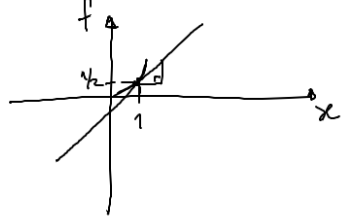
Clase Rocha lunes 8 de mayo

Práctico 2

4) f y g derivables, $f(1)=\frac{1}{2}$, $f'(1)=1$, $g'(1)=2$

$h: h(x) = g\left(2x f(x)\right)$ $\boxed{h \text{ derivable y hallar } h'(1)}$



$$h(x) = g\left(2x . f(x)\right)$$

$$\begin{tikzcd} x \arrow[r, "f"] \arrow[rrr, bend right, "h(x)"'] & f(x) \arrow[r] & \underbrace{2xf(x)}_{y} \arrow[r] & g(y) \end{tikzcd}$$

$g \circ f$ $\quad f: A \to \mathbb{R}$; $g: B \to \mathbb{R}$

$$f(x) \in B \quad \forall x \in A$$

$$\begin{tikzcd} x \arrow[r, "f"] \arrow[d] & f(x) \arrow[r, "g"] \arrow[d] & g(f(x)) \\ A & B & \end{tikzcd}$$

$$h(x) = g\left(2xf(x)\right) \Rightarrow h(1) = g\left(\underbrace{2.1.f(1)}\right) = g(1)$$

$$2.1.f(1) = 2.\frac{1}{2} = 1$$

f es deriv. en $\underline{1}$ $\Rightarrow j: j(x) = \underbrace{2.x}_{\text{der.}}.\underbrace{f(x)}_{\text{der.}}$ es derivable en 1

y g es derivable en $j(1)=1$ (porque $g'(1)=2$) $\Rightarrow h$ es derivable en 1.

$h(x) = g(2x \cdot f(x))$

$h'(x) = g'(\underbrace{2x \cdot f(x)}) \cdot (\underbrace{2x \cdot f(x)}_{j(x)})' \longrightarrow j'(x) = 2x \cdot f'(x) + 2 \cdot f(x)$

$j'(1) = 2 \cdot 1 \cdot f'(1) + 2 \cdot f(1) =$

$= 2 \cdot 1 + 2 \cdot \frac{1}{2} =$

$= 2 + 1 = 3$

$h'(1) = g'(\underbrace{2 \cdot 1 \cdot f(1)}) \cdot 3$

$h'(1) = g'(1) \cdot 3 = 2 \cdot 3 = 6$

5) $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$

$\text{senh}(x) = \dfrac{e^x - e^{-x}}{2}$

$\tanh(x) = \dfrac{\text{senh}(x)}{\cosh(x)}$

$(e^x)' = e^x$

$(\ln(x))' = (L(x))' = \frac{1}{x}$

$e^{-x} = \frac{1}{e^x}$

$\left(e^x - \frac{1}{e^x}\right)' =$

a) $(\text{senh}\, x)' = \cosh(x)$

$\text{Senh}(x) = \dfrac{e^x - e^{-x}}{2} = \underbrace{\frac{1}{2}} \cdot \underbrace{(e^x - e^{-x})}$

Si queremos calcular $\left(e^x - \frac{1}{e^x}\right)'$,

tenemos:

$$\left(e^x - \frac{1}{e^x}\right)' = e^x - \left(\frac{0 \cdot e^x - 1 \cdot e^x}{e^{2x}}\right) =$$

$$= e^{x} + \frac{e^{x}}{e^{x}\cdot e^{x}} = e^{x} + \frac{1}{e^{x}} = e^{x} + e^{-x}$$

$$(k\cdot f(x))' = \underbrace{k'}_{=0}\cdot f(x) + k\cdot f'(x) = k\cdot f'(x)$$

$k \in \mathbb{R}$

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e^{-x}

f	f'
e^{x}	e^{x}
$e^{f(x)}$	$e^{f(x)}\cdot f'(x)$

$$(\text{senh}(x))' = \left(\frac{e^{x} - e^{-x}}{2}\right)' = \frac{1}{2}\cdot\left(e^{x} - e^{-x}\right)' =$$

$$= \frac{1}{2}\cdot\left(e^{x} - e^{-x}\cdot(-1)\right) = \frac{1}{2}\cdot\left(e^{x} + e^{-x}\right) = \frac{e^{x}+e^{-x}}{2} = \cosh(x)$$

$$(\cosh x)' = \text{senh}(x)$$

$$\tanh(x) = \frac{\text{senh}(x)}{\cosh(x)}$$

$$(\text{senh}(x))' = \cosh(x)$$

$$(\cosh(x))' = \text{senh}(x)$$

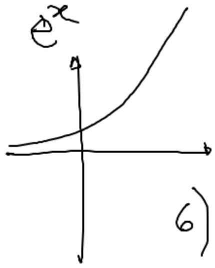
$$(\tanh(x))' = 1 - \tanh^2(x)$$
$$= \frac{1}{\cosh^2(x)}$$

