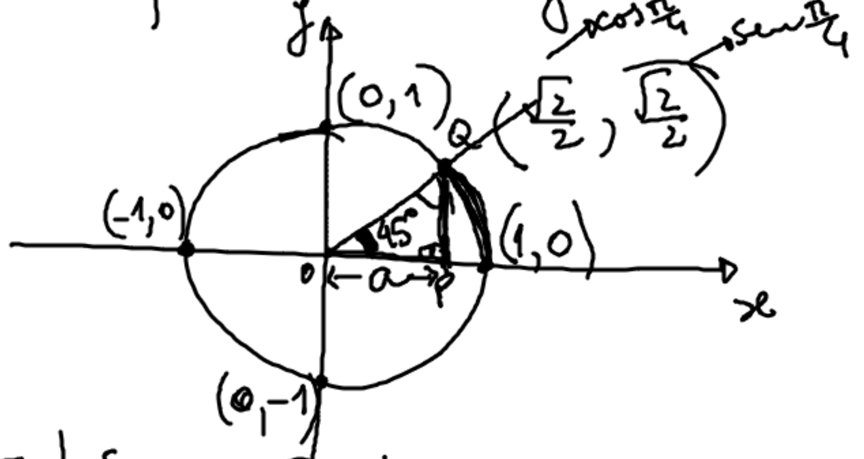
Funciones trigonométricas

$$\left[\begin{array}{l} x+x=2x \\ 1x+1x=(1+1)x \end{array}\right]$$

Circunferencia trigonométrica



$$a^2 + a^2 = 1^2$$

$$a^2 + a^2 = 1$$

$$2a^2 = 1$$

$$a^2 = \frac{1}{2}$$

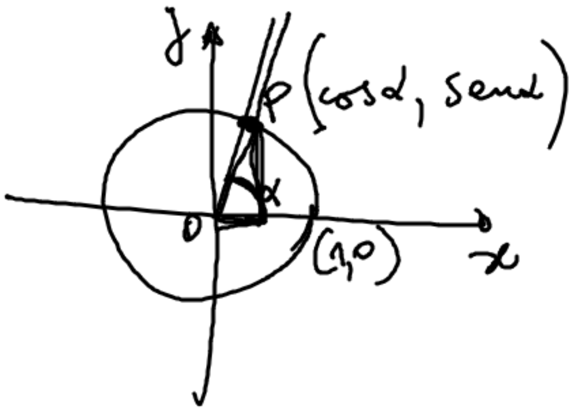
$$a = \sqrt{\frac{1}{2}}$$

$$a = \sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} =$$

$$= \frac{1 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2}}{2}$$

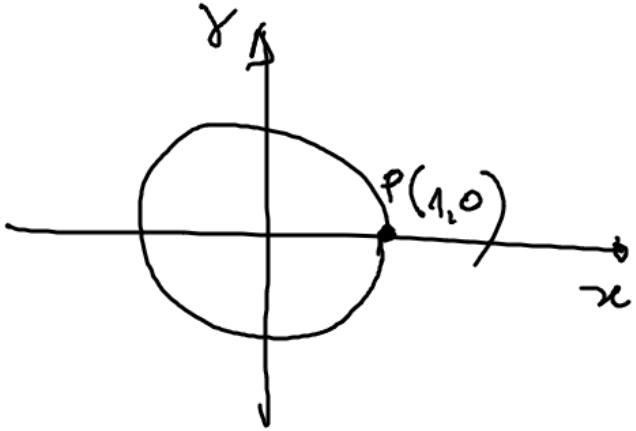
Grados | Radianes

$45° \longrightarrow \frac{\pi}{4}$ rad



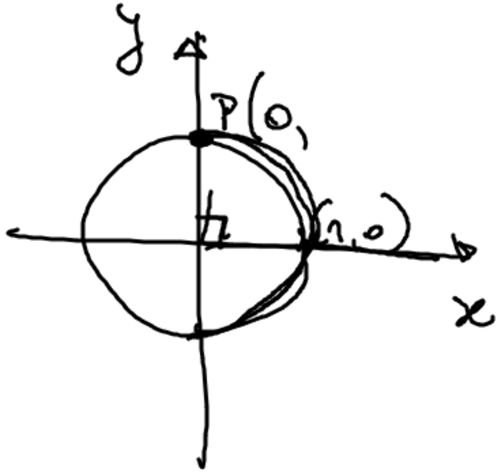
$\text{sen}(\alpha) = \text{ordenada de } P = y_P$

