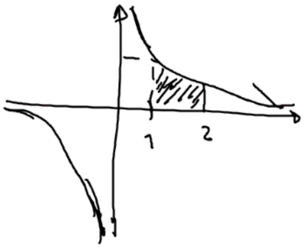
Clase de consulta – 29 de junio

d) $y = \frac{1}{x}$ entre $x=1$ y $x=2$

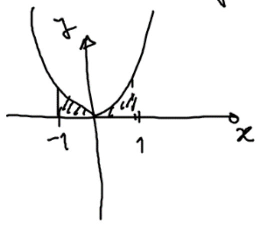


$$\lim_{x\to\pm\infty} \frac{1}{x} = 0^{\pm}$$

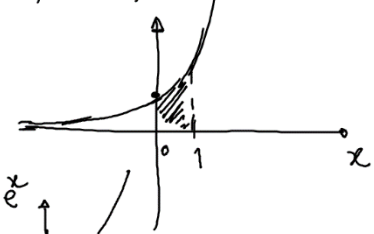
$$A = \int_1^2 \frac{1}{x}\,dx = \log(x)\Big|_1^2 = \log(2) - \overbrace{\log(1)}^{0} = \log(2)$$

e) $f(x) = x^4$ entre $x=-1$ y $x=1$

$$A = \int_{-1}^{1} x^4\,dx = \frac{x^5}{5}\Big|_{-1}^{1}$$

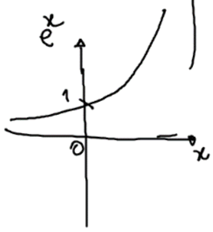


f) $f(x) = e^{2x}$ entre 0 y 1



$$f'(x) = 2e^{2x}$$

$$A = \int_0^1 e^{2x}\,dx = \frac{1}{2}e^{2x}\Big|_0^1 = \frac{1}{2}\cdot e^2 - \frac{1}{2}\cdot e^0 = \frac{e^2-1}{2}$$



8)

a) arcsen(x)

$$\int f' \cdot g = f \cdot g - \int f \cdot g'$$

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

$$\cancel{\int \underbrace{x}_{f'} \cdot \underbrace{e^x}_{g} \, dx = \frac{x^2}{2} \cdot e^x - \underbrace{\int \frac{x^2}{2} \cdot e^x \, dx}}$$

$$\cancel{f(x) = \frac{x^2}{2} \qquad g'(x) = e^x}$$

$$\int \underbrace{x}_{g} \cdot \underbrace{e^x}_{f'} \, dx = e^x \cdot x - \int e^x \cdot 1 \, dx =$$

$$g'(x) = 1 \qquad f(x) = e^x \qquad = e^x \cdot x - \underbrace{\int e^x \, dx} =$$

$$= e^x \cdot x - e^x \Rightarrow F(x) = \underline{e^x \cdot x - e^x}$$

Las primitivas son de la forma $F(x) = e^x \cdot x - e^x + C$, $C \in \mathbb{R}$

<u>Verificación</u>: $F'(x) = e^x \cdot x + \cancel{e^x \cdot 1} - \cancel{e^x} = e^x \cdot x$

$f(x) = \arcsin(x)$

$\left\{ \int f' \cdot g = f \cdot g - \int f \cdot g' \right.$

$\int \arcsin(x)\,dx = \int \underset{f'}{1} \cdot \underset{g}{\arcsin(x)}\,dx =$

$(4 = 4.1)$

$f(x) = x \qquad g'(x) = \dfrac{1}{\sqrt{1-x^2}}$

$= 1 \cdot \arcsin(x) + \int (-2x)\,\dfrac{1}{2\sqrt{1-x^2}}\,dx =$

$= \arcsin(x) + \sqrt{1-x^2}$

$\cancel{\left(\arcsin(x) \cdot \frac{x^2}{2}\right)'} =$

$\left(\sqrt{1-x^2}\right)' = \dfrac{1}{2\sqrt{1-x^2}} \cdot (-2x)$

d) $x^2 \cdot e^x$

$\cancel{\int \underset{f'}{x^2} \cdot \underset{g}{e^x}\,dx = \frac{x^3}{3} \cdot e^x - \underbrace{\int \frac{x^3}{3} \cdot e^x\,dx}}$

$\cancel{f(x) = \frac{x^3}{3} \qquad g'(x) = e^x}$

$\left\{ \begin{array}{l} (k \cdot f(x))' = k \cdot f'(x) \\ \int f' \cdot g = f \cdot g - \int f \cdot g' \end{array} \right.$

$\int \underset{g}{x^2} \cdot \underset{f'}{e^x}\,dx = e^x \cdot x^2 - \underbrace{\int e^x \cdot 2x\,dx} =$

$g'(x) = 2x \qquad f(x) = e^x$

$\int e^x \cdot 2x\,dx = 2 \cdot \int \underset{f'}{e^x} \cdot \underset{g}{x}\,dx = 2 \cdot \left(e^x \cdot x - \int e^x \cdot 1\,dx\right) = 2\left(e^x \cdot x - e^x\right)$

$g \to g'(x) = 1$

$$\int \overbrace{x^2 e^x}^{h(x)} dx = e^x \cdot x^2 - \int e^x \cdot 2x\,dx = e^x \cdot x^2 - 2\left(e^x \cdot x - e^x\right) =$$

$$\int e^x \cdot 2x\,dx = 2\left(e^x \cdot x - e^x\right)$$

$$= e^x \cdot x^2 - 2xe^x + 2e^x =$$

$$= e^x \cdot \left(x^2 - 2x + 2\right)$$

$$H(x) = e^x \cdot \left(x^2 - 2x + 2\right)$$

$$H'(x) = e^x \cdot (2x-2) + e^x \cdot \left(x^2 - 2x + 2\right) = e^x \cdot \left(\cancel{2x} \cancel{-2} + x^2 \cancel{-2x} \cancel{+2}\right) =$$

$$= e^x \cdot x^2$$

e) $\int \overbrace{x \cdot \text{sen}(x)}^{h(x)}\, dx = -\cos(x) \cdot x - \int -\cos(x) \cdot 1\, dx =$

$g \to g'(x) = 1$ $\quad f' \to f(x) = -\cos(x)$

$$\int f' \cdot g = f \cdot g - \int f \cdot g'$$

$$= -\cos(x) \cdot x + \int \cos(x)\, dx =$$

$$= -\cos(x) \cdot x + \text{sen}(x)$$

$$H(x) = \underline{-\cos(x) \cdot x} + \underline{\text{sen}(x)}$$

$$H'(x) = \text{sen}(x) \cdot x - \cancel{\cos(x)} + \cancel{\cos(x)} = x \cdot \text{sen}(x)$$

Si $\overbrace{f'(x)}^{g(x)} = x + 2$

¿cuál puede ser f?

$\underbrace{f(x)}_{G(x)} = \frac{x^2}{2} + 2x$

$$\frac{x^2}{\sqrt{1+x^3}} = \frac{2}{3} \cdot \frac{3x^2}{2\sqrt{1+x^3}} \qquad \int \frac{x^2}{\sqrt{1+x^3}}\,dx = \frac{2}{3} \cdot \sqrt{1+x^3}$$

$$\sqrt{u} \longrightarrow \frac{1}{2\sqrt{u}} \cdot u'$$

$$u = 1+x^3 \longrightarrow u'(x) = 3x^2$$

$$\int_1^2 \frac{x^2}{\sqrt{1+x^3}}\,dx = \frac{2}{3}\sqrt{u}\,\Big|_{1+1^3}^{1+2^3}$$