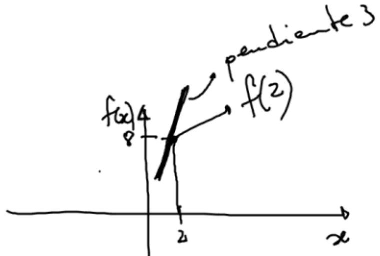
Clase de Rocha por zoom 20 de abril

$y = x^3$ ó $f(x) = x^3$

Ecuación de la tangente paralela a la recta $y = 3x$

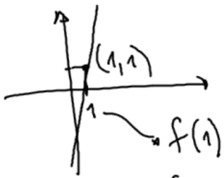
$\underbrace{f'(x) = 3x^2}$



$\left.\begin{array}{l} y = mx + n \\ m = 3 \end{array}\right\} \Rightarrow y = 3x + n$

$f'(x) = 3 \rightarrow 3x^2 = 3 \rightarrow x^2 = \frac{3}{3} \rightarrow x^2 = 1 \rightarrow x = \pm 1$

1) $x = 1$



$f(1) = 1$

$f(x) = x^3$

$y = 3x + n$

$1 = 3.1 + n$

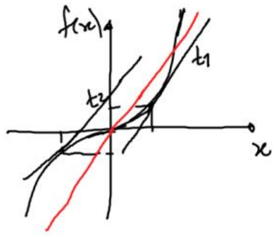
$1 = 3 + n$

$1 - 3 = n$

$-2 = n$

La ecuación de la tangente es $y = 3x - 2$

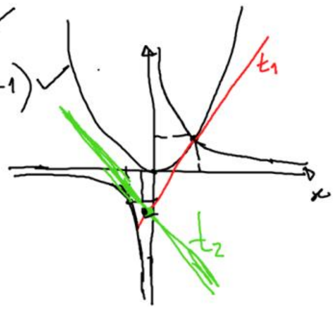
Hacerlo mismo para $x = -1$



2) $y = x^2$ Recta tangente en $(1,1)$ ✓

$y = \frac{1}{x}$ Recta tangente en $(-1,-1)$ ✓

$y(1) = 1^2 = 1$ $y(-1) = \frac{1}{-1} = -1$



$y = x^2$ Recta tangente en $(1,1)$

$y' = 2x$

$t_1)$ $y = mx + n$

$\underbrace{1}_{} = \underbrace{2 \cdot 1}_{} + n \Rightarrow 1 = 2 + n \Rightarrow 1 - 2 = n \Rightarrow -1 = n$

$y'(1) = 2 \cdot 1 = 2$

$t_1)$ $y = 2x - 1$

$y = \frac{1}{x}$ Recta tangente en $(-1, -1)$

$y = mx + n$

$y = \frac{1}{x}$ $\qquad \left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$

$y' = \frac{(1)' \cdot x - 1 \cdot (x)'}{x^2} = \frac{0 \cdot x - 1 \cdot 1}{x^2} = \frac{-1}{x^2} \longrightarrow x = -1$

$y' = \frac{-1}{(-1)^2} = \frac{-1}{1} = -1$

$y = mx + n \qquad y = -1x + n \longleftarrow m = -1$

$y = -1x + n \qquad (-1, -1)$

$-1 = -1(-1) + n \Rightarrow -1 = 1 + n \Rightarrow -1 - 1 = n \Rightarrow -2 = n$

$t_2)$ $y = -x - 2$

Punto de intersección

$t_1)$ $y = 2x - 1$

$t_2)$ $y = -x - 2$

$\begin{cases} y = 2x - 1 \\ y = -x - 2 \end{cases}$

$2x - 1 = -x - 2$

$2x + x = -2 + 1$

$3x = -1$

$x = -1/3$

$y = 2\left(-\frac{1}{3}\right) - 1$

$y = -\frac{2}{3} - 1 \longrightarrow y = -\frac{2}{3} - \frac{3}{3} = -\frac{5}{3}$

$P = \left(-\frac{1}{3}, -\frac{5}{3}\right)$

$$\begin{cases} y = 2x - 1 \\ y = -x - 2 \end{cases} \quad (2)$$

$$\begin{aligned} y &= 2x - 1 \\ + \quad 2y &= -2x - 4 \\ \hline 3y &= 0x - 5 \\ 3y &= -5 \\ y &= -\frac{5}{3} \end{aligned}$$