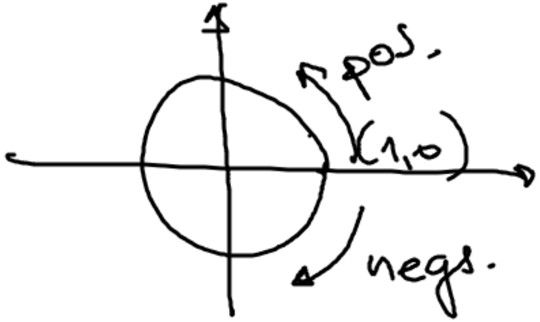
$\cos(\alpha) = \text{abscisa de } P = x_P$

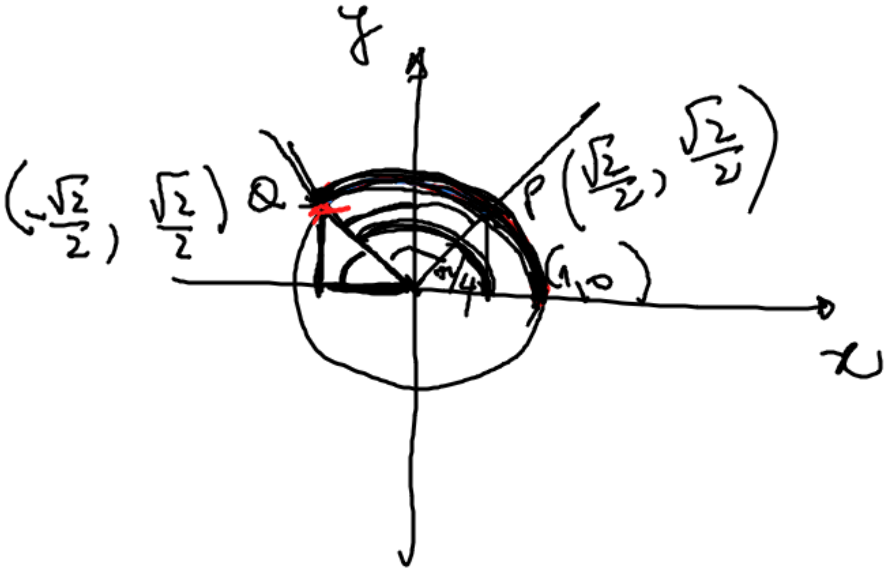


$\text{sen}\, 0 = 0$

$\cos 0 = 1$







$$\frac{\pi}{4}+\frac{\pi}{2}=\frac{\pi}{4}+\frac{2\pi}{4}=\frac{3\pi}{4}$$

$$\frac{3\pi}{4}$$

$$\cos\left(\frac{3\pi}{4}\right)=-\frac{\sqrt{2}}{2}$$

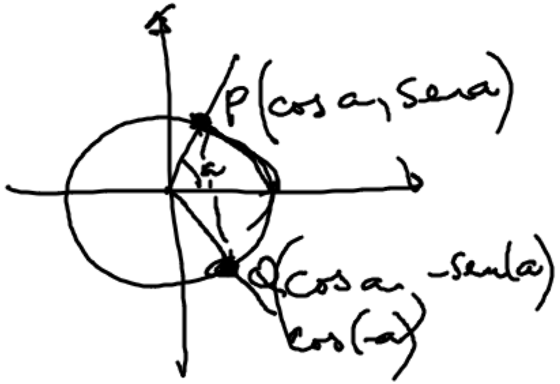
$$\text{sen}\left(\frac{3\pi}{4}\right)=\frac{\sqrt{2}}{2}$$

$$\operatorname{sen}(a) = \operatorname{sen}(a + 2\pi)$$

$$\cos(a) = \cos(a + 2\pi)$$

$$\operatorname{sen}(-a) = -\operatorname{sen}(a)$$

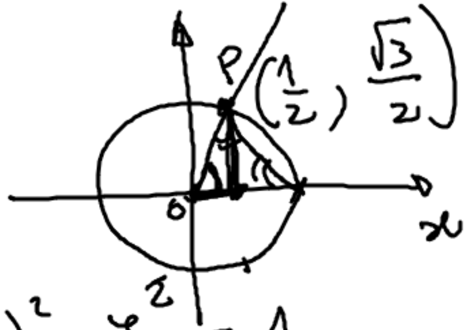
$$\cos(-a) = \cos(a)$$



$$\cos(a+b) = \cos(a).\cos(b) - \operatorname{sen}(a).\operatorname{sen}(b)$$

$$\operatorname{sen}(a+b) = \cos(a).\operatorname{sen}(b) + \operatorname{sen}(a).\cos(b)$$

