Práctico 8

22/7

(5)

b) $a_n = 1 + \frac{(-1)^n}{n}$, $n \geq 1$ $n \in \mathbb{N}$

Estudiar monótona, acotación y convergencia

$a_{n+1} \leq a_n$ (monotona decreciente)

$a_{n+1} \geq a_n$ (monotona creciente)

$\Rightarrow a_{n+1} = 1 + \frac{(-1)^{n+1}}{n+1} \leq 1 + \frac{(-1)^{n+1}}{n+1}$

$= 1 + \frac{(-1)^n}{n} \cdot (-1) = \frac{1 - (-1)^n}{n}$

* Si $n = \dot{2}$ $\Rightarrow a_{n+1} \leq a_n$

* Si $n \neq \dot{2}$ $\Rightarrow a_{n+1} \geq a_n$

$\Rightarrow a_n$ no es monótona

Acotación: $\lim_n a_n = \lim 1 + \underbrace{\frac{(-1)^n}{n}}_{\to 0} = 1$

Convergencia: Dado limite finito $\Rightarrow$ Converge

7) $a_1 = 3 \qquad a_{n+1} = \dfrac{3(1+a_n)}{3+a_n}$

$\forall n \in \mathbb{N}, n \geq 1$

$$a_2 = \frac{3(1+a_1)}{3+a_1}$$

a) Demostrar $a_n \geq \sqrt{3} \quad \forall n \geq 1$

PI) $a_1 = 3 \geq \sqrt{3}$ ✓

HI) $a_n \geq \sqrt{3}$

TI) $a_{n+1} \geq \sqrt{3}$

$$a_{n+1} = \frac{3(1+a_n)}{3+a_n} \geq \sqrt{3} \Rightarrow 3 + 3a_n \geq 3\sqrt{3} + \sqrt{3}\,a_n$$

$$3a_n - \sqrt{3}\,a_n \geq 3\sqrt{3} - 3 \Rightarrow a_n(3-\sqrt{3}) \geq 3\sqrt{3} - 3$$

$$\Rightarrow a_n \geq \frac{3\sqrt{3}-3}{3-\sqrt{3}} = \frac{\sqrt{3}\,\cancel{(3-\sqrt{3})}}{\cancel{3-\sqrt{3}}} = \sqrt{3}$$

$\boxed{a_n \geq \sqrt{3}}$

b) $a_{n+1} \geq a_n \qquad a_{n+1} = \dfrac{3(1+a_n)}{3+a_n}$

$$\frac{3(1+a_n)}{3+a_n} \geq a_n \Rightarrow 3(1+a_n) \geq a_n^2 + 3a_n$$

$$3 + \cancel{3a_n} \geq a_n^2 + \cancel{3a_n} \Rightarrow 3 \geq a_n^2$$

$$\Rightarrow a_n \not\leq \sqrt{3} \quad ?$$

$a_{n+1} \leq a_n$ monotona decreciante

$$a_{n+1} = \frac{3(1+a_n)}{3+a_n}$$

$a_1 = 3$

$a_2 = \overset{?}{=} = 2$

$$a_2 = \frac{3(1+3)}{3+3} = \frac{12}{6} = 2$$

c) $a_n \geq \sqrt{3}$ y a_n monotona decreciente

$\Rightarrow$ el limite es $\sqrt{3}$

Practica 9

Decir termino general de la serie

2 d) $1 - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \frac{1}{9^2} - \frac{1}{11^2} \dots$

impar $2k+1$ con $k \in \mathbb{Z}$

$(-1)^k$ con $k \in \mathbb{Z}$

$\Rightarrow \sum_{k \geq 0} \frac{(-1)^k}{(2k+1)^2}$

b) $\overbrace{1/2 + 1/2}^{} + \overbrace{3/8 + 4/16 + 5/32}^{} \dots$

$1/2 + 2/4 + 3/8 + 4/16 + 5/32$

$\sum_{n \geq 1}^{+\infty} \frac{n}{2^n}$ $\qquad$ $\sum_{m \geq 0}^{+\infty} \frac{m+1}{2^{m+1}}$

