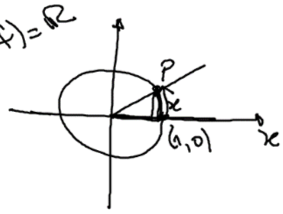
Clase Rocha (Zoom) 22 de junio

Práctico 4

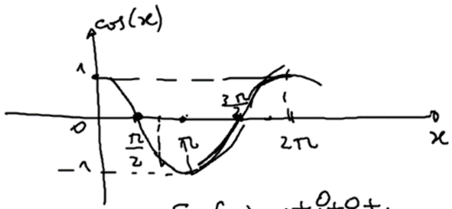
$\rightarrow \mathrm{Dom}(f) = \mathbb{R}$



2) d) $f(x) = \cos^2(x)$ en $[0, 2\pi]$

$\cos^2(x) = 0$

Int. con Oy: $f(0) = \cos^2(0) = (\cos(0))^2 = 1^2 = 1$



Int. con Ox: $\cos^2(x) = 0$

$\underbrace{\cos(x)}_{0} \cdot \cos(x) = 0 \iff x = \frac{\pi}{2}$ o $x = \frac{3\pi}{2}$

Sg $f(x)$: $+ \; 0 \; + \; 0 \; +$; $0 \quad \frac{\pi}{2} \quad \frac{3\pi}{2} \quad \pi$ x

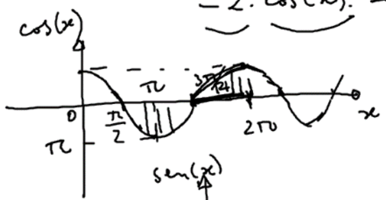
$f(x) = \cos^2(x) = (\cos(x))^2$

u^2, $u = \cos(x)$
(Regla de la cadena)

$f'(x) = 2 \cdot \cos(x) \cdot (-\mathrm{sen}(x))$

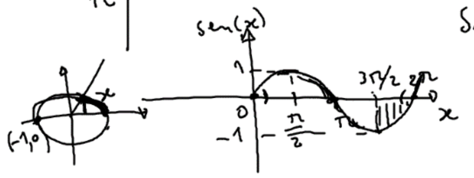
$f'(x) = -2 \cdot \cos(x) \cdot \mathrm{sen}(x)$

$-2 \cdot \cos(x) \cdot \mathrm{sen}(x) = 0 \iff \begin{cases} \cos(x) = 0 \\ \text{o} \\ \mathrm{sen}(x) = 0 \end{cases} \iff \begin{cases} x = 0, \\ x = \frac{\pi}{2}, \\ x = \pi, \\ x = \frac{3\pi}{2} \\ x = 2\pi \end{cases}$



Sg $\cos(x)$: $+ \; 0 \; - \; 0 \; +$; $0 \quad \frac{\pi}{2} \quad \frac{3\pi}{2} \quad 2\pi$ x

Sg $\mathrm{sen}(x)$: $0 \quad 0 \quad 0$; $0 \quad \pi \quad 2\pi$



$-2\cos(x)\mathrm{sen}(x)$: $0 \; - \; 0 \; + \; 0 \; - \; 0 \; + \; 0$; $0 \quad \frac{\pi}{2} \quad \pi \quad \frac{3\pi}{2} \quad 2\pi$ x

m.r   MA   m.r

$\cos(x)$ es periódica

$\forall k \in \mathbb{R}$, $\cos(x) = \cos(x + 2k\pi)$

$\cos(x) = \cos(x + 2\pi)$

$\cos^2(x) = \cos^2(x + 2\pi)$

$$f'(x) = -2 \cdot \cos(x) \cdot \operatorname{sen}(x)$$

$$f''(x) = -2 \cdot \left[-\operatorname{sen}(x) \cdot \operatorname{sen}(x) + \cos(x) \cdot \cos(x) \right]$$

$$= -2 \cdot \left(-\operatorname{sen}^2(x) + \cos^2(x) \right) = -2 \cdot \left(\cos^2(x) - \operatorname{sen}^2(x) \right)$$

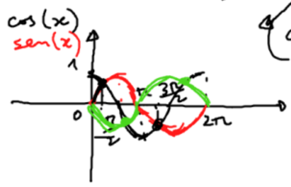
$$\cos^2(x) - \operatorname{sen}^2(x) = 0 \iff \cos^2(x) = \operatorname{sen}^2(x)$$

$$a^2 - b^2 = (a+b) \cdot (a-b)$$

$$(\cos(x) + \operatorname{sen}(x)) \cdot \underline{(\cos(x) - \operatorname{sen}(x))} = 0$$

