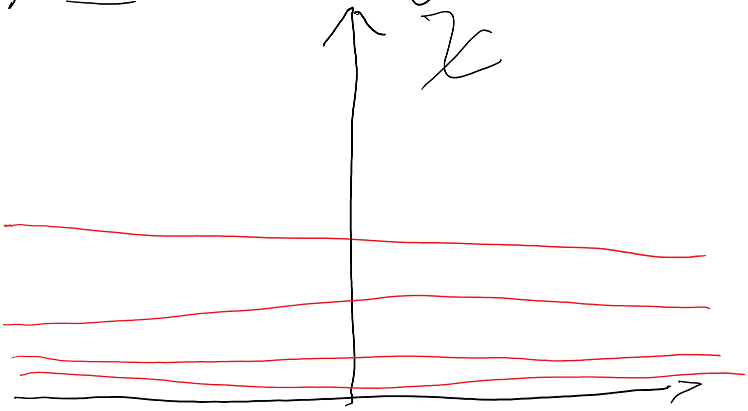
$$\int_X (\liminf f_n) \le \liminf \int_X f_n$$



$X = \mathbb{R}$

$f_n(x) = \frac{1}{n}$ $\qquad f \equiv 0$

$$\int_{\mathbb{R}} (\liminf f_n)\, d\mu = \int_{\mathbb{R}} 0 \, d\mu = 0$$

$$\liminf \int_{\mathbb{R}} f_n \, d\mu = \liminf +\infty = +\infty$$

Si f y g son simples

$$f = \sum_{i=0}^{n} a_i \chi_{E_i}, \quad g = \sum_{i=0}^{m} b_i \chi_{M_i}$$

$$0 \le f \le g$$

$A_{ji} = E_j \cap M_i$ medible

$B_j = M_j \cap \left(\bigcup_{i=0}^{n} E_i \right)^c$ medible

$\int_X f \, d\mu = \sum \int_{A_{ij}} f \, d\mu + \sum \int_{B_i} f \, d\mu$

$\leq \sum \int_{A_{ij}} g \, d\mu + \sum \int_{B_i} g \, d\mu =$

$\int_X g \, d\mu$

f, g son funciones int. cualquieras.

$\int_X f \, d\mu = \sup \left\{ \int_X s \, d\mu ; s \text{ simple y } s \leq f \right\}$

s simple y $\underbrace{s \leq f \leq g}$

$\int s \, d\mu \leq \int f \, d\mu$

$$\int_X s \, d\mu \leq \int_X f \, d\mu$$

s_n simples y $s_n \leq f$ tal que

$$\int_X s_n \, d\mu \longrightarrow \int_X f \, d\mu$$

$$s_n \leq f \leq g$$

s_n simples y $s_n \leq g$

1) $\exists$ s_0 simple y $s_0 \leq g$

tal que $\int_X s_0 \, d\mu > \lim \int_X s_n \, d\mu$

$$\int_X g \, d\mu > \int_X f \, d\mu$$

2) Que no pase eso.

$$\int s_n \, d\mu \longrightarrow \int f \, d\mu$$

$$\int_X g\,d\mu = \int_X f\,d\mu$$

$\forall$ S simple: $s \leq g$ entonces

$$\int_X s\,d\mu \leq \int_X f\,d\mu$$

$A \subset B$ medibles y $f \geq 0$

ver que

$$\int_A f\,d\mu \leq \int_B f\,d\mu$$

$$\cancel{\int_B f\,d\mu = \int_A f\,d\mu + \int_{A^c \cap B} f\,d\mu}$$

S simple en B no neg.

$$s = \sum^{n} a_i \chi_{I_i} \quad I_i \cap B \ldots$$

$$s = \sum_{i=0} a_i \chi_{E_i}, \quad E_i \subset B \text{ medible}$$

$$\int_B s \, d\mu = \sum_{i=0}^{n} a_i \mu(E_i)$$

$$\int_A s \, d\mu = \sum_{i=0}^{n} a_i \mu(A \cap E_i)$$

$f \geq 0$, E medible queremos ver que

$$\int_E f \, d\mu = \int_X \chi_E f \, d\mu$$

$$\int_X \chi_E f \, d\mu = \sup\left\{ \int s \, d\mu ; \ s \leq \chi_E f \right\}$$

Si f es simple en X y

Si φ es simple en X y $r \leq f$, entonces $\chi_E r$ es simple y $\chi_E r \leq \chi_E f$.

$\cancel{\mathcal{A}}$ s simple

$$\int_E s\, d\mu = \int_X \chi_E s\, d\mu$$

$$\sup\left\{\int_X \chi_E r\, d\mu : r \leq f \right\} \leq \int_X \chi_E f\, d\mu$$

r simple

r_n simples tal que $r_n \leq f$ y

$$\int_X r_n\, d\mu \longrightarrow \int_X f\, d\mu$$

$$\int_X \chi_E \varphi_n \, d\mu = \int_E \varphi_n \, d\mu$$

$$¿\int_E \varphi_n \, d\mu \longrightarrow \int_E f \, d\mu ?$$

$\varphi_n|_E$ simple / $\varphi_n|_E \leq f|_E$

$$\int_E \varphi_n \leq c < \int_E f \, d\mu$$

$$\chi_E f \, d\mu = \sup \left\{ \int_E s \, d\mu : \begin{array}{c} s \text{ simple y} \\ s \leq f \end{array} \right\}$$

Sea s_0 en $\{ s$ simple en E y $s \leq f \}$

entonces $\exists$ s' en $\{ s$ simple en X y $s \leq f \}$

(en particular podemos tomar $s_0 = s'$)

tal que $s_0 = \chi_E s'$.

$\sup\{\int_X \chi_E s\,d\mu : s \text{ simple q } s \leq f\} =$
$\int_X \chi_E f\,d\mu$

$\chi_E s : s$ simple q $s \leq f$

$r : r$ simple q $r \leq \chi_E f$

$\chi_E s$: simple q $\chi_E s \leq \chi_E f$

$r = s$, r es simple q $r \leq f$

$\chi_E r = r$

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s simple

$\int_E s\,d\mu = \sum_{i=0}^{n} a_i \mu(E \cap E_i) \overset{\text{pr? } E\cdot\mu}{=}$

$\int \chi_E s\,d\mu$

$$\int_X \chi_E \, s \, d\mu$$