$n \geq 2$ natural

Teo

$$\int_a^b \operatorname{sen}^n(x)\,dx = -\frac{1}{n}\operatorname{sen}^{n-1}(x)\cos(x)\Big|_a^b + \frac{n-1}{n}\int_a^b \operatorname{sen}^{n-2}(x)\,dx$$

$$\int_a^b \cos^n(x)\,dx = \frac{1}{n}\cos^{n-1}(x)\operatorname{sen}(x)\Big|_a^b + \frac{n-1}{n}\int_a^b \cos^{n-2}(x)\,dx.$$

dem

Veamos la primera.

Llamemos $I_n = \int_a^b \operatorname{sen}^n(x)\,dx.$

$$I_n = \int_a^b \operatorname{sen}^{n-1}(x)\operatorname{sen}(x)\,dx = -\operatorname{sen}^{n-1}(x)\cos(x)\Big|_a^b + (n-1)\int_a^b \operatorname{sen}^{n-2}(x)\cos^2(x)\,dx$$

$$= -\operatorname{sen}^{n-1}(x)\cos(x)\Big|_a^b + (n-1)\int_a^b \operatorname{sen}^{n-2}(x)\left(1-\operatorname{sen}^2(x)\right)dx$$

$$= -\operatorname{sen}^{n-1}(x)\cos(x)\Big|_a^b + (n-1)\int_a^b \operatorname{sen}^{n-2}(x)\,dx - (n-1)\underbrace{\int_a^b \operatorname{sen}^n(x)\,dx}_{I_n}.$$

$$nI_n = -\sin^{n-1}(x)\cos(x)\Big|_a^b + (n-1)\int_a^b \sin^{n-2}(x)\,dx.$$ $\boxed{a}$

Ej $$\int_a^b \underbrace{\cos^2(x)}_{1-\sin^2(x)}\sin^2(x)\,dx = \int_a^b \sin^2(x)\,dx - \int_a^b \sin^4(x)\,dx$$ $(*)$

Calculemos $\int_a^b \sin^2(x)\,dx$. Vimos $\sin^2(x) = \dfrac{1-\cos(2x)}{2}$

$$\left[\begin{array}{l} \cos(2x) = \cos^2(x) - \sin^2(x) \\ \cos(a+b) = \cos a \sin b \pm \sin a \cos b \end{array}\right]$$

$$\int_a^b \sin^2(x)\,dx = \frac{1}{2}\left[\int_a^b 1\,dx - \int_a^b \cos(2x)\right] = \frac{1}{2}\left[x\Big|_a^b - \frac{\sin(2x)}{2}\Big|_a^b\right]$$

$f : \mathbb{R} \to \mathbb{R} \quad f(x) = 2x$

$g : \mathbb{R} \to \mathbb{R} \quad g(u) = \cos u .$

$f'(x) = 2$

$$\int_a^b \cos(2x)\,dx = \frac{1}{2}\int_a^b g(f(x))\,\overset{f'(x)}{\overline{2}}\,dx$$

$$= \frac{1}{2}\int_{f(a)}^{f(b)} g(u)\,du = \frac{1}{2}\int_{2a}^{2b} \cos(u)\,du = \frac{\operatorname{sen} u}{2}\Big|_{2a}^{2b}$$

$$= \frac{\operatorname{sen}(2x)}{2}\Big|_b^a$$

Falta $\displaystyle\int_a^b \operatorname{sen}^4(x)\,dx$

Usamos:

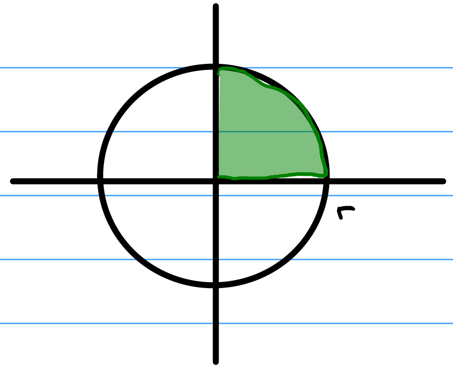
$$\int_a^b \operatorname{sen}^n(x)\,dx = -\frac{1}{n}\operatorname{sen}^{n-1}(x)\cos(x)\Big|_a^b + \frac{n-1}{n}\int_a^b \operatorname{sen}^{n-2}(x)\,dx$$

$$\int_a^b \operatorname{sen}^4(x)\,dx = -\frac{1}{4}\operatorname{sen}^3(x)\cos(x)\Big|_a^b + \frac{3}{4}\int_a^b \operatorname{sen}^2(x)\,dx .$$

$$(*) = \int_a^b \operatorname{sen}^2(x)\,dx - \left(\frac{1}{4}\operatorname{sen}^3(x)\cos(x)\Big|_a^b + \frac{3}{4}\int_a^b \operatorname{sen}^2(x)\,dx\right)$$

$$= -\frac{1}{4}\operatorname{sen}^3(x)\cdot\cos(x)\Big|_a^b + \frac{1}{4}\underbrace{\int_a^b \operatorname{sen}^2(x)\,dx}_{\text{la calculamos antes}}$$

Ej



Calcular el área del círculo de radio r.

