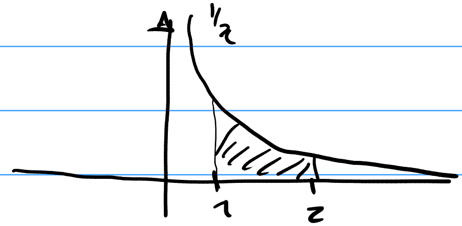
Práctico 4

5 (d) Encontrar el área debajo de la curva $y = \frac{1}{x}$

entre $x = 1$ y $x = 2$



$$\int_1^2 \frac{1}{x}\,dx = \log(x)\Big|_1^2 = \log(2) - \underbrace{\log(1)}_{0} = \log(2)$$

6(b) $$\int_{-1}^{1} x^{1/3}\,dx = \frac{1}{\frac{1}{3}+1}\, x^{\frac{1}{3}+1}\Big|_{-1}^{1} = \frac{3}{4}\left(1 - \underbrace{(-1)^{\frac{4}{3}}}_{=1}\right) = 0.$$

Si $f(x) = x^a$, $a>0$, entonces $F(x) = \frac{1}{a+1}\, x^{a+1}$ es primitiva de f.

$$\left[\; F'(x) = \frac{1}{\cancel{a+1}}\,\cancel{(a+1)}\, x^{a+\cancel{1}-\cancel{1}} = x^a \;\right]$$

$$(-1)^{\frac{4}{3}} = (-1)^{4\cdot\frac{1}{3}} = \left((-1)^4\right)^{1/3} = 1^{\frac{1}{3}} = 1$$

$$\int_a^b g(f(x))\, f'(x)\, dx = \int_{f(a)}^{f(b)} g(u)\, du$$

6 (c) $\displaystyle\int_{-\pi}^{\pi} \overbrace{\text{sen}}^{g}(\overbrace{2x}^{f(x)})\, dx$.

$g(x) = \text{sen}(x)$

$f(x) = 2x \Rightarrow f'(x) = 2$

$$\frac{1}{2}\int_{-\pi}^{\pi} \underbrace{\text{sen}(2x)}_{g(f(x))}\, \underbrace{2}_{f'(x)}\, dx = \frac{1}{2}\int_{f(-\pi)}^{f(\pi)} g(u)\, du = \frac{1}{2}\int_{-2\pi}^{2\pi} \text{sen}(u)\, du = \frac{1}{2}\left(-\cos(u)\right)\Big|_{-2\pi}^{2\pi} = 0$$

[sen(2x) tiene primitiva $-\frac{1}{2}\cos(2x)$]

Alternativamente: $h(x) = \text{sen}(2x)$, $h: \mathbb{R} \to \mathbb{R}$. Es impar.

$h(-x) = -h(x)$.

Si h es impar $\displaystyle\int_{-a}^{a} h = 0$

[$h(-x) = \text{sen}(-2x) = -\text{sen}(2x) = -h(x)$]

Ej 7 (b). Hallar una primitiva de $\dfrac{\log(x)}{x}$.

$$\int_a^x \underline{g(f(u))\, f'(u)}\, dx = \int_{f(a)}^{f(x)} g(u)\, du$$

una primitiva es $F(x) = \int_a^x \frac{\log(v)}{v}\,dv$

$$\overbrace{\frac{1}{v}}^{}\ \overbrace{\log(v)}^{f(v)\ \text{“}g(f(v))\text{”}} \qquad \frac{1}{v} = f'(v)$$

$g(x) = x$

$f(v) = \log(v)$

$$= \int_a^x g(f(v))\,f'(v)\,dv$$

$$= \int_{f(a)}^{f(x)} g(u)\,du = \int_{\log(a)}^{\log(x)} u\,du = \frac{u^2}{2}\Bigg|_{\log(a)}^{\log(x)} = \frac{(\log(x))^2}{2} - \underbrace{\frac{(\log(a))^2}{2}}_{\text{número}}$$

Así que $G(x) = \frac{(\log(x))^2}{2}$ es primitiva de $\frac{\log(x)}{x}$.

