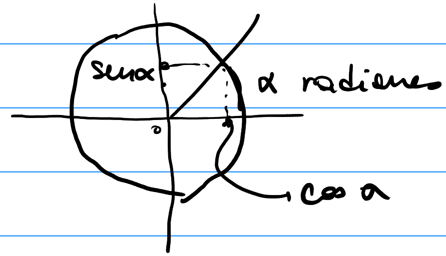
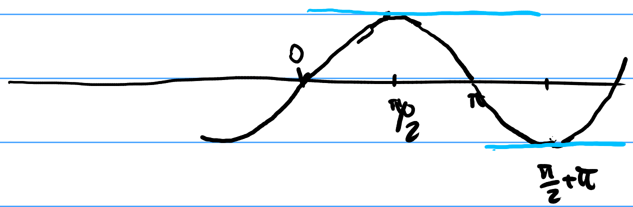
Hemos visto: $\quad \text{sen}, \cos : \mathbb{R} \to \mathbb{R}$



son derivables y $\quad \text{sen}'(x) = \cos(x)$

$\cos'(x) = -\text{sen}(x)$

En qué puntos $\text{sen}(x) = 0$? Esto es $\cos(x) = 0$, o sea $x = \frac{\pi}{2} + n\pi \quad n \in \mathbb{Z}$



En qué puntos $\cos(x) = 0$. Esto es $\text{sen}(x) = 0$.

Así que $x = n\pi \quad n \in \mathbb{Z}$.

Ej: Calculemos $\tan'(x)$.

Recordar que $\tan(x) = \dfrac{\operatorname{sen}(x)}{\cos(x)}$, $x \neq \frac{\pi}{2} + n\pi$, $n \in \mathbb{Z}$

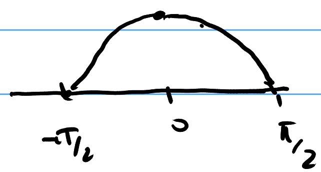
Recordar que $\left(\dfrac{f}{g}\right)'(x) = \dfrac{f'(x)g(x) - f(x)g'(x)}{g^2(x)} = \dfrac{f'(x)}{g(x)} - \dfrac{f(x)g'(x)}{g^2(x)}$.

$$\tan'(x) = \frac{\cos(x)}{\cos(x)} - \frac{\operatorname{sen}(x)\,(-\operatorname{sen}(x))}{\cos^2(x)}$$

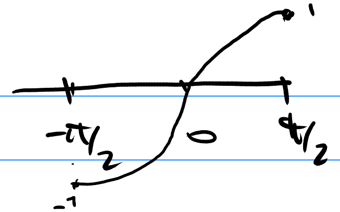
$$= 1 + \frac{\operatorname{sen}^2(x)}{\cos^2(x)} = 1 + \tan^2(x).$$

Dibujemos una gráfica de tan, en $I = (-\frac{\pi}{2}, \pi/2)$

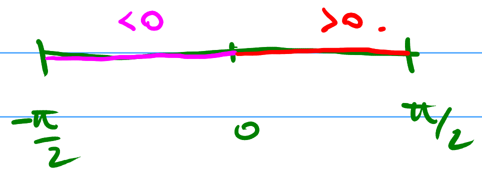
En I, cos tiene gráfica

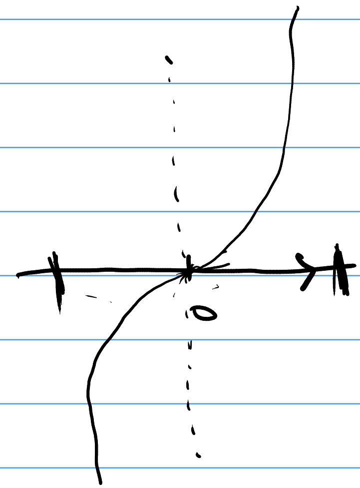


$\cos(x) > 0$ si $x \in I$

En I_1 sen tiene gráfico 

Así que $\tan(x) = \dfrac{\operatorname{sen}(x)}{\cos(x)}$ va a ser $\begin{cases} >0 & \text{si } 0 < x < \pi/2 \\ 0 & \text{si } x = 0 \\ <0 & \text{si } -\frac{\pi}{2} < x < 0 \end{cases}$





$$\tan(-x) = \frac{\operatorname{sen}(-x)}{\cos(-x)} = \frac{-\operatorname{sen}(x)}{\cos(x)} = -\tan(x).$$

$$\lim_{x \to \pi/2^-} \tan(x) = \lim_{x \to \pi/2} \frac{\overbrace{\operatorname{sen}(x)}^{\to 1}}{\underbrace{\cos(x)}_{\to 0^+}} = +\infty.$$

$$\lim_{x\to -\frac{\pi}{2}^+} \tan(x) = \lim_{x\to -\pi/2^+} \frac{\overbrace{\operatorname{sen}(x)}^{\to -1}}{\underbrace{\cos(x)}_{\to 0^+}} = -\infty.$$

