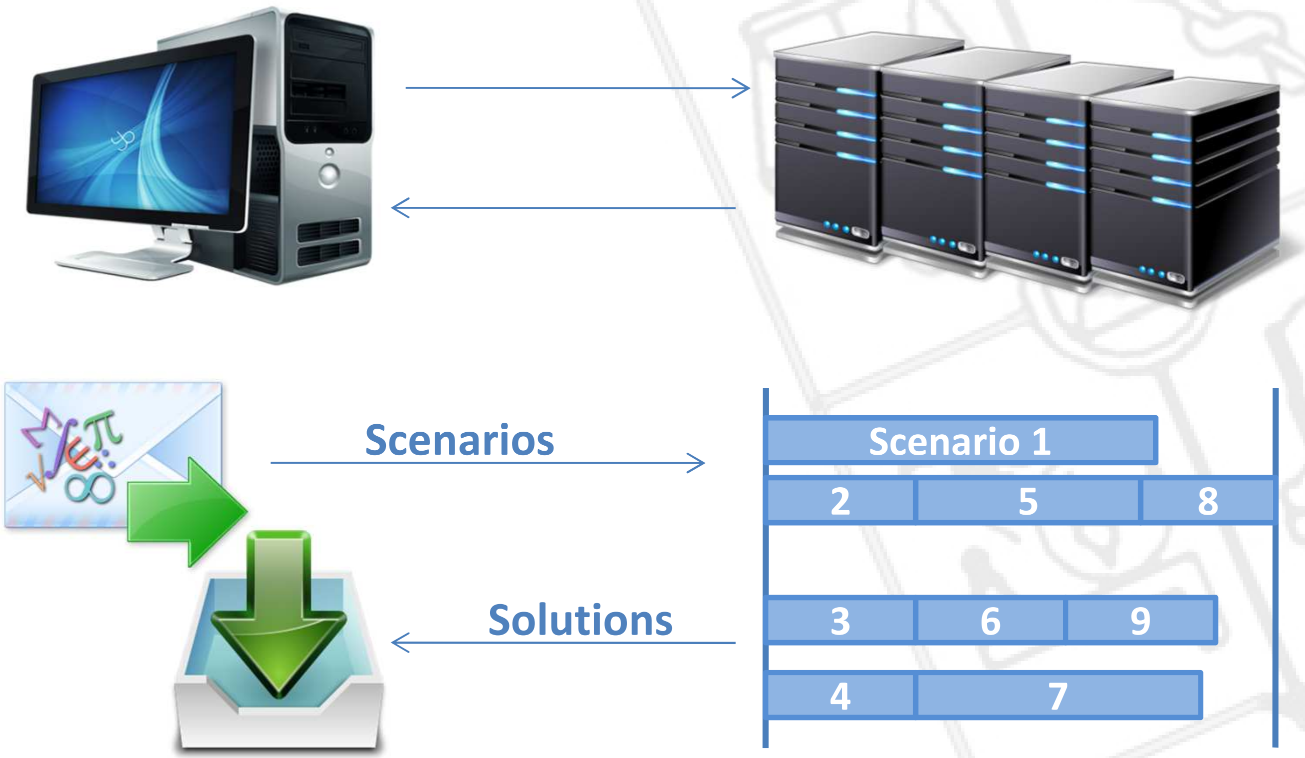
scenarios	1	18	64	144	162	216	324
				Implicit EF			
binary cols	156	1,209	3,315	7,410	9,867	10,920	16,185
linear cols	84	651	1,785	3,990	5,313	5,880	8,715
all columns	240	1,860	5,100	11,400	15,180	16,800	24,900
Rows	179	2,812	9,746	21,916	25,048	32,824	49,186
non-zeros	1,860	10,844	38,052	85,592	97,076	128,288	192,332
				Explicit EF			
binary cols	156	2,808	9,984	22,464	25,272	33,696	50,544
linear cols	84	1,512	5,376	12,096	13,608	18,144	27,216
all columns	240	4,320	15,360	34,560	38,880	51,840	77,760
Rows	179	5,682	21,716	48,936	52,698	73,704	110,856
non-zeros	1,860	16,584	61,992	139,632	152,376	210,048	315,672

Results

Scenarios	Cplex EF value (1hr)	Cplex EF gap (1hr)	PH+ value	PH+ Gap	PH+ Run time	PH+ vs. Cplex % value	Total Fixed Variables
18	$4,928,180	0.31%	$4,920,078	0.47%	4m22s	-0.16%	2
64	$5,357,780	1.29%	$5,386,971	0.74%	13m1s	0.54%	1719
144	$5,266,830	2.12%	$5,287,935	1.68%	21m42s	0.40%	5051
162	$5,187,040	3.10%	$5,242,032	1.98%	22m54s	1.05%	823
216	$5,332,550	3.92%	$5,437,714	1.87%	37m11s	1.93%	1828
324	$5,545,260	2.91%	$5,536,196	2.99%	71m39s	-0.16%	740

Future Improvements

- **Large number of scenarios lead to decomposition**
- **Need to parallelize**



Conclusion

- Spatial characteristics in forest planning lead to MIP problems
- Most are difficult to solve
- Actual use mostly heuristics
- Algorithmic challenges

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Combinatorial Challenges in Forest Management Modelling

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