Práctico 6,1

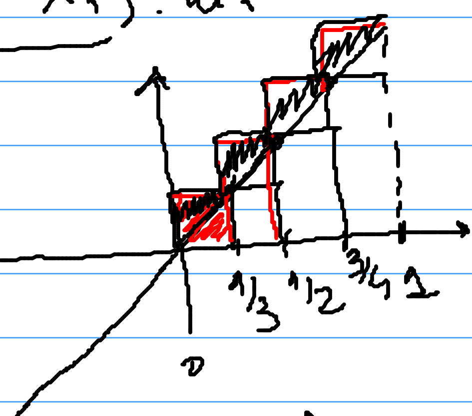
1 Sea $f:[0,1]\to\mathbb{R}$ dado $f(x)=x$

a) $\underline{S}(f,P)$; $\overline{S}(f,P)$ con $P=\{0,1/3,1/2,3/4,1\}$ en $[0,1]$

b) Sea $P=\{0=x_0, x_1 \dots x_{n-1}, x_n=1\}$ de $[0,1]$

$$\underline{S}(f,P) < 1/2 < \overline{S}(f,P)$$

a) $\underline{S}(f,P) = \sum_{i=0}^{i=n-1} \underbrace{(x_{i+1}-x_i)}\, m_i$



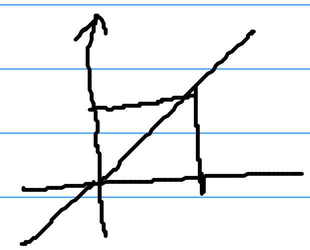
$a = (x_1 - x_0)\, m_0 = 0$

$b = (x_2 - x_1)\, m_1 = (1/2 - 1/3)\, f(1/3)$

$c = (x_3 - x_2)\, m_2 = (3/4 - 1/2)\, f(1/2)$

$d = (x_4 - x_3)\, m_3 = (1 - 3/4)\, f(3/4)$

$\underline{S}(f,P) = a+b+c+d$



$\overline{S}(f,P) = \sum_{i=0}^{i=n-1} (x_{i+1}-x_i)\, M_i$

$A = (x_1 - x_0)\, M_0 = (1/3 - 0) \cdot f(1/3)$

$B = (x_2 - x_1)\, M_1 = (1/2 - 1/3) \cdot f(1/2)$

$C = (x_3 - x_2)\, M_2 = (3/4 - 1/2)\, f(3/4)$

$D = (x_4 - x_3)\, M_3 = (1 - 3/4)\, f(1)$

$\overline{S}(f,P) = A+B+C+D$

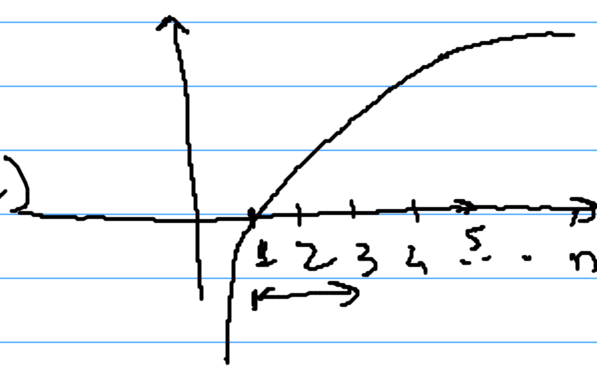
3) $f(x) = Lx \quad , \quad x>0$ P del intervalo $[1,n]$

$P = \{1, 2, 3, \dots n-1, n\} \quad n \in \mathbb{N} \quad , \quad n>1$

a) $\underline{S}(f,P) \quad , \quad \overline{S}(f,P)$

$$\underline{S}(f,P) = \sum_{i=0}^{n-1} (x_{i+1} - x_i)\, m_i$$

$$= L(1) + L(2) + \dots + L(n-1)$$



$$\overline{S}(f,P) = L(2) + L(3) + \dots + L(n)$$

b) $\int_1^n f(x)\,dx \qquad e\left(\frac{n}{e}\right)^n \leq n! \leq ne\left(\frac{n}{e}\right)^n$

$n \in \mathbb{N}, n>1$

$$\int_1^n 1.L(x)\,dx = (xLx - x)\Big|_1^n$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$\int f'g = f.g - \int f.g'$$

$f(x)$ cont $[a,b]$

$$\int_1^n 1.L(x)\,dx = n(Ln - 1) + 1$$

$$f(x) = F'(x)$$

$$\int f(x)\,dx = F(x)$$

$$\underline{S}(f,P) = L(2) + \dots + L(n-1)$$
$$= L(2.3.\dots n-1) = L((n-1)!)$$

$$n! = n(n-1)(n-2)\dots 1$$

$$\overline{S}(f,P) = L(2) + \dots + L(n)$$
$$= L(2\dots n) = L(n!)$$

Demostrar $e\left(\frac{n}{e}\right)^n \leq n! \leq n e\left(\frac{n}{e}\right)^n$ $n \in \mathbb{N}$, $n \geq 2$

