Solución entregable 3 (2021)

Ej 1 Gradiente: $\boxed{\nabla F = (f_x, f_y)}$

$F(x,y) = (x+1)e^{y} + (y-4)^2 e^{x}$

$\frac{\partial F}{\partial x} = F_x = e^{y} + (y-4)^2 e^{x}$

$\frac{\partial F}{\partial y} = F_y = (x+1)e^{y} + e^{x} 2(y-4) = (x+1)e^{y} + 2e^{x}(y-4)$

$$\left[\nabla F = \left(e^{y} + (y-4)^2 e^{x},\ (x+1)e^{y} + 2e^{x}(y-4)\right)\right]$$

Ej 2

$f(x,y) = x^4 + y^4$

a)

$\nabla F = (f_x, f_y) \rightarrow f_x = 4x^3 \rightarrow \nabla F = (4x^3, 4y^3)$

$f_y = 4y^3$

b)

Puntos críticos $\nabla F = (0,0)$

$\Rightarrow f_x = 4x^3 = 0 \rightarrow x = 0$ punto crít $= (0,0)$

$f_y = 4x^3 = 0 \rightarrow y = 0$

c) Criterio de Hessiana $H = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix}$

$f_{xx} = 12x^2 \quad f_{yy} = 12y^2$

$f_{xy} = 0$ todos continuas $\Rightarrow f \in C^2 \Rightarrow f_{xy} = f_{yx}$

entonces $F_{yx} = 0$

$$H = \begin{pmatrix} 12(0)^2 & 0 \\ 0 & 12(0)^2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$|H| = 0$ es el criterio no concluye pero por ser $f(x,y) = x^4 + y^4$ una función que siempre es positiva y su valor mínimo es 0, alcanzado para $y = x = 0$ [podemos ver $x^4 + y^4$ es siempre positivo entonces su mínimo es 0]

Podemos concluir que el punto crítico es un mínimo.

Ej 3, 4, 5, 6 ver en apunte de Varinia

Ej 7: $x(t) = \text{Sen}(t)\,\mu(t)$

$$\int_{-\infty}^{\infty} \text{Sen}(t)\,\mu(t)\, e^{-st}\, dt \qquad \text{Sen}(t) = \frac{e^{it} - e^{-it}}{2i}$$

$$\int_{0}^{\infty} \frac{e^{+it} - e^{-it}}{2i}\, e^{-st}\, dt$$

por $\mu(t)$

$$\frac{1}{2i}\int_0^\infty e^{it}e^{-st}\,dt - \frac{1}{2i}\int_0^\infty e^{-it}e^{-st}\,dt$$

$$\frac{1}{2i}\left[\int_0^\infty e^{t(i-s)}\,dt - \int_0^\infty e^{-t(i+s)}\,dt\right]$$

$$\frac{1}{2i}\left[\frac{1}{i-s}[0-1] - -\frac{1}{i+s}[0-1]\right]$$

$$\downarrow \qquad\qquad \downarrow$$

ROC:

$$e^{-t(i+\sigma+i\omega)} =$$

$$\underbrace{e^{-t(\sigma)}}_{\sigma>0}\,\underbrace{e^{-ti(1+\omega)}}_{|1|=1}$$

ROC:

$$e^{t(i-\sigma-i\omega)} = \underbrace{e^{-t\sigma}}_{\sigma>0}\,e^{ti(1-\omega)}$$

$$\left[\begin{array}{c} \Rightarrow \text{ROC } \sigma>0 \\ \mathcal{L}(x(t)) = \frac{1}{2i}\left[\frac{-1}{i-s} + \frac{1}{i+s}\right] \end{array}\right]$$

Usando propiedad

$$\mathcal{L}(-t\,x(t)) = \frac{d}{ds}\mathcal{L}(x(t))$$

$$\Rightarrow \mathcal{L}(-t\,\text{sen}(t)\,u(t)) = \frac{1}{2i}\left[-\frac{1}{(i-s)^2} + \frac{1}{(i+s)^2}\right]$$

Ej 8

$$X(s) = \frac{s}{(s+1)(s-2)} \qquad Re\{s\} > 2.$$

$$\Rightarrow \frac{s}{(s+1)(s-2)} = \frac{1}{3(s+1)} + \frac{2}{3(s-2)}.$$

$$\mathcal{L}^{-1}\left\{\frac{1}{3(s+1)}\right\} = \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}$$

- $e^{-t}u(t)$ $\quad Re\{s\} > -1$
- $-e^{-t}u(-t)$ $\quad Re(s) < -1$

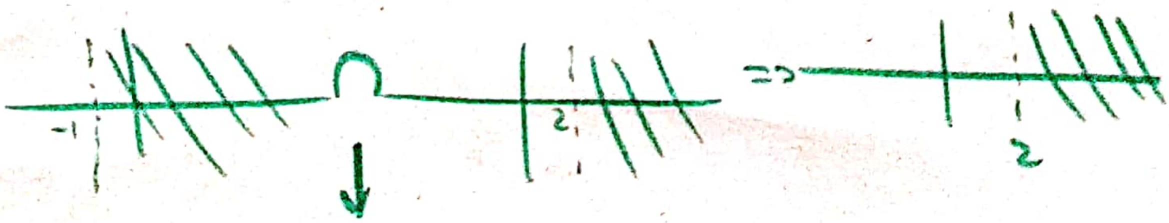
$$\mathcal{L}^{-1}\left\{\frac{2}{3}\,\frac{1}{s-2}\right\} = \frac{2}{3}\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\}$$

- $e^{2t}u(t)$ $\quad Re(s) > 2$
- $-e^{2t}u(-t)$ $\quad Re(s) < 2$

Como la ROC $Re\{s\} > 2$:

$$\left[\underbrace{\frac{1}{3}e^{-t}u(t)}_{Re\{s\} > -1} + \underbrace{\frac{2}{3}e^{2t}u(t)}_{Re\{s\}}\right] = x(t)$$

$Re(s) > 2$



intersección