7(d) Hallar primitiva de $\frac{2x+1}{x^2+x+1}$.

$$\boxed{\int_a^x g(f(v))\,f'(v)\,dv = \int_{f(a)}^{f(x)} g(u)\,du.}$$

$$\int_a^x \frac{2v+1}{v^2+v+1}\,dv = \int_a^x \frac{f'(v)}{f(v)}\,dv$$

$$= \int_a^x \underbrace{\frac{1}{f(v)}}_{g(f(v))} \cdot f'(v)\,dv = \int_a^x g(f(v))\,f'(v)\,dv$$

$$= \int_{f(a)}^{f(x)} g(u)\,du = \int_{f(a)}^{f(x)} \frac{1}{u}\,du$$

→ $f(v) = v^2+v+1$

$f'(v) = 2v+1$

$g(u) = \frac{1}{u}$

$$= \log(u) \Big|_{f(a)}^{f(x)} = \lg f(x) - \underbrace{\lg(f(a))}_{\text{número}}$$

Así que $F(x) = \log(f(x))$
$= \lg(x^2+x+1)$ es primitiva.

Comentario.

$\dfrac{f'}{f}$ tiene primitiva $\log(f)$.

$$(\log \circ f)'(x) = \log'(f(x))\, f'(x) = \frac{1}{f(x)} f'(x)$$

8 (b) Hallar una primitiva usando "partes".

$e^{-4x} \cos(2x)$.

$$\int_a^x f'(u) g(u)\, du = f(u) g(u) \Big|_a^x - \int_a^x f(u) g'(u)\, du.$$

Una primitiva es:

$$F(x) = \int_0^x \underbrace{e^{-4u}}_{g(u)} \underbrace{\cos(2u)}_{f'(u)} du = \frac{1}{2} \operatorname{sen}(2u)\, e^{-4u} \Big|_0^x - \int_0^x \cancel{\frac{1}{2}} \operatorname{sen}(2u) (-\cancel{4}^{\,2}) e^{-4u}\, du \quad (*)$$

Si $f'(u) = \cos(2u)$, podemos usar $f(u) = \frac{1}{2} \operatorname{sen}(2u)$ || Si $g(u) = e^{-4u} \Rightarrow g'(u) = -4 e^{-4u}$

$$(*) = \underline{\frac{1}{2}\sin(2x)e^{-4x}} - 2\underbrace{\int_0^x \sin(2u)e^{-4u}\,du}\quad (**) \leftarrow$$

$$\int_0^x \underbrace{\sin(2u)}_{f'(u)}\underbrace{e^{-4u}}_{g(u)}du = \underbrace{-\frac{\cos(2u)}{2}}_{f(u)}\underbrace{e^{-4u}}_{g(u)}\Big|_0^x + \int_0^x +\frac{\cos(2u)}{\cancel{2}}(-\cancel{4}^{2})e^{-4u}\,du$$

$$= -\frac{\cos(2x)}{2}e^{-4x} - \left(-\frac{1}{2}\cdot 1\right) + 2\underbrace{\int_0^x \cos(2u)e^{-4u}\,du}_{\substack{\shortparallel\\ F(x)}}$$

$$F(x) = \frac{1}{2}\sin(2x)e^{-4x} - 2\left[-\frac{\cos(2x)}{2}e^{-4x} + \frac{1}{2} + 2F(x)\right]$$

$$= \frac{1}{2}\sin(2x)e^{-4x} + \cos(2x)e^{-4x} - 1 - 4F(x)$$

$$\Rightarrow 5F(x) = \frac{1}{2}\sin(2x)e^{-4x} + \cos(2x)e^{-4x} - 1 \Rightarrow F(x) = \underbrace{\frac{1}{5}e^{-4x}\left(\frac{\sin(2x)+\cos(2x)}{2}\right)}_{\text{primitive.}} - \underbrace{\frac{1}{5}}_{\text{unitree}}$$

8 (e)

$x \operatorname{sen}(x)$

$$\int_0^x \underbrace{u}_{g(u)}\, \underbrace{\operatorname{sen}(u)}_{f'(u)}\,du = \overbrace{u}^{g(u)}\overbrace{(-\cos u)}^{f(u)}\Big|_0^x - \int_0^x \overbrace{(-\cos u)}^{f(u)} \cdot \overbrace{1}^{g'(u)}\,du$$

$$= -x\cos(x) + \int_0^x \cos u\,du = -x\cos(x) + \operatorname{sen} u\Big|_0^x$$

$$= -x\cos x + \operatorname{sen} x \;..$$