

Práctico 2

Ej 1:

$$B = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{pmatrix}$$

$$|B| = \begin{vmatrix} 2 & 3 & 1 & 2 & 3 & 1 & 2 \\ 1 & 2 & 2 & 1 & 2 & 3 & 1 \\ 3 & 1 & 2 & 3 & 1 & 2 & 3 \\ 6 & -6 & -6 & & & & \\ & & & +1 & +2 & +2 & 7 \end{vmatrix} = 18$$

$$|B| = (-1)^2 \cdot 1 \cdot \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} + (-1)^3 \cdot 2 \cdot \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} + (-1)^4 \cdot 3 \cdot \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix}$$

$$\begin{pmatrix} \dots & \dots & 0 \\ \dots & \dots & \dots \\ 0 & \dots & \dots \end{pmatrix} = \begin{pmatrix} \dots & \dots & \dots \\ \dots & \dots & \dots \\ 0 & \dots & \dots \end{pmatrix} (A-A)$$
$$= -5 + 2 + 21 = 18 \quad (A-A)$$

$$F = \begin{pmatrix} 0 & 1 & 11 & 1 \\ 0 & 2 & 19 & -1 \\ 0 & 0 & 12 & 0 \\ -1 & 1 & 10 & 2 \end{pmatrix}$$

$$(-1)^5 \cdot (-1) \cdot \begin{vmatrix} 1 & 11 & 1 \\ 2 & 19 & -1 \\ 0 & 12 & 0 \end{vmatrix} = (-1)^5 \cdot 12 \cdot \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -12 \cdot -3 = 36$$

Ex 2:

$$D = \begin{pmatrix} 1 & 2 & 2 & 1 \\ 1 & 8 & 6 & 1 \\ -2 & 6 & 6 & 1 \\ 1 & 3 & 3 & -1 \end{pmatrix}$$

$$|D| = \begin{vmatrix} 2 & 5 & 5 & 0 \\ 2 & 11 & 9 & 0 \\ -1 & 9 & 9 & 0 \\ 1 & 3 & 3 & -1 \end{vmatrix} = (-1) \cdot (-1) \cdot \begin{vmatrix} 2 & 5 & 5 \\ 2 & 11 & 9 \\ -1 & 9 & 9 \end{vmatrix}$$

$$= (-1) \cdot \begin{vmatrix} 0 & 23 & 23 \\ 0 & 27 & 27 \\ -1 & 9 & 9 \end{vmatrix} = (-1) \cdot (-1) \cdot (-1) \cdot \begin{vmatrix} 23 & 23 \\ 27 & 27 \end{vmatrix}$$

$$= 23 \cdot 27 - 23 \cdot 27 = 23(27 - 29) = 23 \cdot (-2) = -46$$

Ex 4:

$$A = \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \dots & a_{nn} \end{pmatrix}$$

PROP:

$$|B| = \begin{vmatrix} b_{11} & \dots & b_{1n} \\ \vdots & & \vdots \\ \lambda b_{j1} & \dots & \lambda b_{jn} \\ \vdots & & \vdots \\ b_{n1} & \dots & b_{nn} \end{vmatrix} = \lambda \begin{vmatrix} b_{11} & \dots & b_{1n} \\ \vdots & & \vdots \\ b_{j1} & \dots & b_{jn} \\ \vdots & & \vdots \\ b_{n1} & \dots & b_{nn} \end{vmatrix}$$

$$|\lambda A| = \begin{vmatrix} \lambda a_{11} & \dots & \lambda a_{1n} \\ \vdots & & \vdots \\ \lambda a_{n1} & \dots & \lambda a_{nn} \end{vmatrix} = \lambda \cdot \begin{vmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \dots & a_{nn} \end{vmatrix} = \lambda^2 \begin{vmatrix} a_{11} & \dots & a_{1n} \\ a_{21} & \dots & a_{2n} \\ \vdots & & \vdots \\ a_{n1} & \dots & a_{nn} \end{vmatrix}$$

$$\dots = \lambda^n \begin{vmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \dots & a_{nn} \end{vmatrix} = \lambda^n \cdot |A|$$

$$C_{ij} = (-1)^{i+j} \cdot |A_{dij}|$$

$$A^{-1} = \frac{1}{|A|} \cdot [cof(A)]^T$$

$$x^3 = 1$$

$$x = 1$$

Ej 5:

$$a) A^3 = I_n; \quad |A^3| = |I_n| = 1 = |A| \cdot |A| \cdot |A| \Rightarrow |A| = 1$$

$$c) |A \cdot A^T| = |A| \cdot |A^T| = |A|^2 = 16 \Rightarrow |A| = \pm 4$$

$$d) |A^2 \cdot A^T \cdot A^{-1}| = |A|^2 \cdot |A^T| \cdot |A^{-1}| = |A| \cdot |A^T| = |A|^2 = 9 \Rightarrow |A| = \pm 3$$

Ej 6:

$$1) \begin{array}{cc|cc} 2/2 & -2/2 & 1/2 & 0 \\ -1 & 3 & 0 & 1 \end{array} \xrightarrow{F_2 + F_1} \begin{array}{cc|cc} 1 & -1 & 1/2 & 0 \\ 0 & 2/2 & 1/2 & 1/2 \end{array} \xrightarrow{F_1 + F_2} \begin{array}{cc|cc} 1 & 0 & 3/4 & 1/2 \\ 0 & 1 & 1/4 & 1/2 \end{array}$$

$$2) |A| = 4; \quad A^{-1} = \frac{1}{4} \cdot \begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}^T = \begin{pmatrix} 3/4 & 1/2 \\ 1/4 & 1/2 \end{pmatrix}$$

Ej 7:

$$\underbrace{A^{-1} \cdot A}_{I_d} \begin{pmatrix} x \\ y \end{pmatrix} = A^{-1} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 5 \\ 3 \end{pmatrix} = \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$