Práctico 4

2015

1c) $f(x) = \text{Sen}(x)$

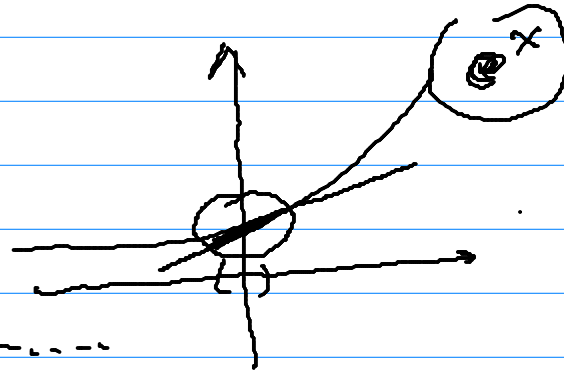
$$\Rightarrow P_{2n+1,0}(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\Rightarrow P_{2n+1,\varphi}(x) \Rightarrow ?$$

$$\left[P_{n,a}(x) = \frac{f(a)}{0!} + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!} + \frac{f^{(n)}(a)(x-a)^n}{n!} \right]$$

$f(x) = e^x$

$$\Rightarrow \underbrace{P_{n,0}(x) = 1 + 1x}_{} + \frac{1 \cdot x^2}{2!} + \dots$$



$f(x) = \text{Sen}(x)$ $\quad f''(x) = -\text{Sen}(x)$

$f' = \cos(x)$

$$P_{2n+1,a}(x) = \overbrace{\text{Sen}(a)}^{=0} + \cos(a)(x-a) + \frac{-\overbrace{\text{Sen}(a)}^{=0}(x-a)^2}{2!}$$

en $a = 0$

En ej 3; desarrolle los 3 primeros terminos

mc Laurin

$$f(x) = \frac{1}{1-x}$$

$$f'(x) = \frac{-1}{(1-x)^2} \; ; \; f''(x) = \frac{-1\,\left[(1-x)^2\right]'}{(1-x)^4}$$

$$\left((1-x)^2\right)' = (2x-1)\cdot 1$$

$$= 2(1-x). \qquad \frac{2\,\cancel{(1-x)}}{(1-x)^{\cancel{4}\,3}}$$

$$f''(x) = \frac{2}{(1-x)^3} \; ; \; f'''(x) = \frac{-2\left[(1-x)^3\right]'}{(1-x)^6}$$

$$f'''(x) = \frac{-2\cdot 3\,\cancel{(1-x)}^{2}\cdot -1}{(1-x)^{\cancel{6}\,4}} = \frac{2\cdot 3}{(1-x)^4}$$

$$f^{iv}(x) = \frac{2\cdot 3\cdot 4}{(1-x)^5} \qquad f^{n}(x) = \frac{n!}{(1-x)^{n+1}}$$

$x=0$ $\qquad f^{n}(x=0) = n!$

$$f^{iv}(x=0) = 4!$$

$P_{n,0}(x)$ de $f(x) = \frac{1}{1-x}$

es $$P_{n,0}(x) = 1 + \frac{\cancel{1!}}{\cancel{1!}}(x-0)^1 + \frac{\cancel{2!}}{\cancel{2!}}(x-0)^2 + \dots + \frac{\cancel{4!}}{\cancel{4!}}(x-0)^4 + \dots + \frac{\cancel{n!}}{\cancel{n!}}(x-0)^n$$

a) $f(x) = \frac{1}{1-x} \Rightarrow P_n(x) = 1 + x + x^2 + x^3 + \dots + x^n$

b) $f(x) = L(1+x) \Rightarrow P_n(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^{n-1}\frac{x^n}{n}$

$f'(0), f''(0), f'''(0), \dots f^{(n)}(0)$

3c $\lim_{x\to 0} \frac{x - L(1+x)}{x^2} \Rightarrow 0/0$ indet

Uso el desarrollo de $L(1+x)$

$P_n(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots (-1)^{n}\frac{x^n}{n}$

Usar $P_2(x)$

$\lim_{x\to 0} \frac{\cancel{x} - (\cancel{x} - x^2/2)}{x^2} = \lim_{x\to 0} \frac{\cancel{x^2}}{2\cancel{x^2}} = 1/2$

5a Aproximar e con error $< 10^{-5}$

Usando vean teo resto Lagrange.

Lo ideal

$e = 2{,}7182818 28$

$f(x) = e^x \xrightarrow{x=1} P_n(x) = \overbrace{1 + x}^{=2} + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} + R_n(x)$

$= 2{,}5$