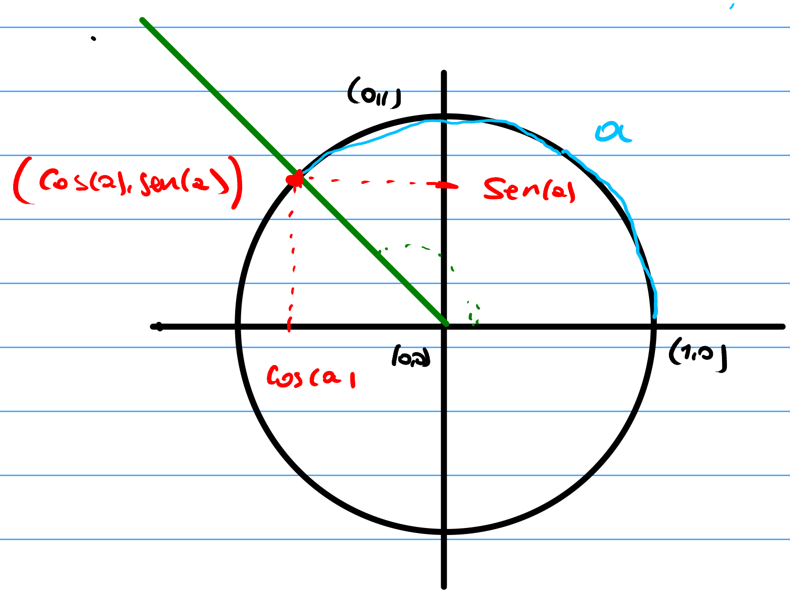
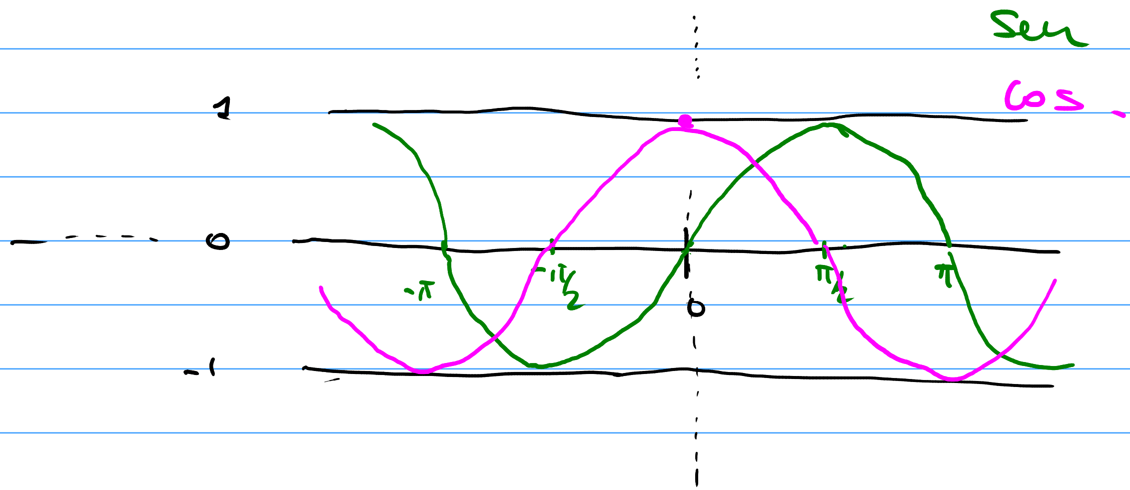




$$\cos(a) = \operatorname{sen}(a + \pi/2)$$
$$\operatorname{sen}(b) = \cos(b - \pi/2)$$

$\boxed{b = a + \pi/2}$

$$\cos(-a) = \cos(a)$$
$$\operatorname{sen}(-a) = -\operatorname{sen}(a)$$

$$\cos(a+\pi) = -\cos(a)$$
$$\operatorname{sen}(a+\pi) = -\operatorname{sen}(a)$$