$$(\tanh(x))' = \left(\frac{\text{senh}(x)}{\cosh(x)}\right)' = \frac{\cosh(x)\cdot\cosh(x) - \text{senh}(x)\cdot\text{senh}(x)}{\cosh^2(x)} =$$

$$= \frac{\cosh^2(x) - \text{senh}^2(x)}{\cosh^2(x)} = \frac{\cosh^2(x)}{\cosh^2(x)} - \underbrace{\frac{\text{senh}^2(x)}{\cosh^2(x)}}_{=1-\tanh^2(x)} = 1 - \left(\frac{\text{senh}(x)}{\cosh(x)}\right)^2 =$$

$$\operatorname{senh}(x) = \frac{e^x - e^{-x}}{2} \qquad \operatorname{Dom}(\operatorname{senh}(x)) = \mathbb{R}$$

e^x



$$L(x) = \log_e x \underset{not.}{=} \log(x)$$

6) i) $f(x) = \underbrace{e^{\operatorname{sen}(3x)}}$

$f'(x) = e^{\operatorname{sen}(3x)} \cdot \cos(3x) \cdot 3$

$$\left\{ \begin{array}{l} (e^x)' = e^x \\ (\operatorname{sen}(x))' = \cos x \\ (3x)' = 3 \end{array} \right.$$

e^x	
$e^{f(x)}$	$e^{f(x)} \cdot f'(x)$

ii) $f(x) = \operatorname{sen}(e^x + \operatorname{sen} x)$

$f'(x) = \cos(e^x + \operatorname{sen} x) \cdot (e^x + \cos x)$

iii) $f(x) = \operatorname{sen}(\underbrace{e^{x+2}}_{u})$

$f'(x) = \cos(e^{x+2}) \cdot e^{x+2} \cdot 1$

f	f'
$\operatorname{sen}(x)$	$\cos(x)$
$\operatorname{sen}(f(x))$	$\cos(f(x)) \cdot f'(x)$
$\operatorname{sen} u$	$\cos(u) \cdot u'$
$\operatorname{sen} u$	$\cos(u) \cdot u'$
$e^u = e^{x+2}$	$e^u \cdot u'$

v) $f(x) = e^{x} \cdot \underbrace{\cos(3x+5)}$

$$\cos(u) \mid -\operatorname{sen}(u) \cdot u'$$

$$\left(\cos(3x+5)\right)' = -\operatorname{sen}(3x+5) \cdot 3$$

$a \cdot b - a \cdot c = a \cdot (b-c)$

$$f'(x) = e^{x} \cdot \cos(3x+5) + e^{x} \cdot \left(-\operatorname{sen}(3x+5) \cdot 3\right)$$
$$= e^{x} \cdot \cos(3x+5) - e^{x} \cdot 3\operatorname{sen}(3x+5) =$$
$$= e^{x} \cdot \left[\cos(3x+5) - 3\operatorname{sen}(3x+5)\right]$$

vii) $f(x) = e^{e^{x}}$

$$f'(x) = e^{e^{x}} \cdot e^{x} = e^{e^{x}+x}$$

$$e^{u} \mid e^{u} \cdot u' \qquad u = e^{x}$$

viii) $f(x) = \tan(e^{x})$

(Hacerla)

$$\tan(u) \mid \frac{1}{\cos^2(u)} \cdot u' \qquad (1+\tan^2(u)) \cdot u'$$

b) $f(x) = \log(\overbrace{\operatorname{sen}(x)}^{u})$

$$f'(x) = \frac{1}{\operatorname{sen}(x)} \cdot \cos(x) = \frac{\cos(x)}{\operatorname{sen}(x)} = \frac{1}{\tan(x)}$$

$$\log(x) \mid \frac{1}{x}$$

$$\log(u) \mid \frac{1}{u} \cdot u' = \frac{u'}{u}$$

Repaso práctico 1

Hallar la ecuación de la recta tangente a la curva $y = x^2 - 3x + 2$ en $(3, f(3))$

$f : \mathbb{R} \to \mathbb{R} \qquad x \to f(x)$

$f' : A \to \mathbb{R} \qquad x \to f'(x)$

en $(3, f(3))$

$y = x^2 - 3x + 2$

$f'(x) = 2x - 3$

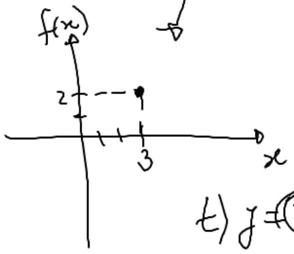
$f(u) = u^2 - 3u + 2$

$f(x) = x^2 - 3x + 2$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} =$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 3.(x+h) + 2 - (x^2 - 3x + 2)}{h}$$

(Terminar)



el valor de $f'(3)$

$f(3) = 3^2 - 3.3 + 2 = 2$

t) $y = m x + n$

$m = f'(3) = 2.3 - 3 = 3 \to$ t) $y = 3x + n$

$2 = 3.3 + n$

$2 - 9 = n \to n = -7$

t) $y = 3x - 7$