6

Clasificar la serie, en caso convergente hallar su suma.

a) $\sum_{n\geq 0} \frac{2^n+3^n}{6^n}$

$$\sum_{n\geq 0} \frac{2^n}{6^n} + \sum_{n\geq 0} \frac{3^n}{6^n}$$

$$\sum_{n\geq 0} \frac{2^n}{(2\cdot 3)^n} + \sum_{n\geq 0} \frac{3^n}{(2\cdot 3)^n} = \sum_{n\geq 0} \frac{2^n}{2^n \cdot 3^n} + \sum_{n\geq 0} \frac{3^n}{2^n \cdot 3^n}$$

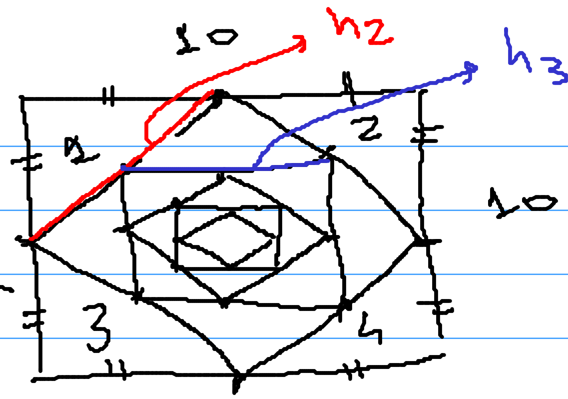
$$= \sum_{n\geq 0} \frac{1}{3^n} + \sum_{n\geq 0} \frac{1}{2^n} = \sum_{n\geq 0} (1/3)^n + \sum_{n\geq 0} (1/2)^n$$

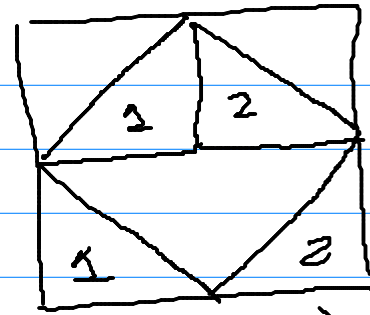
Serie geométrica $\sum_{n\geq 0} ar^n \Rightarrow \frac{a}{1-r}$

$|r| < 1$

$$= \frac{1}{1-1/3} + \frac{1}{1-1/2} = 3/2 + 2 = 7/2$$

8





Calcular la suma de las áreas de los cuadrados de la figura sabiendo que la dimensión del cuadrado de mayor área es 10x10.

1

$$h_2^2 = 2(5)^2 \Rightarrow h_1 = 5\sqrt{2}$$

$$h_3^2 = 2\left(\frac{5\cdot\sqrt{2}}{2}\right)^2 \Rightarrow h_2 = \frac{\sqrt{2}\cdot 5\cdot\sqrt{2}}{2^2}$$

2

$$\sum_{n\geq 0}\left(\frac{5\cdot\sqrt{2}^n}{2^{n-1}}\right)^2 = \sum_{n\geq 0}\frac{25\cdot 2^n}{(2^{n-1})^2}$$

$$\sum_{n\geq 0}\frac{25\cdot 2^n}{\underbrace{2^{2n-2}}_{2^{2n}\cdot 2^{-2}}} = \sum_{n\geq 0}\frac{100\cdot \cancel{2^n}}{(2^n)^2} = \boxed{\sum_{n\geq 0}\frac{100}{2^n}}$$

$$2^{2n} = (2^n)^2$$

$$\sum_{n\geq 0}\frac{5^2\cdot 2^2}{2^n} = \boxed{\sum_{n\geq 0}\frac{5^2}{2^{n-2}}}$$

$100 = 5^2\cdot 2^2$ (teo. fundamental aritmética)

para saber el área total $n=0,\ldots,5$

$$\sum_{n=0}^{5}\frac{5^2}{2^{n-2}}$$