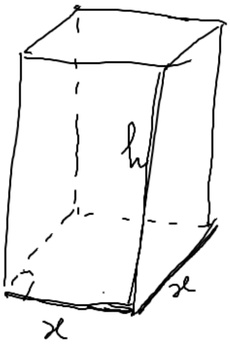
Clase práctico de Rocha – 5 de junio



$S_{vp} = \underline{C} \to$ nº conocido

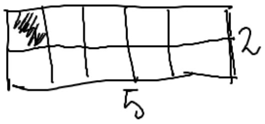
Volumen máximo

$V = x^2 \cdot h$

$A_b = x^2 + 4xh = C$

$4xh = C - x^2$

$h = \dfrac{C - x^2}{4x}$



5x2

$$\left.\begin{array}{l} V = x^2 \cdot h \\ h = \dfrac{C - x^2}{4x} \end{array}\right\} \Rightarrow V = x^2 \cdot \frac{C - x^2}{4x} = \frac{x^{\cancel{2}}(C - x^2)}{4\cancel{x}}$$

$$V(x) = \frac{x(C - x^2)}{4} = \frac{1}{4} \cdot x(C - x^2)$$

$\underline{x \in \mathbb{R}^+}$

$(\alpha \cdot f)' = \alpha \cdot f'$

$$V'(x) = \frac{1}{4} \cdot \left(x(C - x^2)\right)'$$

Estudio de funciones

$$f(x) = \frac{3x-5}{2x+1}$$

Dom (f)

Intersección con los ejes

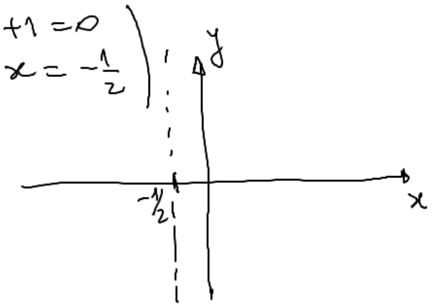
Derivada primera

Derivada segunda

Límites cuando $x \to +\infty$ y $x \to -\infty$

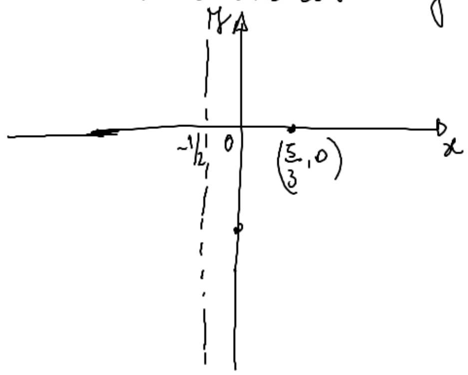
$$Dom(f) = \mathbb{R} - \left\{-\frac{1}{2}\right\}$$

$$\begin{pmatrix} 2x+1=0 \\ x = -\frac{1}{2} \end{pmatrix}$$



$$f(x) = \frac{3x-5}{2x+1}$$

Intersecciones con los ejes



$$3x - 5 = 0$$

$$x = \frac{5}{3} \rightarrow f\left(\frac{5}{3}\right) = 0$$

$\left(\frac{5}{3}, 0\right)$ int. con Ox

$$f(0) = \frac{-5}{1} = -5$$

$(0, -5)$ int. con Oy

Derivada 1ª

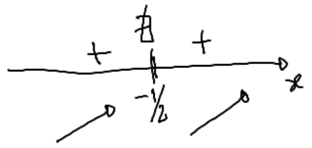
$$f'(x) = \frac{3(2x+1) - 2(3x-5)}{(2x+1)^2} = \frac{13}{(2x+1)^2}$$

$$f'(x) = \frac{13}{(2x+1)^2}$$

$$(2x+1)^2 = (2x+1)(2x+1)$$

$$\left.\begin{array}{l} (2x+1)^2 > 0 \quad \forall x \neq -\frac{1}{2} \\ 13 > 0 \end{array}\right\} \Rightarrow f'(x) > 0 \quad \forall x \neq -\frac{1}{2}$$

Sg



Derivada 2ª

$u^2 \longrightarrow 2uu'$

$$f'(x) = \frac{13}{(2x+1)^2}$$

$$\left((2x+1)^2\right)' = 2(2x+1)\cdot 2 = 4(2x+1)$$

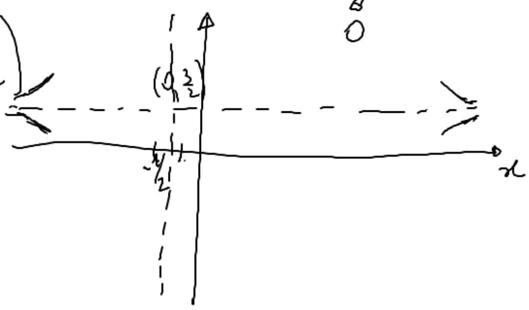
$$f''(x) = \frac{0\cdot(2x+1)^2 - 13\cdot 4(2x+1)}{(2x+1)^4} = \frac{-52\cancel{(2x+1)}}{(2x+1)^{\cancel{4}\,3}} = \frac{-52}{(2x+1)^3}$$