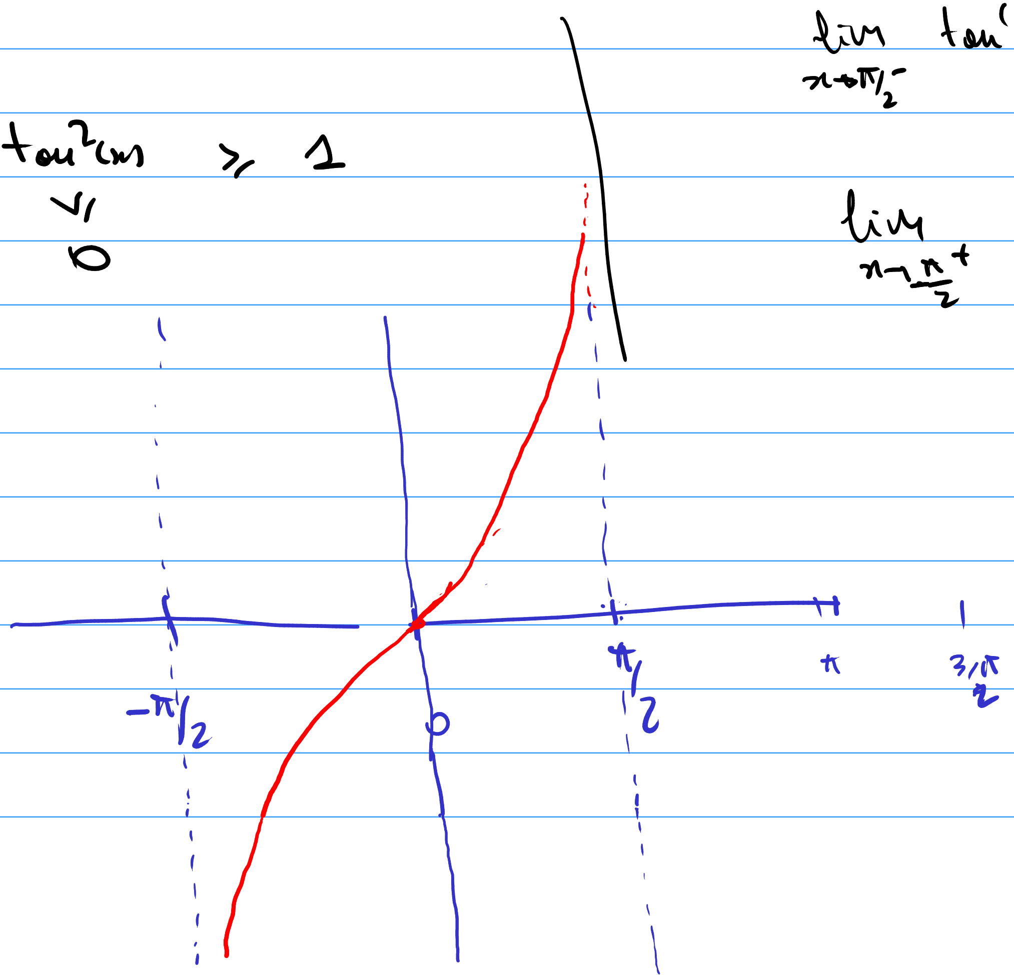
¿Qué pasa con $\tan'(x) = 1+\tan^2(x)$?

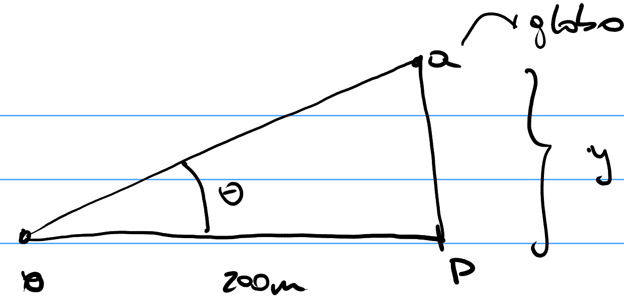
$$\tan'(0) = 1 + \overbrace{\tan^2(0)}^{0} = 1$$

$$\lim_{x\to\pi/2^-} \tan'(x) = +\infty$$

$$\tan'(x) = 1 + \underbrace{\tan^2(x)}_{\geq 0} \geq 1$$

$$\lim_{x\to -\frac{\pi}{2}^+} \tan'(x) = 1 + \lim_{x\to -\pi/2^+} \tan^2(x) = +\infty$$





Ej

Se suelta un globo desde el punto P.
Un observador en O mide el ángulo θ
que forma el globo con el horizonte.

Distancia entre O y P $= 200m$.
El ángulo aumenta $\frac{1}{20}$ radianes por segundo.

¿A qué velocidad está subiendo el globo cuando $\theta = \pi/4$?

y es una función del tiempo t (en segundos). $y(t)$

θ " " " " t. $\theta(t)$.

Como el globo sube, $\theta(t)$ aumenta cont.: Existe un t_0 donde $\theta(t_0) = \pi/4$.