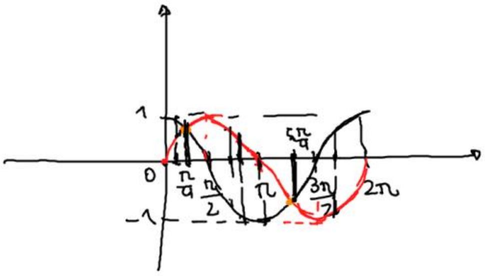
$$\cos(x) = \operatorname{sen}(x)$$

$$\cos(x) = -\operatorname{sen}(x)$$



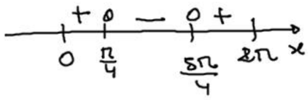
$x = \frac{\pi}{4}$, $x = \frac{5\pi}{4}$

$x = \frac{3\pi}{4}$ $\quad x = \frac{3\pi}{2} + \frac{\pi}{4} = \frac{7\pi}{4}$



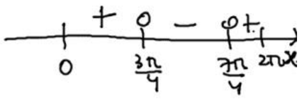
$$f''(x) = -2 \cdot (\cos(x) + \operatorname{sen}(x)) \cdot (\cos(x) - \operatorname{sen}(x))$$

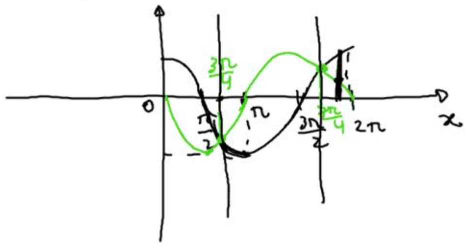
Sg $\cos(x) - \operatorname{sen}(x)$



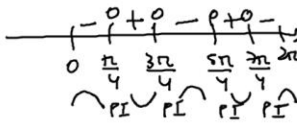
Sg $\cos(x) + \operatorname{sen}(x)$

$\cos(x) - (-\operatorname{sen}(x))$





Sg $-2 \cdot (\quad)$



$$f(x)=\cos^2(x)$$

No existen $\lim\limits_{x\to\pm\infty} f(x)$

Sg $f'(x)$: 0 $-$ $\frac{\pi}{2}$ $+$ π $-$ $\frac{3\pi}{2}$ $+$ 2π ; x

m.r.

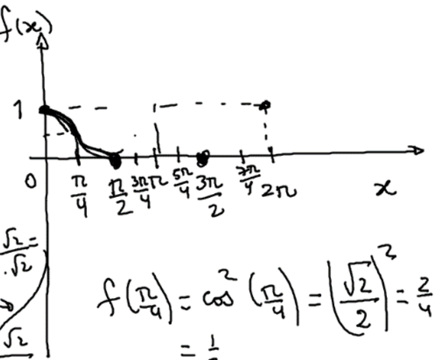
$f\left(\frac{\pi}{2}\right)=0$

$f(\pi)=\cos^2(\pi)=1$

$f\left(\frac{3\pi}{2}\right)=0$

$f(0)=1$

$f(2\pi)=\cos^2(2\pi)=1$



$x^2+x^2=1$

$2x^2=1$

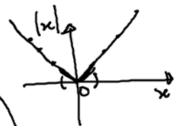
$x^2=\frac{1}{2}$

$x=\sqrt{\frac{1}{2}}=\frac{1\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}}=\frac{\sqrt{2}}{2}$

$f\left(\frac{\pi}{4}\right)=\cos^2\left(\frac{\pi}{4}\right)=\left(\frac{\sqrt{2}}{2}\right)^2=\frac{2}{4}=\frac{1}{2}$

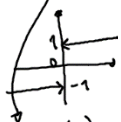
Sg $f'(x)$: 0 $-$ $\frac{\pi}{4}$ $+$ $\frac{3\pi}{4}$ $-$ $\frac{5\pi}{4}$ $+$ $\frac{7\pi}{4}$ $-$ 2π ; x

e) $f(x)=\dfrac{\sqrt{x^2}}{x+1}$ $\qquad f(x)=\dfrac{|x|}{x+1}$



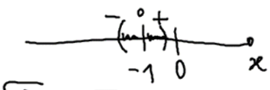
Dominio $\quad \mathrm{Dom}(f)=\mathbb{R}-\{-1\}$ $\qquad \sqrt{x^2}=(x^2)^{1/2}=|x|$

