Ejercicio 7 - Práctica 6.1

8/6

a) $0 \le x \le \pi/2$ $\quad \displaystyle\int_0^x \frac{\cos(t)}{\operatorname{Sen}^2(t) - 5\operatorname{Sen}(t) + 6}\, dt$

Calcular integral.

$$\int \frac{\cos(t)\, dt}{\operatorname{Sen}^2(t) - 5\operatorname{Sen}(t) + 6} \underset{CV}{\Rightarrow} \quad u = \operatorname{Sen}(t), \quad du = dt \cos(t)$$

$$\int \frac{du}{u^2 - 5u + 6} = \int \frac{du}{(u-2)(u-3)} \qquad u^2 - 5u + 6 = (u-2)(u-3)$$

$$\frac{1}{(u-2)(u-3)} = \frac{A}{(u-2)} + \frac{B}{(u-3)} \qquad A = 1 \quad B = 1$$

$$A(u-3) + B(u-2) = u(A+B) - 3A - 2B = 1$$

$$\begin{cases} A + B = 0 \Rightarrow A = -B \\ -3A - 2B = 1 \Rightarrow 3A - 2B = 1 \Rightarrow B = 1 \end{cases} \quad A = -1$$

$$\frac{1}{(u-2)(u-3)} = \frac{-1}{(u-2)} + \frac{1}{(u-3)} \qquad \int \frac{f'}{f} = L|f|$$

$$\int \frac{du}{(u-2)(u-3)} = -\boxed{\int \frac{1 \cdot du}{(u-2)}} + \int \frac{du}{(u-3)} =$$

$$= -L|u-2| + L|u-3|$$

$$(e^x)' = e^x \qquad \left(e^{x^3}\right)' = e^{x^3} \cdot 3x^2 \cdot \underbrace{\ln e}_{=1}$$

$$(e^f)' = e^f \cdot f' \qquad 2^{x^3} = 2^{x^3} \cdot 3x^2 \cdot \ln 2$$

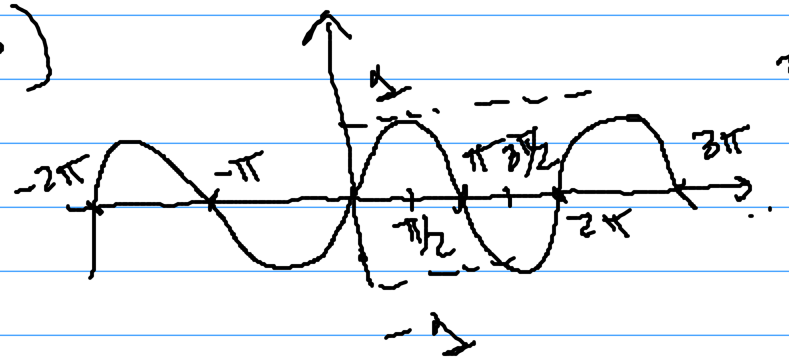
$$\int \frac{1}{(u-2)(u-3)} \cdot \partial u = -L|u-2| + L|u-3|$$

$u = \text{sen}(t) \Rightarrow$

$$\int \underbrace{\frac{\cos(t)\, \partial t}{\text{sen}^2(t) - 5\,\text{sen}(t) + 6}}_{= f(t)} = -L|\text{sen}(t)-2| + L|\text{sen}(t)-3|$$

$$\int_0^{\pi/2} f(t)\, \partial t = \overbrace{-L|\text{sen}(t)-2| + L|\text{sen}(t)-3|}^{F(t)} \Big|_0^{\pi/2}$$

$$= F(\pi/2) - F(0)$$



a) $\int \frac{1}{x} \operatorname{sen}^3(1+\log(x))\, dx$ = du

u

CV $u = 1+\log(x)$

$du = dx \; \frac{1}{x}$

$$\int \frac{1}{x} \operatorname{sen}^3(1+\log(x))\, dx = \int \operatorname{sen}^3(u)\, du$$

$$\int \operatorname{sen}^2(u) \cdot \operatorname{sen}(u)\, du = \int (1-\cos^2(u))\operatorname{sen}(u)\, du$$

