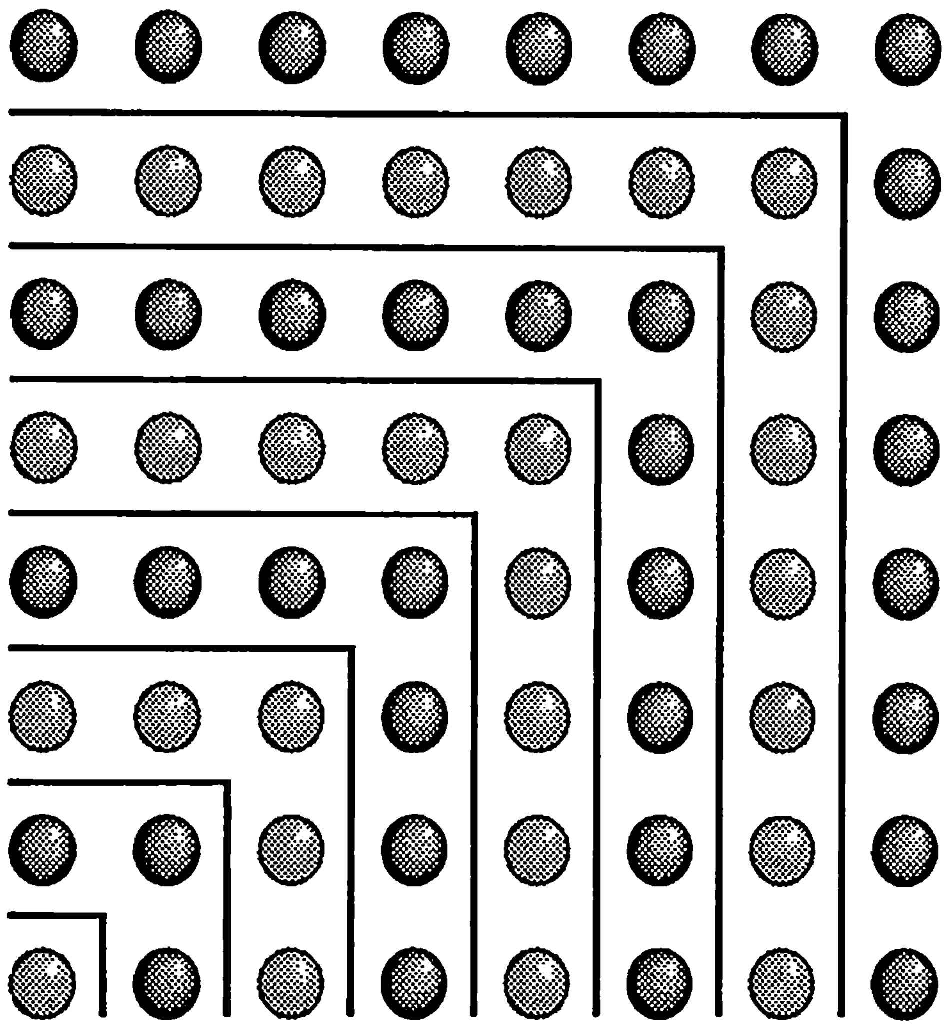
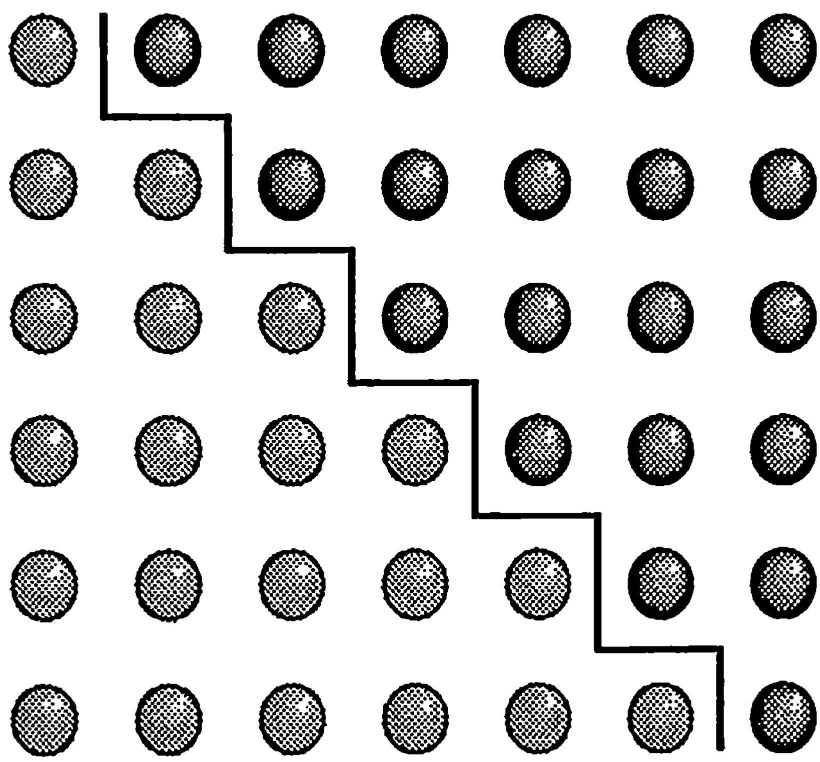
Sums of Odd Integers I



$$1+3+5+\cdots+(2n-1) = n^2$$

—Nicomachus of Gerasa (circa A.D. 100)

Sums of Integers I



$$1 + 2 + \cdots + n = \frac{1}{2}n(n+1)$$

—"The ancient Greeks"
(as cited by Martin Gardner)

Sums of Squares III

$$3(1^2+2^2+\cdots+n^2) = \frac{1}{2}n(n+1)(2n+1)$$

$$\begin{array}{ccccc} n & n & \cdots & n & n \\ n-1 & n-1 & \cdots & n-1 & \\ \vdots & \vdots & \iddots & & \\ 2 & 2 & & & \\ 1 & & & & \end{array} + \begin{array}{ccccc} n & n-1 & \cdots & 2 & 1 \\ n & n-1 & \cdots & 2 & \\ \vdots & \vdots & \iddots & & \\ n & n-1 & & & \\ n & & & & \end{array} + \begin{array}{ccccc} 1 & 2 & \cdots & n-1 & n \\ 2 & 3 & \cdots & n & \\ \vdots & \vdots & \iddots & & \\ n-1 & n & & & \\ n & & & & \end{array}$$

$$= \begin{array}{ccccc} 2n+1 & 2n+1 & \cdots & 2n+1 & 2n+1 \\ 2n+1 & 2n+1 & \cdots & 2n+1 & \\ \vdots & \vdots & \iddots & & \\ 2n+1 & 2n+1 & & & \\ 2n+1 & & & & \end{array}$$

—Sidney H. Kung