Sg $f''(x)$: $+$ | $\nexists$ en $-1/2$ | $-$; $\cup$ $\quad -1/2 \quad$ $\cap$; x

$$\lim_{x\to\pm\infty} \frac{3x-5}{2x+1} = \lim_{x\to\pm\infty} \frac{3x\left(1-\frac{5}{3x}\right)}{2x\left(1+\frac{1}{2x}\right)} = \lim_{x\to\pm\infty} \frac{3\cancel{x}}{2\cancel{x}} = \frac{3}{2}$$

$$\frac{5}{3x} \to 0, \quad \left(1-\frac{5}{3x}\right) \to 1, \quad \frac{1}{2x} \to 0$$

$$3x-5 = 3x\left(1-\frac{5}{3x}\right)$$

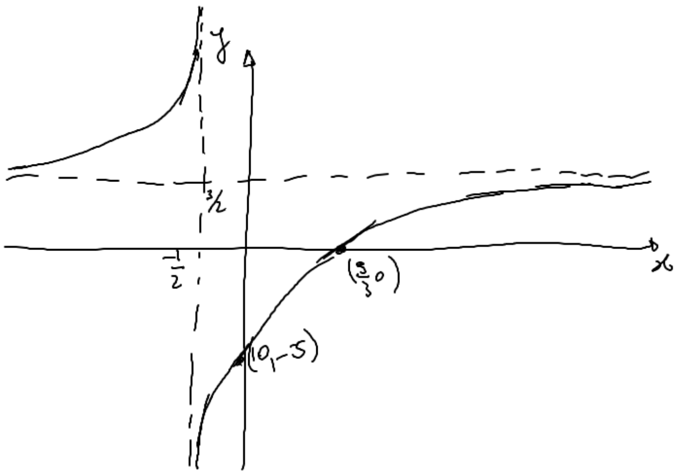


$y = \frac{3}{2}$

Asintota horizontal

$$\lim_{x \to -\frac{1}{2}^{\pm}} \frac{\overbrace{3x-5}^{\to -13/2}}{\underbrace{2x+1}_{\to 0^{\pm}}} = \mp\infty$$

$Sg(2x+1)$: $-$ 0 $+$ at $-1/2$



Realizar el estudio de las funciones:

1) $f(x) = \dfrac{x^2}{x+3}$

2) $f(x) = (x-2) \cdot e^x$

3) $f(x) = \log \dfrac{2x+1}{x}$

4) $f(x) = \log \left| \dfrac{2x+1}{x} \right|$

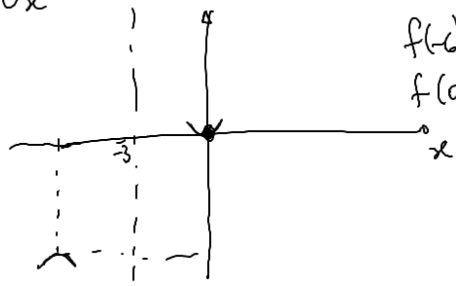
$$f(x) = \frac{x^2}{x+3}$$

$\mathrm{Dom}(f) = \mathbb{R} - \{-3\}$

Raíces: $(0,0)$ int. con O_y y con O_x

$$f'(x) = \frac{x^2+6x}{(x+3)^2}$$

$\mathrm{Sg}\, f'(x)$: $+$ 0 $-$ $\nexists$ $-$ 0 $+$; x: -6, -3, 0 ; $\nearrow$ $\searrow$ $\searrow$ $\nearrow$



$f(-6) = -12$

$f(0) = 0$

$$\frac{12}{8} = \frac{\cancel{2}\times 6}{\cancel{2}\times 4} = \frac{6}{4}$$

$$f'(x) = \frac{x^2+6x}{(x+3)^2}$$

$$f''(x) = \frac{(2x+6)(x+3)^2 - (x^2+6x).2(x+3)}{(x+3)^4} =$$

$$= \frac{\cancel{(x+3)}.\left((2x+6)(x+3) - (x^2+6x).2\right)}{(x+3)^{\cancel{4}3}} =$$

