(1)

Entregable:

1 a) $b = i+2$

binomial $= i+2$

polar $= r e^{i\varphi}$ $\quad r = \sqrt{a^2+b^2}$

$\varphi = \text{Arctan}\left(\frac{b}{a}\right)$

$r = \sqrt{1^2+2^2} = \sqrt{5}$

$\varphi = \text{Arctan}\left(\frac{1}{2}\right) \Rightarrow \sqrt{5}\, e^{\text{Arctan}\left(\frac{1}{2}\right)}$

$\Rightarrow \left[b^3 = (\sqrt{5})^3\, e^{3\arctan(1/2)} \right]$ (polar)

$\left[\text{binomial} : (\sqrt{5})^3 \left[\cos(3\,\text{Arctan}(1/2)) + i\,\text{sen}(3\,\text{Arctan}(1/2))\right] \right]$

b) i^{125}

potencias de $i \rightarrow i^a = i^r$ donde r es el resto de dividir a entre 4

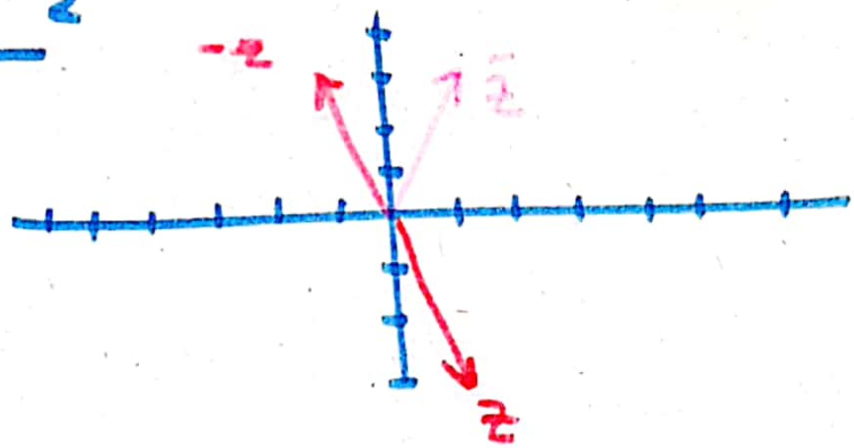
$\Rightarrow i^{125} = i^1 = i$

$$\begin{array}{r|l} 125 & 4 \\ \hline 05 & 31 \\ 1 & \end{array}$$

$\Rightarrow i^{1000} = i^0 = 1$

$$\begin{array}{r|l} 1000 & 4 \\ \hline 20 & 250 \\ 00 & \\ 0 & \end{array}$$

Ej 2



→ el opuesto está sobre la misma línea pero distinta dirección (ref según el origen)

→ el conjugado está reflejado resp. al eje x.

Ej 3

$a = 2e^{i\pi} \rightarrow a = 2\underbrace{\cos(\pi)} + 2i\,\text{sen}(\pi)$

$\boxed{a = -2}$

$b = 2e^{i\pi/4} \rightarrow b = 2\cos(\pi/2) + 2i\,\text{sen}(\pi/4)$

$2\,\frac{\sqrt{2}}{2} + 2i\,\frac{\sqrt{2}}{2}$

$\boxed{b = 1 + \sqrt{2}\,i}$

~~Ej 4~~

- $a+b = -2 + 1 + \sqrt{2}\,i = \underbrace{-1 + \sqrt{2}\,i}$
- $a.b = 2e^{i\pi} . 2e^{i\pi/4} = 4e^{5/4\pi}$
- $a/b = 2e^{i\pi} / 2e^{i\pi/4} = 1e^{i3/4\pi}$
- $\frac{a}{(a+b)} = \frac{-2}{-1+\sqrt{2}i} \cdot \frac{-1-\sqrt{2}i}{-1-\sqrt{2}i} = \frac{2+2\sqrt{2}\,i}{3} = \underbrace{\frac{2}{3} + \frac{2}{3}\sqrt{2}i}$

Para multip y divisiones es conveniente la forma polar ②
Para suma y restas la binomial

Ej 4:

$(2-i)(3+bi) = 6+2bi-3i+b$

$= \underbrace{6+b}_{Re} + \underbrace{(2b-3)}_{im} i$

$\Rightarrow$ a) imaginario puro $\Rightarrow Re = 0$

$6+b=0$

$\boxed{b=-6}$

b) real $\Rightarrow im = 0$

$2b-3=0$

$2b=3$

$\boxed{b=3/2}$

Ej 5:

$\frac{2x+1}{x^3-x}$ descompongo el denom

$x^3-x = x(x^2-1)$

$x=0$

$x^2-1=0$

$x^2=1$

$x=\pm 1$

$\Rightarrow$ raíces $x=0$

$x=1$

$x=-1$

Escaneado con CamScanner

$$\Rightarrow \frac{2x+1}{x^3-x} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x}$$

multip por x y $x=0$

$$\frac{2x+1}{x^2-1} = \frac{A\,x}{x-1} + \frac{B\,x}{x+1} + C \Big|_{x=0}$$

$$\boxed{-1 = C}$$

multip $x-1$ y $x=1$

$$\frac{2x+1\,\cancel{(x-1)}}{x(x+1)\cancel{(x-1)}} = A + \frac{B\,(x-1)}{x+1} + \frac{C}{x}(x-1)$$

en $x=1$

$$\boxed{\frac{3}{2} = A}$$

multip $x+1$

$$\frac{(2x+1)\ \cancel{(x+1)}}{x\,\cancel{(x+1)}\,(x-1)} = \frac{A}{x-1}(x+1) + B + \frac{C}{x}(x+1)$$

$x=-1$

$$\boxed{\frac{-1}{2} = B} \Rightarrow \frac{2x+1}{x^3-x} = \frac{3}{2(x-1)} - \frac{1}{2(x+1)} - \frac{1}{x}$$

Ej6) $\dfrac{4}{(x+2)(x^2+9)}$

raíces $\cdot\, x = -2$

$x^2+9=0$ $\quad \cdot\, x = -3i$

$x^2 = -9$ $\quad \cdot\, x = +3i$

$x = \pm 3i$

$$\frac{4}{(x+2)(x^2+9)} = \frac{A}{x+2} + \frac{Mx+N}{x^2+9}$$

Multip $x+2$:

$$\frac{4}{x^2+9} = A + \left(\frac{Mx+N}{x^2+9}\right)(x+2)$$

en $x=-2$

$$\left[\frac{4}{13} = A\right]$$

multip por x^2+9

$$\frac{4}{x+2} = \frac{A}{x+2}(x^2+9) + Mx+N$$

en $x = 3i$

$$\frac{4}{3i+2} = 3Mi + N \Rightarrow \frac{4}{3i+2}\cdot\frac{-3i+2}{-3i+2} = \frac{8-12i}{13}$$

$$\Rightarrow \frac{8}{13} - \frac{12}{13}i = 3Mi + N$$

$$\boxed{N = \frac{8}{13}}$$

$3M = -\frac{12}{13}$

$\boxed{M = -\frac{4}{13}}$

Escaneado con CamScanner

$$\frac{4}{(x+2)(x^2+9)} = \frac{4}{13(x+2)} + \frac{-4x+8}{13(x^2+9)}$$

Ej 7

$$\frac{5}{(x-3)^2(x^2+2x+5)}$$

$x=3$ raiz dble

$x^2+2x+5=$

$$\frac{-2 \pm \sqrt{4-4.1.5}}{2.1} \quad \begin{matrix} \nearrow -1-2i \\ \searrow -1+2i \end{matrix}$$

$$\frac{5}{(x-3)^2(x^2+2x+5)} = \frac{A}{x-3} + \frac{B}{(x-3)^2} + \frac{Mx+N}{(x+1)^2+4}$$

mult por $(x-3)^2$ y reemp $x=3$

$$\frac{5}{9+6+5} = B$$

$$\frac{5}{20} = \boxed{\frac{1}{4} = B}$$

Para sacar A mult por x y $x \to \infty$

$$\frac{5x}{(x-3)^2(x^2+2x+5)} = \frac{Ax}{x-3} + \frac{Bx}{(x-3)^2} + \frac{Mx^2+N}{(x+1)^2+4}$$

$\lim\limits_{x \to \infty} \to \quad 0 = A + 0 + M \to \underline{A = -M}$

para M y N mult $(x+1)^2+4$ y $x=-1-2i$

$$\frac{5}{(x-3)^2} = Mx+N \Rightarrow \frac{5}{(-1-2i-3)^2} = -M-2Mi+N$$

$\Rightarrow M = 1/10 \quad N = 1/4 \quad A = -1/10 \quad B = 1/4$