Sustituyo en alguna de las ecuaciones

$$\begin{aligned} y &= 2x - 1 \quad (1) \\ y &= -x - 2 \quad (-1) \end{aligned}$$

$$\begin{aligned} y &= 2x - 1 \\ -y &= x + 2 \\ \hline 0y &= 3x + 1 \end{aligned}$$

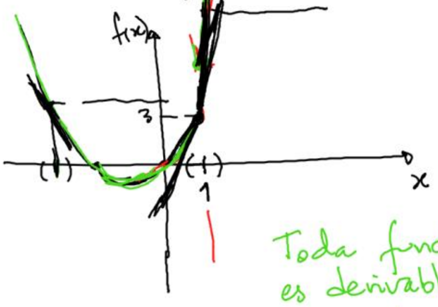
$$3x + 1 = 0$$

$$x = -\frac{1}{3}$$

3)

$$f(x) = \begin{cases} x^2 + 2x & \text{si } x \leq 1 \\ x^3 + 4x & \text{si } x > 1 \end{cases}$$

$$f'(x) = \begin{cases} 2x + 2 & \text{si } x < 1 \\ -3x^2 + 4 & \text{si } x > 1 \end{cases}$$



$$x^2 + 2x = 0$$

$$x \cdot (x + 2) = 0$$

$$x = 0$$

ó

$$x + 2 = 0 \iff$$

$$\iff x = -2$$

$$f(1) = 1^2 + 2.1 = 3$$

$$x^3 + 4x = 0$$

$$x \cdot (x^2 + 4) = 0 \quad \begin{cases} x = 0 \\ x^2 + 4 = 0 \end{cases}$$

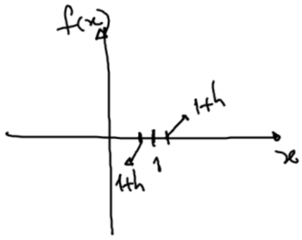
no hay raíces

Toda función polinómica es derivable en todos los reales.

$$f(x)=\begin{cases} x^2+2x & \text{si } x\le 1 \\ -x^3+4x & \text{si } x>1 \end{cases}$$

f es derivable en todos los $x<1$ o $x>1$
(por estar definida con expresiones polinómicas)

Estudio $\lim\limits_{h\to 0} \dfrac{f(1+h)-f(1)}{h}$



$$f(1)=1^2+2\cdot 1=1+2=3$$

$$f(x)=\begin{cases} x^2+2x & \text{si } x\le 1 \\ -x^3+4x & \text{si } x>1 \end{cases}$$

cuadrado del binomio

$$\lim_{\substack{h\to 0^-\\(h<0)}} \frac{f(1+h)-f(1)}{h}=\lim_{h\to 0^-}\frac{(1+h)^2+2(1+h)-3}{h}=$$

$$=\lim_{h\to 0^-}\frac{1+2h+h^2+2+2h-3}{h}=\lim_{h\to 0^-}\frac{h^2+4h}{h}=$$

$$=\lim_{h\to 0^-}\frac{\cancel{h}(h+4)}{\cancel{h}}=4$$

$$\lim_{\substack{h\to 0^+\\ (h>0)}} \frac{f(1+h)-\overbrace{f(1)}^{=3}}{h} =$$

$$f(1+h) = -(1+h)^3 + \underbrace{4(1+h)}_{} = -(1+h)^2\cdot(1+h)+4+4h =$$

$$= -(1+2h+h^2)\cdot(1+h)+4+4h = -(1+h+2h+2h^2+h^2+h^3)+4+4h =$$

$$= -1-h-2h-2h^2-h^2-h^3+4+4h =$$

$$= -h^3-3h^2+h+3$$

$$\lim_{h\to 0^+} \frac{-h^3-3h^2+\overbrace{h}^{h\cdot 1}+\cancel{3}-\cancel{3}}{h} = \lim_{h\to 0^+} \frac{\cancel{h}\cdot(-\overbrace{h^2}^{\to 0}-\overbrace{3h}^{\to 0}+1)}{\cancel{h}} = 1$$

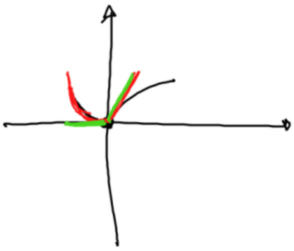
Como $\lim_{h\to 0^-} \frac{f(1+h)-f(1)}{h} = 4$ y

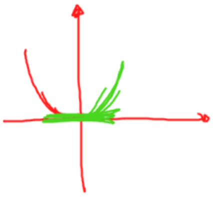
$\lim_{h\to 0^+} \frac{f(1+h)-f(1)}{h} = 1$ entonces