$\Rightarrow$

$$\cos(a+2\pi) = \cos(a)$$
$$\operatorname{sen}(a+2\pi) = \operatorname{sen}(a),$$

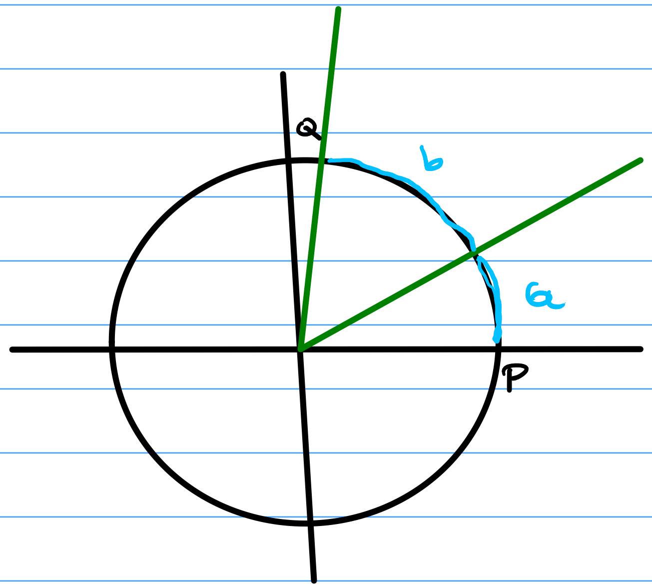
$$\tan(a) = \frac{\operatorname{sen}(a)}{\cos(a)} \quad \text{cuando } \cos(a) \neq 0.$$

Hoy queremos: - fórmulas para $\text{sen}(a+b)$ y $\cos(a+b)$.

- $\text{sen}' = \cos$ y $\cos' = -\text{sen}$.

Teorema $\left|\begin{array}{l} \cos(a+b) = \cos(a)\cos(b) - \text{sen}(a)\,\text{sen}(b) \\ \text{sen}(a+b) = \cos(a)\,\text{sen}(b) + \text{sen}(a)\cos(b) \end{array}\right|$

¿Porqué es verdad? Primero vemos la fórmula para $\cos(a+b)$.

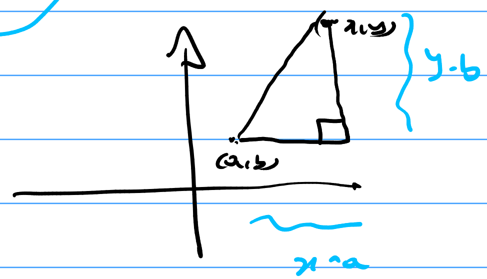


$P = (1,0)$

$Q = (\cos(a+b), \text{sen}(a+b))$

(x,y) (a,b)

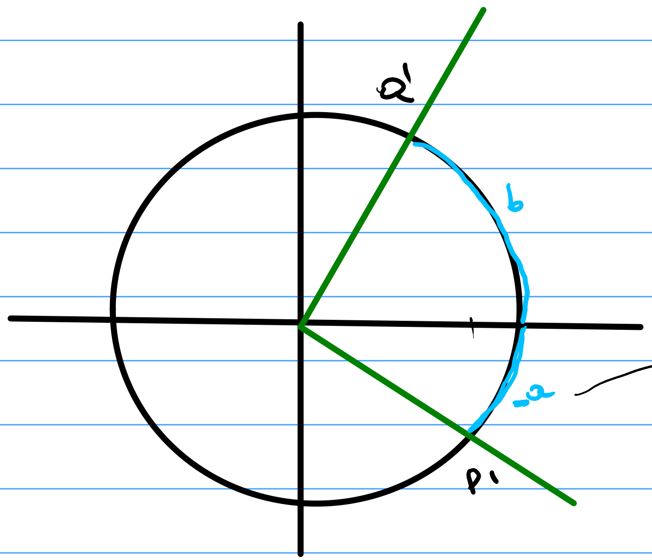
$(x-a)^2 + (y-b)^2$



$(\text{distancia entre } P \text{ y } Q)^2 = (\cos(a+b) - 1)^2 + (\text{sen}(a+b) - 0)^2$ (*)