Raices. $\quad f(x)=0 \qquad x=0$ $\qquad f(-4)=\sqrt{(-4)^2}=\sqrt{16}=4$



$f(0)=0$

$|x|'=\mathrm{sg}(x)$



$$\lim_{x\to-1^\pm}\frac{\sqrt{x^2}}{x+1}=\pm\infty$$

$\sqrt{x^2}\to\sqrt{u}\ ,\ u=x^2$

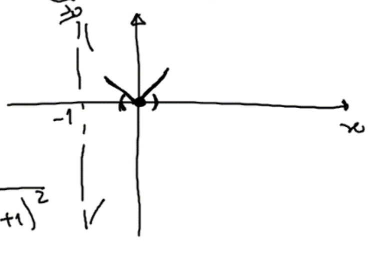
$\frac{1}{2\sqrt{u}}\cdot u'=\frac{u'}{2\sqrt{u}}$

$$f'(x)=\frac{(\sqrt{x^2})'\cdot(x+1)-\sqrt{x^2}\cdot(x+1)'}{(x+1)^2}=\frac{\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2}}(x+1)-\sqrt{x^2}\cdot 1}{(x+1)^2}=$$

$$f'(x) = \frac{\frac{x(x+1)}{\sqrt{x^2}} - \sqrt{x^2}}{(x+1)^2} = \frac{\frac{x(x+1)}{\sqrt{x^2}} - \frac{x^2}{\sqrt{x^2}}}{(x+1)^2} =$$

$$= \frac{x^2+x-x^2}{\sqrt{x^2}\cdot(x+1)^2} = \frac{x}{\sqrt{x^2}\cdot(x+1)^2} = \frac{x}{\underbrace{|x|}_{>0}\cdot\underbrace{(x+1)^2}_{>0}}$$

$Sg\ f'(x)$	$-$	$\nexists$ (-1)	$-$	$\nexists$ (0)	$+$
	$\searrow$		$\searrow$	m.r.	$\nearrow$



$f(0)=0$

$$\lim_{x\to 0^{+}_{-}} \frac{x}{|x|\cdot(x+1)^2}$$

$0 \in Dom(f)$ pero $0 \notin Dom(f')$

b) $f(x) = \dfrac{\cos(2x)}{3}$

$F(x) = \dfrac{1}{6}\cdot \operatorname{sen}(2x) + K$, $K\in\mathbb{R}$

$F'(x) = \dfrac{1}{6}\cdot\cos(2x)\cdot 2 = \dfrac{1}{\cancel{6}_3}\cdot\cancel{2}\cdot\cos(2x) = \dfrac{\cos(2x)}{3}$

$(\operatorname{sen}(2x))' = \underline{2\cdot\cos(2x)}$

$\dfrac{\cos(2x)}{3} = \dfrac{2\cdot\cos(2x)}{2\cdot 3} = \dfrac{1}{6}\cdot \underline{2\cos(2x)}$

e) $f(x) = e^{2x} - 3e^{x} + 2$

$F(x) = \frac{1}{2}e^{2x} - 3e^{x} + 2x$

$F(x) = 2x - 3e^{x} + \dfrac{e^{2x}}{2}$

$F'(x) = \frac{1}{\cancel{2}}\cdot\cancel{2}\cdot e^{2x} - 3e^{x} + 2 = f(x)$

$e^{x} \xrightarrow{f'} e^{x}$

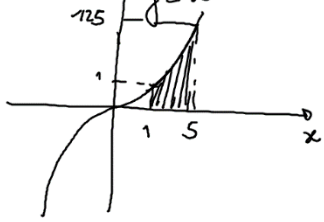
$e^{u} \rightarrow e^{u}\cdot u'$

f) $f(x) = \dfrac{1}{\sqrt{1-x^2}}$

$F(x) = \text{arcsen}(x)$

5) Área debajo de la curva

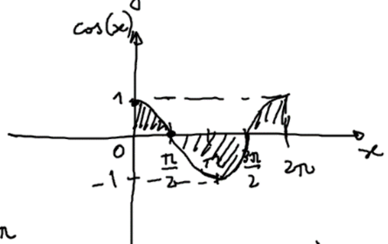
$y = x^3$ entre $x=1$ y $x=5$



$A = \int_1^5 x^3\,dx = \dfrac{x^4}{4}\Big|_1^5 = \dfrac{5^4}{4} - \dfrac{1^4}{4} = 156$

$\left(\dfrac{x^4}{4}\right)' = \left(\dfrac{1}{4}\cdot x^4\right)' = \dfrac{1}{\cancel{4}}\cdot \cancel{4}x^3 = x^3$

c) $y = \cos(x)$ entre $x=0$ y $x=2\pi$



$\int_0^{2\pi} \cos(x)\,dx = 0$

$\int_0^{\pi/2} \cos(x)\,dx - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos(x)\,dx + \int_{\frac{3\pi}{2}}^{2\pi} \cos(x)\,dx$

$= \text{sen}(x)\Big|_0^{\frac{\pi}{2}} - \text{sen}(x)\Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + \text{sen}(x)\Big|_{\frac{3\pi}{2}}^{2\pi} =$

$= \text{sen}\left(\frac{\pi}{2}\right) - \text{sen}(0) - \left(\text{sen}\,\frac{3\pi}{2} - \text{sen}\,\frac{\pi}{2}\right) + \text{sen}\,2\pi - \text{sen}\,\frac{3\pi}{2}$

$= 1 - 0 - (-1-1) + 0 + 1 = 1+2+1 = 4$