$$\underline{S}(f,P) = L((n-1)!) \quad ; \quad \overline{S}(f,P) = L(n!)$$

$$\int_1^n Lx\, dx = n(Ln - 1) + 1$$

$$\underline{S}(f,P) \leq \int_1^n Lx\, dx \leq \overline{S}(f,P)$$

$$L((n-1)!) \leq n(Ln-1) + 1 \leq L(n!)$$

elimino lado e

$$\underbrace{(n-1)! \leq e^{n(Ln-1)} \cdot e}_{2} \leq n!$$

$$e^{n(Ln-1)} \cdot e \underbrace{\leq n!}_{1}$$

1

$$n! \geq e^{n(Ln-1)} \cdot e = e^{nLn} \cdot e^{-n} \cdot e$$

$$= (e^{Ln})^n \cdot e^{-n} \cdot e = n^n \cdot e^{-n}\, e = \frac{n^n}{e^n} \cdot e$$

$$= \left(\frac{n}{e}\right)^n \cdot e \quad \checkmark \quad \rightarrow \quad n! \geq \left(\frac{n}{e}\right)^n \cdot e$$

2

$$(n-1)! \leq \underbrace{e^{n(Ln-1)} \cdot e}$$

por 1 $\quad (n-1)! \leq \left(\frac{n}{e}\right)^n \cdot e$

$$n! = n\,\underbrace{(n-1)(n-2) - \cdots\cdots 1}_{(n-1)!}$$

$$n! = n(n-1)! \Rightarrow \frac{n!}{n} = (n-1)!$$

$$(n-1)! = \frac{n!}{n} \leq \left(\frac{n}{e}\right)^n \cdot e \Rightarrow n! \leq n \cdot e \left(\frac{n}{e}\right)^n \quad \checkmark$$

c) $\lim\limits_{n\to\infty} \frac{e}{n}\sqrt[n]{n!}$

$\left(\text{sugerencia } \lim\limits_{n\to+\infty} \sqrt[n]{n} = 1 \right)$

$\lim\limits_{n} \sqrt[n]{n} = \lim\limits_{n} (n)^{1/n}$; $n! \leq ne\left(\frac{n}{e}\right)^n$

$\lim\limits_{n} \frac{e}{n}\sqrt[n]{n!} \leq \lim\limits_{n} \frac{e}{n}\sqrt[n]{ne\left(\frac{n}{e}\right)^n}$

$\lim\limits_{n} \frac{e}{n}\left(ne\left(\frac{n}{e}\right)^n\right)^{1/n} = \lim\limits_{n} \frac{\cancel{e}}{\cancel{n}}\left((ne)^{1/n} \cdot \frac{\cancel{n}}{\cancel{e}}\right)$

$(e^a)^b = e^{a.b}$

$= \lim\limits_{n} (ne)^{1/n} = \lim\limits_{n\to+\infty} \underbrace{\sqrt[n]{n}}_{=1} \cdot e^{1/n} = 1$

$* \int \operatorname{Sen}^2(x)\,dx = \int \underbrace{\operatorname{Sen}(x)}_{f'} \cdot \underbrace{\operatorname{Sen}(x)}_{g}\,dx$

$\int f' \cdot g = f \cdot g - \int f \cdot g'$

$f = -\operatorname{Cos}(x) \quad f' = \operatorname{Sen}(x)$
$g = \operatorname{Sen}(x) \quad g' = \operatorname{Cos}(x)$

$\int \operatorname{Sen}^2(x)\,dx = -\operatorname{Sen}(x) \cdot \operatorname{Cos}(x) + \int \operatorname{Cos}^2(x)\,dx$

$\int \operatorname{Cos}^2(x)\,dx = \int \overbrace{\operatorname{Cos}(x)}^{f'} \cdot \overbrace{\operatorname{Cos}(x)}^{g}\,dx =$

$$\int f' \cdot g = f \cdot g - \int f \cdot g'$$

$$\int \operatorname{sen}^2(x) = -\operatorname{sen}(x)\cos(x) + \int \cos^2(x)\,dx$$

usando $\cos^2(x) + \operatorname{sen}^2(x) = 1$

$$\Rightarrow \cos^2(x) = 1 - \operatorname{sen}^2(x)$$

$$\Rightarrow \int \operatorname{sen}^2(x)\,dx = -\operatorname{sen}(x).\cos(x) + \int 1 - \operatorname{sen}^2(x)\,dx$$

$$\int \operatorname{sen}^2(x)\,dx = -\operatorname{sen}(x).\cos(x) + x - \int \operatorname{sen}^2(x)\,dx$$

$$2\int \operatorname{sen}^2(x)\,dx = -\operatorname{sen}(x)\cos(x) + x$$

$$\Rightarrow \int \operatorname{sen}^2(x)\,dx = \frac{1}{2}\left(-\operatorname{sen}(x).\cos(x) + x\right)$$

Otra forma de calcular $\int \operatorname{sen}^2(x)\,dx$ es usar

$$\operatorname{sen}(x)\operatorname{sen}(x) = -\frac{\cos(2x)+1}{2} \Rightarrow$$

$$\int \operatorname{sen}^2(x)\,dx = -\int \frac{\cos(2x)}{2}\,dx + \int \frac{1}{2}\,dx$$

* $$\int \frac{1}{x} \operatorname{sen}^3\left(1 + \log(x)\right)dx$$

* $$\int \frac{dx}{(2x^2+1)(x-1)}$$