$$\cancel{\frac{12}{8} = \frac{8+4}{8}}$$

$$= \frac{18}{(x+3)^3}$$

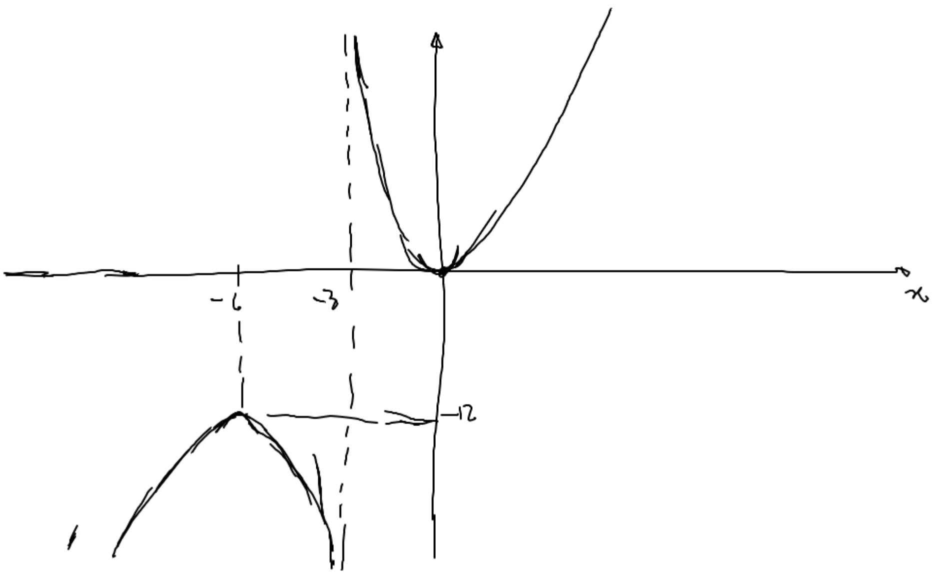
$\mathrm{Sg}\, f''(x)$: $-$ $\nexists$ $+$; x: -3 ; $\cap$ $\cup$

$$f(x) = \frac{x^2}{x+3}$$

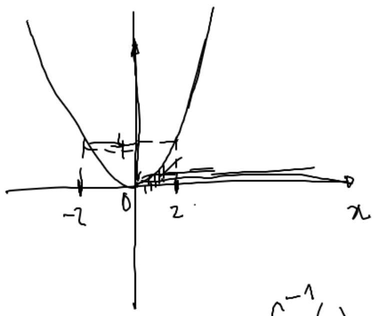
$$\lim_{x\to\pm\infty} \frac{x^2}{x+3} = \lim_{x\to\pm\infty} \frac{x^{\cancel{2}}}{\cancel{x}\cdot\left(1+\frac{3}{x}\right)} = \lim_{x\to\pm\infty} \frac{\overset{\to\pm\infty}{x}}{1+\underset{\to 0}{\frac{3}{x}}} = \pm\infty$$

$$\lim_{x\to -3^\pm} \frac{\overset{\to 9}{x^2}}{\underset{\to 0^\pm}{x+3}} = \pm\infty$$

$sg(x+3)$: $-$ 0 $+$; -3, x



Dudas de teórico sobre la inversa de una función



$f: f(x) = x^2$

$f'(x) = 2x$

$\text{Sg}\, f'(x)$ $\quad - \; 0 \; +$ $\quad 0 \quad x$

Si $f(2) = 4$, $f^{-1}(4) = 2$

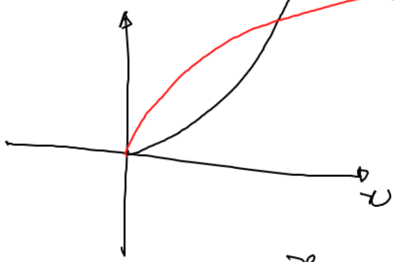
Si $f(x) = y$, entonces $\underline{f^{-1}(y) = x}$

$f(x) = x^2$, $f: [0, +\infty) \to [0, +\infty)$

$x^2 = y \to x = \sqrt{y}$

$f^{-1}(x) = \sqrt{x}$ si $f(x) = x^2$

$f : [0,+\infty) \to [0,+\infty) \,/\, f(x) = x^2$



$f^{-1} : [0,+\infty) \to [0,+\infty) \,/\, f^{-1}(x) = \sqrt{x}$

$$\begin{tikzcd} x \arrow[r, "f"] \arrow[rr, bend right] & x^2 \arrow[r, "f^{-1}"] & x \end{tikzcd}$$

$f^{-1} \circ f = id$

$f^{-1}(f(x)) = x$

$$f(f^{-1}(x)) = x$$

$$f'(f^{-1}(x)) \cdot {f^{-1}}'(x) = 1$$

$${f^{-1}}'(x) = \frac{1}{f'(f^{-1}(x))}$$