$= \cos^2(a+b) - 2\cos(a+b) + 1 + \text{sen}^2(a+b) = 2 - 2\cos(a+b)$

$= 2(1 - \cos(a+b))$



Este es el mismo dibujo pero rotado un ángulo "a" en sentido horario

Como las rotaciones preservan las distancias $\Rightarrow$ $\text{dist}(P', Q') = \text{dis}(P, Q)$

$P' = (\cos(-a), \text{sen}(-a)) = (\cos(a), -\text{sen}(a))$

$Q' = (\cos(b), \text{sen}(b))$

(**) $\text{dist}(P', Q')^2 = (\cos(a) - \cos(b))^2 + (-\text{sen}(a) - \text{sen}(b))^2$

$= \cos^2(a) - 2\cos(a)\cos(b) + \cos^2(b) + \text{sen}^2(a) + 2\,\text{sen}(a)\,\text{sen}(b) + \text{sen}^2(b)$

$= 2 - 2\cos(a)\cos(b) + 2\,\text{sen}(a)\,\text{sen}(b)$

$= 2(1 - \cos(a)\cos(b) + \text{sen}(a)\,\text{sen}(b))$ (**)

Notación $(\cos(a))^2 = \cos^2(a)$

Como $(*) = (**)$ $\quad 2(1-\cos(a+b)) = 2(1-\cos(a)\cos(b)+\operatorname{sen}(a)\operatorname{sen}(b))$

Divido entre 2, cancelo los 1 y cambio el signo $\Rightarrow$ $\cos(a+b) = \cos(a)\cos(b) - \operatorname{sen}(a)\operatorname{sen}(b)$ ✓

Falta la fórmula de $\operatorname{sen}(a+b)$. Sabemos $\operatorname{sen}(x) = \cos\left(x-\frac{\pi}{2}\right)$ $\leftarrow$

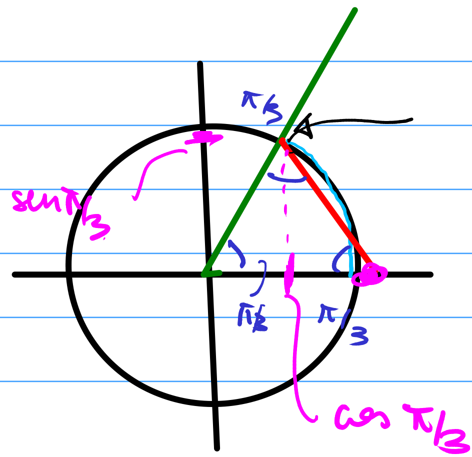
$$\operatorname{sen}(a+b) = \cos\left(a+\left(b-\frac{\pi}{2}\right)\right) = \cos(a)\underbrace{\cos(b-\pi/2)}_{=} - \operatorname{sen}(a)\underbrace{\operatorname{sen}(b-\pi/2)}_{=}$$

lo vimos.