α	sen α	cos α	tan α
$\frac{2\pi}{3}$			



$$\left(\frac{1}{2}\right)^2 + y_P^2 = 1$$

$$\frac{1}{4} + y_P^2 = 1$$

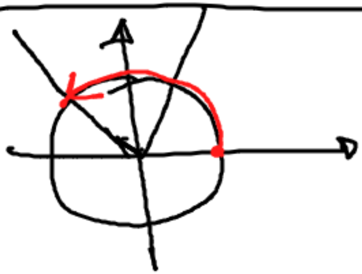
$$y_P^2 = 1 - \frac{1}{4}$$

$$y_P^2 = \frac{3}{4}$$

$$y_P = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$$

$$\text{sen}\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$



$$\frac{2\pi}{3} = \frac{\pi}{3} + \frac{\pi}{3}$$

$$\text{sen}\left(\frac{2\pi}{3}\right) = \text{sen}\left(\frac{\pi}{3} + \frac{\pi}{3}\right)$$

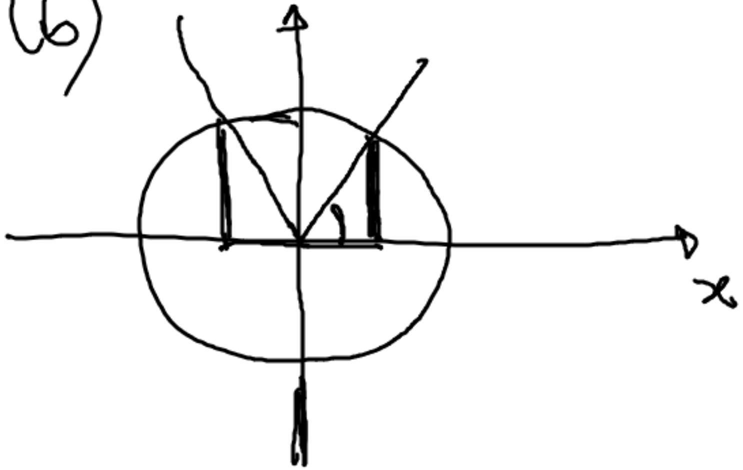
Sabemos que: $\text{sen}\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$ y $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$$\text{sen}\left(\frac{2\pi}{3}\right) = \text{sen}\left(\underbrace{\frac{\pi}{3}}_{a} + \underbrace{\frac{\pi}{3}}_{b}\right) = \cos\left(\frac{\pi}{3}\right)\cdot\text{sen}\left(\frac{\pi}{3}\right) + \text{sen}\left(\frac{\pi}{3}\right)\cdot\cos\left(\frac{\pi}{3}\right)$$

$$= \frac{1}{2}\cdot\frac{\sqrt{3}}{2} + \frac{1}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

$$\text{sen}(a+b) = \cos(a)\cdot\text{sen}(b) + \text{sen}(a)\cdot\cos(b)$$

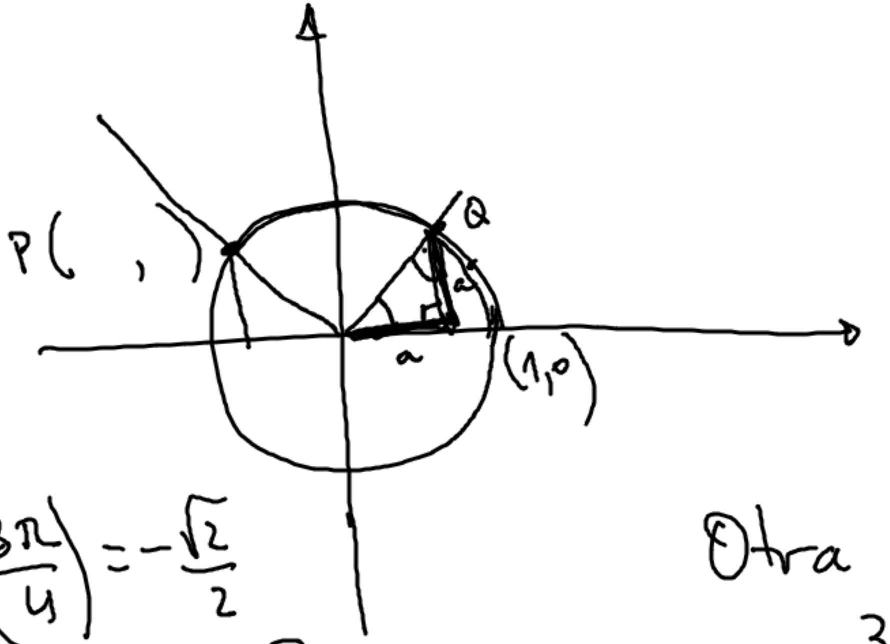
$$\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$$



$$\tan(x) = \frac{\operatorname{sen}(x)}{\cos(x)}$$

$$\tan\left(\frac{2\pi}{3}\right) = \frac{\operatorname{sen}\left(\frac{2\pi}{3}\right)}{\cos\left(\frac{2\pi}{3}\right)}$$