Queremos $y'(t_0)$ ($y'(t)$ es la velocidad)

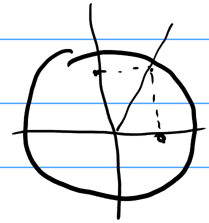
Sabemos $\theta'(t) = \frac{1}{20}$ (ángulo aumenta $\frac{1}{20}$ cada segundo).

$$\tan(\theta) = \frac{y}{200} \Rightarrow y = 200\tan(\theta).$$

$$y'(t) = 200\,(\tan\circ\theta)'(t) = \overbrace{200\,\frac{1}{20}}^{10.}\,(1+\tan^2(\theta(t))).$$

$$\boxed{(g\circ f)'(t) = f'(t)\, g'(f(t))}$$

$$(\tan\circ\theta)'(t) = \theta'(t)\cdot\tan'(\theta(t))$$
$$= \frac{1}{20}\left(1+\tan^2(\theta(t))\right)$$



$$y'(t_0) = 10\left(1+\tan^2(\underbrace{\theta(t_0)}_{\pi/4})\right) = 10\left(1+\overbrace{\tan^2\left(\frac{\pi}{4}\right)}^{1}\right)$$

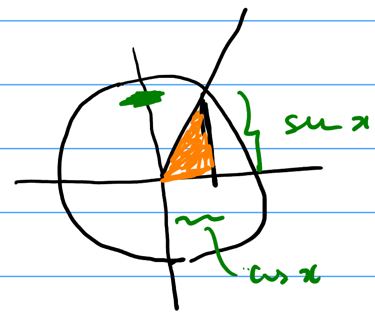
$$= 10\,(1+1) = 20.$$

Así que la velocidad es 20 m/s

Hallar la velocidad del globo cuando el ángulo θ_0 cumple $\operatorname{sen}(\theta_0) = \frac{1}{5}$.

Calculamos $y'(t) = 10(1 + \tan^2\theta(t))$.

$$\tan^2 x = \underbrace{\frac{\operatorname{sen}^2 x}{\cos^2(x)} = \frac{\operatorname{sen}^2(x)}{1-\operatorname{sen}^2(x)}}_{1=\operatorname{sen}^2+\cos^2}$$



$$\tan^2(\theta_0) = \frac{(1/5)^2}{1-(1/5)^2} = \frac{1/\cancel{5^2}}{\frac{5^2-1}{\cancel{5^2}}} = \frac{1}{24}$$

Si t_0 es el tiempo para el cual $\theta(t_0) = \theta_0$ $\Rightarrow y'(t_0) = 10\Big(1 + \underbrace{\tan^2 \overbrace{\theta(t_0)}^{\theta_0}}_{1/24}\Big) = 10(1 + 1/24)$

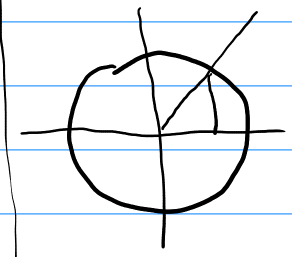
$$= 10\,\frac{25}{24} = \frac{5\cdot 25}{12} = \frac{5^3}{12}.$$

Ej: Hallar recta tangente a $y = \text{sen}(4x)$ en $x = \frac{\pi}{16}$.

$f(x) = \text{sen}(4x) = \text{sen}(k(x))$ donde $k(x) = 4x$.

$$f'(x) = (\text{sen} \circ k)'(x) = k'(x)\, \text{sen}'(k(x))$$
$$= 4 \cos(4x)$$

$$\Rightarrow f'\left(\frac{\pi}{16}\right) = 4 \cos\left(\frac{4\pi}{16}\right) = 4 \cos(\pi/4) = 4 \frac{1}{\sqrt{2}} = 2 \cdot \frac{2}{\sqrt{2}} = 2\sqrt{2}$$



$\cos \pi/4 = \text{sen}\, \pi/4$

$$1 = \text{sen}^2 \frac{\pi}{4} + \cos^2 \frac{\pi}{4} = 2 \cos^2 \frac{\pi}{4} \Rightarrow \frac{1}{2} = \cos^2 \pi/4 \Rightarrow + \frac{1}{\sqrt{2}} = \cos(\pi/4)$$

La recta es $y = 2\sqrt{2}x + b$. Tiene que pasar por $\left(\frac{\pi}{16}, f\left(\frac{\pi}{16}\right)\right) = \left(\frac{\pi}{16}, \text{sen} \frac{\pi}{4}\right)$

$= \left(\frac{\pi}{16}, \frac{1}{\sqrt{2}}\right)$.

$$\Rightarrow \frac{1}{\sqrt{2}} = 2\sqrt{2} \cdot \frac{\pi}{16} + b \Rightarrow b = \frac{1}{\sqrt{2}} - \frac{2\sqrt{2}\pi}{16} = \frac{1}{\sqrt{2}} - \frac{\sqrt{2}\pi}{8}.$$