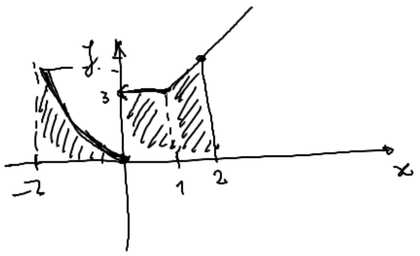
Clase Rocha 26 de junio



$$f: f(x) = \begin{cases} x^2 & \text{si } x \in [-2,0] \\ 3 & \text{si } x \in (0,1] \\ x+2 & \text{si } x \in (1,2] \end{cases}$$

Hallar el área bajo la curva de f

$$\int_{-2}^{0} x^2\,dx + \int_{0}^{1} 3\,dx + \int_{1}^{2} (x+2)\,dx =$$

$$= \frac{x^3}{3}\Big|_{-2}^{0} + 3x\Big|_{0}^{1} + \left(\frac{x^2}{2} + 2x\right)\Big|_{1}^{2} = \frac{0^3}{3} - \frac{(-2)^3}{3} + 3.1 - 3.0 +$$

$$+ \frac{2^2}{2} + 2.2 - \left(\frac{1^2}{2} + 2.1\right) = \frac{8}{3} + 3 + 2 + 4 - \frac{1}{2} - 2 = 7 + \frac{8}{3} - \frac{1}{2}$$

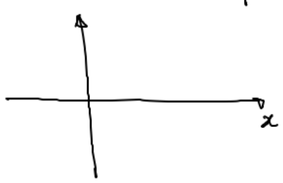
b) a) $\displaystyle\int_{1}^{2} x^5\,dx = \frac{x^6}{6}\Big|_{1}^{2} = \frac{2^6}{6} - \frac{1^6}{6} =$

$$= \frac{64}{6} - \frac{1}{6} = \frac{63}{6}$$

b) $\displaystyle\int_{-1}^{1} x^{\frac{1}{3}}\,dx = \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1}\Bigg|_{-1}^{1} = \frac{x^{4/3}}{4/3}\Bigg|_{-1}^{1}$

$f(x) = \cancel{x}^2$

$F(x) = \dfrac{x^3}{3}$



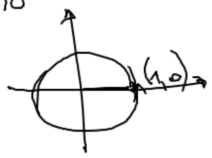
c) $\int_{-\pi}^{\pi} \text{sen}(2x)\,dx$

$f(x) = \text{sen}(2x) \longrightarrow F(x) = -\frac{1}{2}\cos(2x)$

$\Downarrow$

$F'(x) = -\frac{1}{\cancel{2}}\cdot\left(-\text{sen}(2x)\cdot\cancel{2}\right)$

$\int_{-\pi}^{\pi} \text{sen}(2x)\,dx = -\frac{1}{2}\cos(2x)\Big|_{-\pi}^{\pi} = -\frac{1}{2}\cdot\cos(2\pi)+\frac{1}{2}\cos(-2\pi) =$



$= -\frac{1}{2}+\frac{1}{2} = 0$

d) $\int_{-1}^{1} \text{senh}(2x)\,dx = \int_{-1}^{1} \frac{e^{2x}-e^{-2x}}{2}\,dx$

$\text{senh}(2x) = \frac{e^{2x}-e^{-2x}}{2}$ $\Big\} = \frac{1}{2}\cdot\left(\int_{-1}^{1} e^{2x}\,dx - \int_{-1}^{1} e^{-2x}\,dx\right)$

$(\text{senh}(x))' = \cosh(x)$ $\qquad f(x) = \text{senh}(2x)$

$(\cosh(x))' = \text{senh}(x) \longrightarrow F(x) = \frac{1}{2}\cdot\cosh(2x)$

$\Rightarrow \int_{-1}^{1} \text{senh}(x)\,dx = \frac{1}{2}\cosh(2x)\Big|_{-1}^{1}$ (terminar)

$$\int_a^x \underbrace{f'(u)\cdot u'} = \underbrace{f(u)}\Big|_{u(a)}^{u(x)}$$

$$x \xrightarrow{u} u(x) \xrightarrow{f} f(u(x))$$

$f(u) \xrightarrow{\text{derivada}} f'(u)\cdot u'$; primitiva

$\log(u) \longrightarrow \frac{1}{u}\cdot u'$

$e^u \longrightarrow e^u\cdot u'$

$$f(x) = e^{\overbrace{x^2+1}^{=u(x)}} \underbrace{\cdot 2x}_{u'(x)} \Rightarrow F(x) = e^{x^2+1}$$

$$\int_1^2 \underbrace{e^{x^2+1}\cdot 2x}\, dx = e^{x^2+1}\Big|_{1^2+1}^{2^2+1} =$$