$$= \cos(a)\operatorname{sen}(b) - \operatorname{sen}(a)(-\cos(b))$$

$$= \cos(a)\operatorname{sen}(b) + \operatorname{sen}(a)\cos(b).$$ ∎

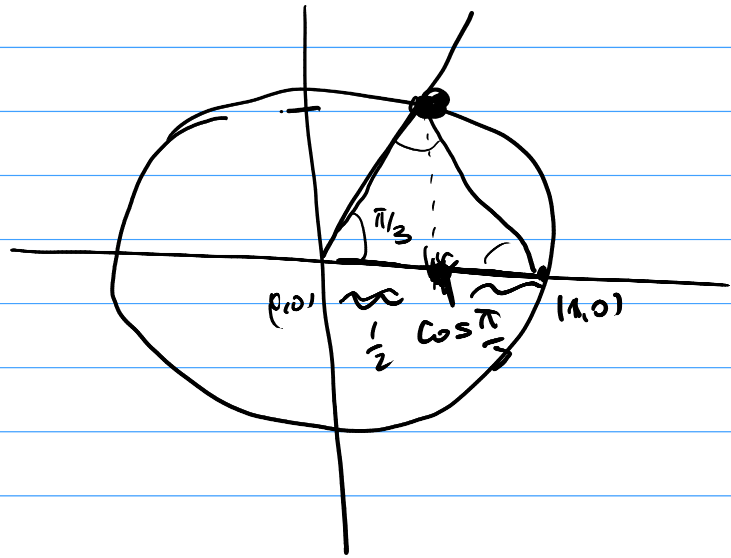
Ej $\quad 60° = \frac{360°}{6} \sim \frac{2\pi}{6} = \frac{\pi}{3}$



Construyo triángulo equilátero.
los lados tienen largo 1
Como el triángulo es equilátero, $\cos\frac{\pi}{3}$ es el punto medio del lado del triángulo.

El lado es el segmento de $(0,0)$ a $(1,0)$ $\Rightarrow$ $\cos \pi/3 = 1/2$

$$\operatorname{sen}\frac{\pi}{3} = \sqrt{1-\underbrace{\cos^2 \pi/3}_{1/4}} = \sqrt{1-(1/2)^2} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}.$$

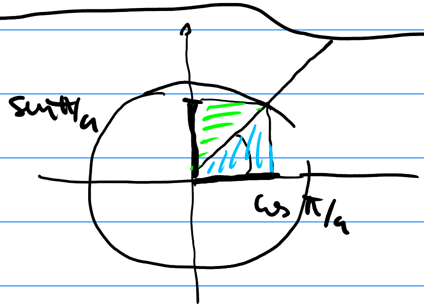


Ej Calculemos $\operatorname{sen}\left(\frac{7\pi}{12}\right)$.

$$\frac{7}{12} = \frac{4}{12} + \frac{3}{12} = \frac{1}{3} + \frac{1}{4}$$

$$\Rightarrow \frac{7\pi}{12} = \frac{\pi}{3} + \pi/4 \quad . \quad \text{Sen}\left(\frac{7\pi}{12}\right) = \text{sen}\left(\frac{\pi}{3} + \frac{\pi}{4}\right) =$$

$$= \text{sen}\,\frac{\pi}{3}\,\cos\frac{\pi}{4} + \cos\frac{\pi}{3}\,\text{sen}\,\frac{\pi}{4} = \frac{\sqrt{3}}{2}\,\frac{1}{\sqrt{2}} + \frac{1}{2}\,\frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}}\left(\sqrt{3}+1\right)$$



$$\cos \pi/4 = \text{sen}\,\pi/4$$

$$1 = \cos^2\pi/4 + \text{sen}^2\,\pi/4 = 2\cos^2\pi/4$$

$$\Rightarrow \frac{1}{2} = \cos^2\pi/4 \Rightarrow \frac{1}{\sqrt{2}} = \cos(\pi/4)$$

<u>Corolario.</u>

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\text{Sen}(2x) = 2\cos(x)\,\text{sen}(x)$$

$$\cos(2x) = \cos(x+x) = \cos(x)\cos(x) - \text{sen}(x)\,\text{sen}(x) = \cos^2(x) - \text{sen}^2(x) = 2\cos^2(x) - 1$$

$$1 = \cos^2 x + \text{sen}^2 x \Rightarrow -\text{sen}^2(x) = \cos^2(x) - 1$$

$\operatorname{sen}(2x) = \operatorname{sen}(x+x) = \operatorname{sen}(x)\cos x + \cos x \operatorname{sen} x = 2\cos x \operatorname{sen} x$

Si queremos calcular $\operatorname{sen}'(x)$, tenemos que calcular $\lim_{h \to 0}$ de

$$\frac{\operatorname{sen}(x+h) - \operatorname{sen}(x)}{h}$$

vamos a usar la fórmula de $\operatorname{sen}(a+b)$.