Calcular para $\frac{3\pi}{4}$ y $\frac{5\pi}{6}$



$\cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\mathrm{sen}\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\alpha = \frac{3\pi}{4}$

$a^2 + a^2 = 1$

$2a^2 = 1$

$\longrightarrow a = \frac{\sqrt{2}}{2}$

Otra forma:

$\frac{3\pi}{4} = \frac{\pi}{4} + \frac{\pi}{2}$

$\mathrm{sen}\left(\frac{3\pi}{4}\right) = \mathrm{sen}\left(\frac{\pi}{4} + \frac{\pi}{2}\right)$

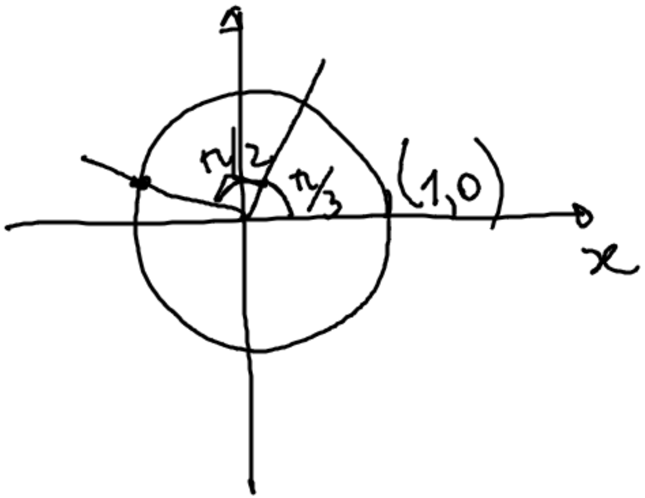
$\mathrm{sen}(a+b)$

$\frac{\cancel{\sqrt{2}}}{\cancel{2}} \cdot \frac{(-\cancel{2})}{\cancel{\sqrt{2}}} = -1$

$\tan\left(\frac{3\pi}{4}\right) =$

$= \frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = -1$

$\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{\sqrt{4}} = \sqrt{\frac{2}{4}} = \sqrt{\frac{1}{2}}$

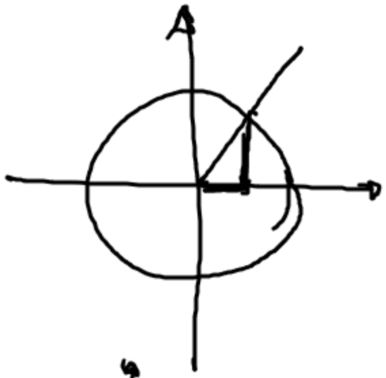


$$\alpha = \frac{5\pi}{6}$$

$$\frac{5\pi}{6} = \frac{\pi}{3} + \frac{\pi}{2}$$

$$\text{sen}\left(\frac{5\pi}{6}\right) = \text{sen}\left(\frac{\pi}{3} + \frac{\pi}{2}\right)$$

$$\text{sen}(a+b)$$



$$\cos(a) = \text{sen}\left(a + \frac{\pi}{2}\right)$$

$$\text{sen}(a) = \cos\left(a + \frac{\pi}{2}\right)$$

$$\text{sen}\left(\frac{\pi}{3} + \frac{\pi}{2}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

$$\cos\left(\frac{\pi}{3} + \frac{\pi}{2}\right) = \text{sen}\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

2) (Práctico 2)

a) $f(x) = \dfrac{1}{\operatorname{sen} x + \cos x}$

$$f'(x) = \frac{\overbrace{1'}^{=0} \cdot (\operatorname{sen} x + \cos x) - 1 \cdot (\operatorname{sen} x + \cos x)'}{(\operatorname{sen} x + \cos x)^2} =$$

$$= \frac{0 - (\cos x - \operatorname{sen} x)}{(\operatorname{sen} x + \cos x)^2} = \frac{-\cos x + \operatorname{sen} x}{(\operatorname{sen} x + \cos x)^2}$$

b) $f(x) = \operatorname{sen}(x^2+5x)$

$$x \xrightarrow{h} \underbrace{x^2+5x}_{y} \xrightarrow{j} \operatorname{sen}(y)$$

$h(x) = x^2 + 5x$

y

$j(x) = \operatorname{sen}(x)$

f es la composición de:

$(j \circ h)(x) = j\big(h(x)\big)$

$(j \circ h)'(x) = j'\big(h(x)\big) \cdot h'(x)$

$$f(x) = \operatorname{sen}\left(x^2 + 5x\right)$$

$$f'(x) = \cos\left(x^2 + 5x\right) \cdot \left(2x + 5\right)$$
$$= \left(2x + 5\right) \cdot \cos\left(x^2 + 5x\right)$$

---

c) $$f(x) = \frac{\operatorname{sen}(2x)}{x}$$

$$f'(x) = \frac{[\operatorname{sen}(2x)]' \cdot x - \operatorname{sen}(2x) \cdot x'}{x^2}$$

$$\left(\operatorname{sen}(2x)\right)' = \cos(2x) \cdot 2 = 2 \cdot \cos(2x)$$

$$x' = 1$$

$$\rightarrow f'(x) = \frac{2 \cdot \cos(2x) \cdot x - \operatorname{sen}(2x) \cdot 1}{x^2} =$$

$$f'(x) = \frac{2x\cos(2x) - \operatorname{sen}(2x)}{x^2}$$

2g)

3b)

3d)

$$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

$$= \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$$

2 g) $f(x) = \dfrac{x+1}{\operatorname{sen}(2x)}$

$$f'(x) = \frac{(x+1)' \cdot \operatorname{sen}(2x) - (x+1) \cdot \left(\operatorname{sen}(2x)\right)'}{\left(\operatorname{sen}(2x)\right)^2} =$$

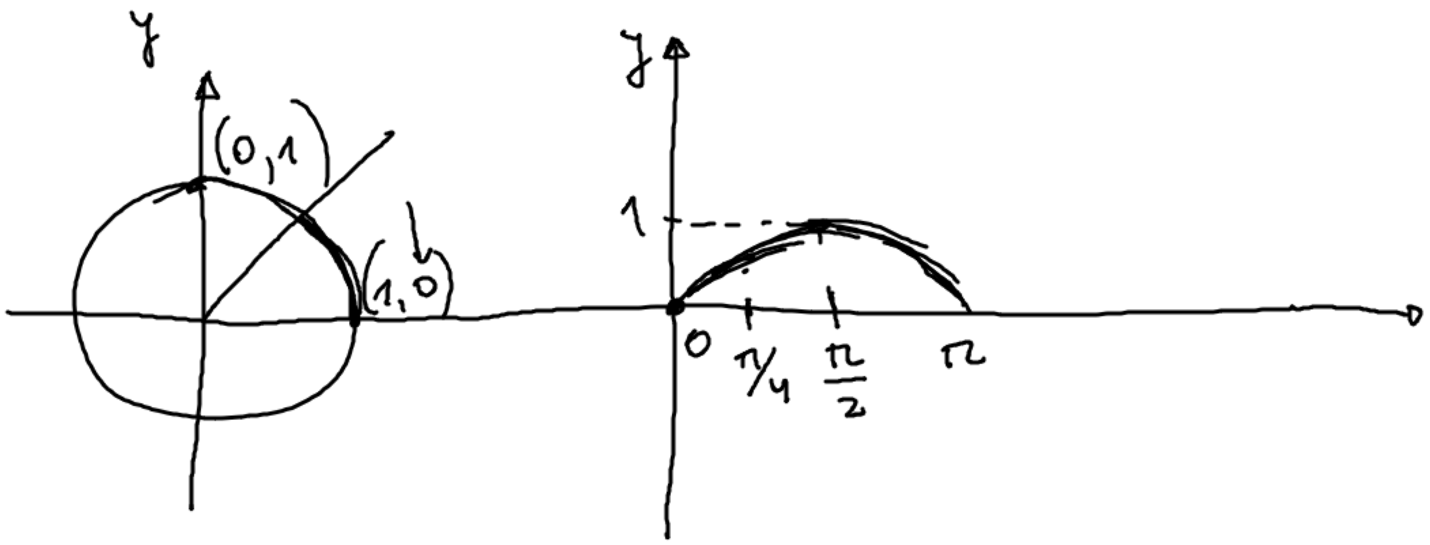
$$= \frac{1 \cdot \operatorname{sen}(2x) - (x+1) \cdot 2 \cdot \cos(2x)}{\operatorname{sen}^2(2x)}$$

3b) $f(x) = \cos(x^3+1)$

$f'(x) = -\operatorname{sen}(x^3+1) \cdot 3x^2$

3d) $f(x) = \operatorname{sen}(\cos x)$

$f'(x) = \cos(\cos x) \cdot (-\operatorname{sen} x)$



Sen (x)