b) $f(x) = \frac{\log(x)}{x}$

$$x \xrightarrow{u} u(x) \xrightarrow{g} g(u(x))$$

$g\circ u$

c) $x^2\cdot(1+x^3)$

d) $\frac{2x+1}{x^2+x+1}$

e) $\mathrm{sen}(x)\cdot\cos(x)$

$(g(u))' = \underline{g'(u)\cdot u'}$

b) $\frac{\log(x)}{x} = \overbrace{\log(x)}^{u(x)}\cdot \frac{1}{x}$ $u'(x)$

$x \xrightarrow{u} \log(x) \xrightarrow{g} \cdot \log(x)$

$g(y) = y$

Si fuera $f(x) = x$, entonces $F(x) = \frac{x^2}{2}$

$u(x) = \log(x) \longrightarrow F(x) = \frac{(\log(x))^2}{2}$

$x \xrightarrow{u} \overbrace{\log(x)}^{u(x)} \longrightarrow u(x) = \log(x)$

derivo

$F'(x) = \frac{1}{\cancel{2}} \cdot \cancel{2} \cdot \log(x) \cdot \frac{1}{x}$

c) $x^2\cdot(1+x^3) = \frac{1}{3}\cdot\overbrace{\underbrace{3x^2}_{u'(x)}}^{u'}\cdot\overbrace{\underbrace{(1+x^3)}_{u(x)}}^{u} \longrightarrow F(x)=\frac{1}{3}\cdot\frac{u^2}{2}$

$u(x)=1+x^3$

$u'(x)=3x^2$

$x \xrightarrow{u(x)} 1+x^3 \xrightarrow{g} 1+x^3$

$g(u(x))=u$

$g\circ u$

f	f'
$\log(u)$	$\frac{1}{u}\cdot u' = \frac{u'}{u}$
e^u	$e^u\cdot u'$
u	u'
u^2	$2u\cdot u'$

f	f'
$\frac{u^2}{2}$	$u\cdot u'$

$F(x)=\frac{1}{3}\,\frac{(1+x^3)^2}{2}$

d) $f(x)=\frac{2x+1}{x^2+x+1} = (2x+1)\cdot\frac{1}{x^2+x+1}$

$u(x)=x^2+x+1$

$u'(x)=2x+1$

$\frac{u'}{u} \longrightarrow \log(u)$

$F(x)=\log(x^2+x+1)$

e) $\operatorname{sen}(x)\cdot\cos(x)$

Método de integración por partes

$$\int (f' \cdot g) = f \cdot g - \int f \cdot g'$$

Ejemplo: Hallar una primitiva de: $h(x) = x \cdot e^x$

Si consideramos $f'(x) = e^x$, $g(x) = x$,

tenemos que $f(x) = e^x$ y $g'(x) = 1$.

Entonces, de la fórmula anterior resulta:

$$\int e^x . x \, dx = e^x . x - \int e^x . 1 \, dx = e^x . x - e^x$$

Es decir: $H(x) = e^x . x - e^x = e^x . (x - 1)$

Pregunta: ¿Era lo mismo considerar $f'(x) = x$ y $g(x) = e^x$?

Ejercicio 8 a)

$h(x) = \operatorname{arc\,sen}(x)$

En este caso consideramos $\operatorname{arcsen}(x) = 1 . \operatorname{arcsen}(x)$

y elegimos $f'(x) = 1$ y $g(x) = \operatorname{arcsen}(x)$

Entonces: $f(x) = x$ y $g'(x) = \frac{1}{\sqrt{1 - x^2}}$

Luego: $\int 1 . \operatorname{arcsen}(x) \, dx = x . \operatorname{arcsen}(x) - \int x . \frac{1}{\sqrt{1 - x^2}} \, dx$

Tenemos que hallar una primitiva de $x \cdot \frac{1}{\sqrt{1-x^2}}$

$$x \cdot \frac{1}{\sqrt{1-x^2}} = -\frac{1}{2}(-2x)\frac{1}{\sqrt{1-x^2}} = -1 \cdot (-2x) \cdot \frac{1}{2\sqrt{1-x^2}}$$

La función $-2x \cdot \frac{1}{2\sqrt{1-x^2}}$ es la derivada de $\sqrt{1-x^2}$

Recapitulando: $\int 1 \cdot \text{arc.sen}(x)\,dx = x \cdot \text{arcsen}(x) - \int x \cdot \frac{1}{\sqrt{1-x^2}}\,dx =$

$= x \cdot \text{arcsen}(x) + \sqrt{1-x^2}$

e) $\text{sen}(x) \cdot \underbrace{\cos(x)}_{u(x)} = -\underbrace{\left(-\text{sen}(x)\right)}_{u'(x)} \cdot \underbrace{\cos(x)}_{u(x)}$

$$F(x) = -\frac{\cos^2(x)}{2}$$

$\underbrace{\text{sen}(x)}_{u(x)} \cdot \underbrace{\cos(x)}_{u'(x)}$

$$F(x) = \frac{\text{sen}^2(x)}{2}$$

Traten de hacer algunas partes del ejercicio 8.