CV: $v = \cos(u)$

$dv = -\operatorname{sen}(u) \cdot du$

$$= \int -(1-v^2)\, dv = -\int 1\, dv + \int v^2\, dv =$$

$$= -v + \frac{v^3}{3} = -\cos(u) + \frac{\cos(u)^3}{3}$$

desarmo CV

desarmo CV $u = 1+\log x$

$$= -\cos(1+\log(x)) + \frac{\cos(1+\log(x))^3}{3} + K$$

Practico 6

6b $\int \frac{dx}{(2x^2+1)(x-1)}$

A =
B =
C =

$$\frac{1}{(2x^2+1)(x-1)} = \frac{Ax+B}{2x^2+1} + \frac{C}{x-1}$$

$$\int \frac{Ax+B}{2x^2+1}\,dx + \int \frac{C}{x-1}\,dx \qquad \int \frac{f'}{f}$$

$$\int \frac{Ax+B}{2x^2+1}\,dx = \frac{1}{4}A \int \frac{4x}{2x^2+1}\,dx + B \int \frac{1}{2x^2+1}\,dx$$

$$B \int \frac{1}{1+2x^2}\,dx \qquad \int \frac{1}{1+x^2}\,dx = \operatorname{arctg}(x)+K$$

$$1+2x^2 = 1+(\underbrace{\sqrt{2}x}_{u})^2$$

CV $u = \sqrt{2}x$

$du = \sqrt{2}\,dx$

$\frac{du}{\sqrt{2}} = dx$

$$B \int \frac{1}{1+(\sqrt{2}x)^2}\,dx = B \int \frac{1}{1+u^2} \cdot \frac{du}{\sqrt{2}} = \frac{B}{\sqrt{2}} \int \frac{1}{1+u^2}\,du$$

$$= \frac{B}{\sqrt{2}} \operatorname{arctg}(u) = \frac{B}{\sqrt{2}} \operatorname{arctg}(\sqrt{2}x)+K \qquad \operatorname{arctg}(u)$$

deshago CV $u = \sqrt{2}x$

Práctica 7 (impropias)

e) $\displaystyle\int_0^{+\infty} \underbrace{(x-1)e^x}_{f(x)}\, dx$

$$\int \underbrace{(x-1)}_{f}\underbrace{e^x}_{g'}\, dx = (x-1).e^x - \underbrace{\int 1.e^x\, dx}_{e^x}$$

$$\int (x-1)e^x\, dx = e^x\,(x-2)$$

$$\int_0^{+\infty} (x-1)\,e^x\, dx = e^x\,(x-2)\Big|_0^b$$

$$= \lim_{b\to+\infty} e^b\,(b-2) - \underbrace{e^0\,(0-2)}_{-2}$$

$$= \lim_{b\to+\infty} e^b\,(b-2) + 2 \Rightarrow \text{Divergente.}$$

ñ)

$$\int \frac{1}{1+\textcircled{x}^2}\,dx$$

$$\int_0^{\pi/2} \frac{\cos(x)}{1-\cos^2(x)}\,dx$$

$\cos^2(x)+\operatorname{sen}^2(x)=1$

$1-\cos^2(x)=\operatorname{sen}^2(x)$

$$\int \frac{\cos(x)}{\operatorname{sen}^2(x)}\,dx$$

CV $u=\operatorname{sen}(x)$ $\Rightarrow \displaystyle\int \frac{du}{u^2} = \int u^{-2}\,du$

$du = dx \cos(x)$

$$\Rightarrow \int u^{-2}\,du = -\frac{1}{u} = \frac{u^{-1}}{-1} \underset{\text{desarm CV}}{\Rightarrow} = \frac{-1}{\operatorname{sen}(x)}$$

$$\int \frac{\cos(x)\,dx}{1-\cos^2(x)} = -\frac{1}{\operatorname{sen}(x)}\Bigg|_0^{\pi/2} \qquad [0,\pi/2]$$

$$= \underbrace{\frac{-1}{\operatorname{sen}(\pi/2)}}_{=-1} + \lim_{x\to 0} \frac{1}{\operatorname{sen}(x)} \Rightarrow \text{Diverge.}$$