Calculamos el área verde.

$g : [0,r] \to \mathbb{R}$ $\qquad g(x) = \sqrt{r^2 - x^2}$

Queremos $\displaystyle\int_0^r \sqrt{r^2-x^2}\,dx$. $\qquad$ " $x = r\,\text{sen}(t)$ "

" $\sqrt{r^2-x^2} = \sqrt{r^2 - r^2\text{sen}^2(t)} = r\sqrt{1-\text{sen}^2 t} = r\sqrt{\cos^2(t)} = r\cos(t)$ "

$f : [0,\pi/2] \to [0,r]$ $\qquad f(t) = r\,\text{sen}(t)$ $\qquad f(0) = r\,\text{sen}(0) = 0$

$f'(t) = r\cos(t)$ $\qquad f(\pi/2) = r\,\text{sen}\frac{\pi}{2} = r$

$$\int_0^{\pi/2} g(f(t))\,f'(t)\,dt = \int_{f(0)}^{f(\pi/2)} g(x)\,dx = \int_0^r g(x)\,dx \leftarrow$$ la integral que queremos calcular

$\|$

$$\int_0^{\pi/2} \sqrt{r^2 - r^2\text{sen}^2(t)}\;r\cos(t)\,dt = r^2\int_0^{\pi/2} \cos^2(t)\,dt \quad (*)$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2} \Rightarrow \int_0^{\pi/2} \cos^2(t)\,dt = \frac{1}{2}\left[\int_0^{\pi/2} 1\,dt + \int_0^{\pi/2} \cos(2t)\,dt\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{2} + \underbrace{\frac{\operatorname{sen}(2t)}{2}\Big|_0^{\pi/2}}_{0-0}\right] = \frac{\pi}{4}$$

$\Rightarrow$ (*) $\frac{r^2\pi}{4}$ Área del círculo $= r^2\pi$.

<u>Def</u>. Una función f es <u>impar</u> si: $f(-x) = -f(x)$.

dijemos $f: [-a,a] \to \mathbb{R}$ ó $f: \mathbb{R} \to \mathbb{R}$

" — — — <u>par</u> si $f(-x) = f(x)$.

<u>Ej</u> sen es impar. , cos es par. $f(x) = x^3$ impar

$f(x) = x^2$ par

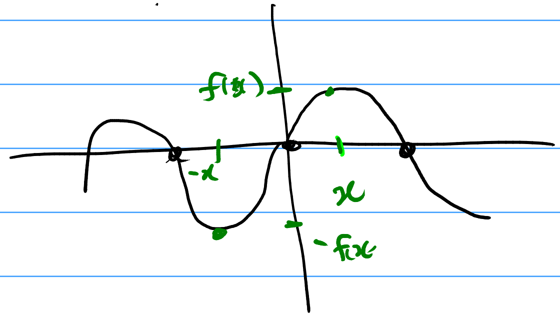
$f(x) = x^n$ es impar si n impar

par si n es par.

$(-x)^n = ((-1)x)^n = (-1)^n x^n$

-1: n impar

1: n par.

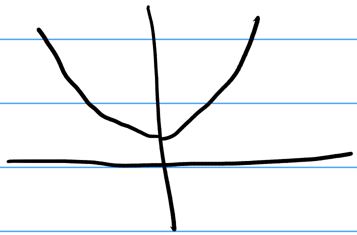


f impar

$(x, f(x))$ está en gráfico

$(-x, -f(x))$ " "

x^2+1



f par $(-x, f(x))$

Imp Si $f: [-a,a] \to \mathbb{R}$ es impar $\Rightarrow \int_{-a}^{a} f = 0$.

dem

Tenemos $i: [-a,a] \to [-a,a]$ $i(t) = -t$.

$i(-a) = a$, $i'(t) = -1$

$i(a) = -a$

$$\int_{i(-a)}^{i(a)} f = \int_{-a}^{a} f(i(t))\, i'(t)\, dt = \int_{-a}^{a} \underbrace{f(-t)}_{=-f(t)} (-1)\, dt = \underbrace{\int_{-a}^{a} f(t)\, dt}_{I}$$

$\parallel$

$$\int_{a}^{-a} f = - \underbrace{\int_{-a}^{a} f}_{I}$$

$I = -I \Rightarrow I = 0.$

Ej $\int_{-a}^{a} \sin(t)\, dt = 0.$

$$\int_1^x \log(t)\,dt = \int_1^x \log(t)\cdot 1\,dt = \log(t)\cdot t\Big|_1^x - \int_1^x \frac{1}{\cancel{t}}\,\cancel{t}\,dt$$

$$= \log(x)\,x - x .$$