$\lim_{h\to 0} \frac{f(1+h)-f(1)}{h} \nexists$ y entonces

f no es derivable en $x=1$.

$$g(x) = \begin{cases} x^2 & \text{si } x<0 \\ x^3 - x^2 & \text{si } x \geq 0 \end{cases} \longrightarrow (0,0)$$





$$g(x) = \begin{cases} x^2 & \text{si } x<0 \\ x^3 - x^2 & \text{si } x \geq 0 \end{cases}$$

$(x^2)' = 2x$

$(x^3-x^2)' = 3x^2-2x$

$$\lim_{x\to 0} g(x) = g(0) = 0$$

$$\lim_{h\to 0^-} \frac{g(0+h)-g(0)}{h} = \lim_{h\to 0^-} \frac{(0+h)^2 - (0^3-0^2)}{h} =$$

$$= \lim_{h\to 0^-} \frac{h^2}{h} = \lim_{h\to 0^-} \frac{h\cdot \cancel{h}}{\cancel{h}} = \lim_{h\to 0^-} h = 0$$

$$\lim_{h\to 0^+} \frac{g(0+h)-g(0)}{h} = 0$$

Conclusión: g es derivable en 0 y $g'(0)=0$

→ Si f es derivable en x entonces f es continua en x.

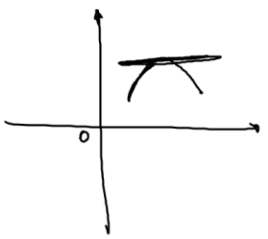
→ Si f no es continua en x, entonces f no es derivable en x.

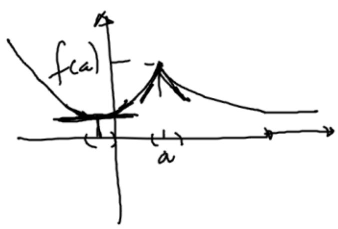
f puede ser continua en x y no ser derivable en x.

Definición de función derivable

f es derivable en x si y solo si $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} =$

$f'(x)$ ← nº real





4) a) $f(x) = 2x^2 + 3x$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{2(x+h)^2 + 3(x+h) - (2x^2+3x)}{h}$$

$$\lim_{h\to 0} \frac{2(x+h)^2+3(x+h)-(2x^2+3x)}{h} =$$

$$= \lim_{h\to 0} \frac{2(x^2+2xh+h^2)+3x+3h-2x^2-3x}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2x^2}+4xh+2h^2+\cancel{3x}+3h-\cancel{2x^2}\cancel{-3x}}{h} =$$

$$= \lim_{h\to 0} \frac{4xh+2h^2+3h}{h} = \lim_{h\to 0} \frac{\cancel{h}\cdot(4x+2h+3)}{\cancel{h}} = 4x+3$$

$$(f+g)'(x) = f'(x) + g'(x)$$

$$(\lambda \cdot f)'(x) = \lambda \cdot f'(x)$$

$\lambda \in \mathbb{R}$

$$(f\cdot g)'(x) = f'(x)\cdot g(x) + f(x)\cdot g'(x)$$

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{g^2(x)}$$

$$\left(4\cdot(x^2+3x)\right)' =$$

$$= 4\cdot(2x+3)$$

5) c)

$$f(x) = \overbrace{(3x^3 - 5)}^{f} \cdot \overbrace{\left(\frac{1}{x^2} + \frac{1}{x}\right)}^{g} \qquad (f \cdot g)' = f' \cdot g + f \cdot g'$$

$$(\lambda \cdot f)' = \lambda \cdot f' \qquad f'(x) = (3 \cdot 3x^2 - 0) \cdot \left(\frac{1}{x^2} + \frac{1}{x}\right) + (3x^3 - 5) \cdot \left(-\frac{2}{x^3} - \frac{1}{x^2}\right)$$

$$g(x) = \frac{1}{x^2} + \frac{1}{x} \qquad \left(\frac{f}{g}\right)' \rightarrow \frac{\overbrace{(1)'}^{=0} \cdot x - 1 \cdot x'}{x^2} = -\frac{1}{x^2}$$

$$\left(\frac{f}{g}\right)' \quad \frac{(1)' \cdot x^2 - 1 \cdot 2x}{(x^2)^2} = \frac{0 - 2x}{x^4} = \frac{-2\cancel{x}}{\cancel{x^4}\,{}_3} = -\frac{2}{x^3}$$