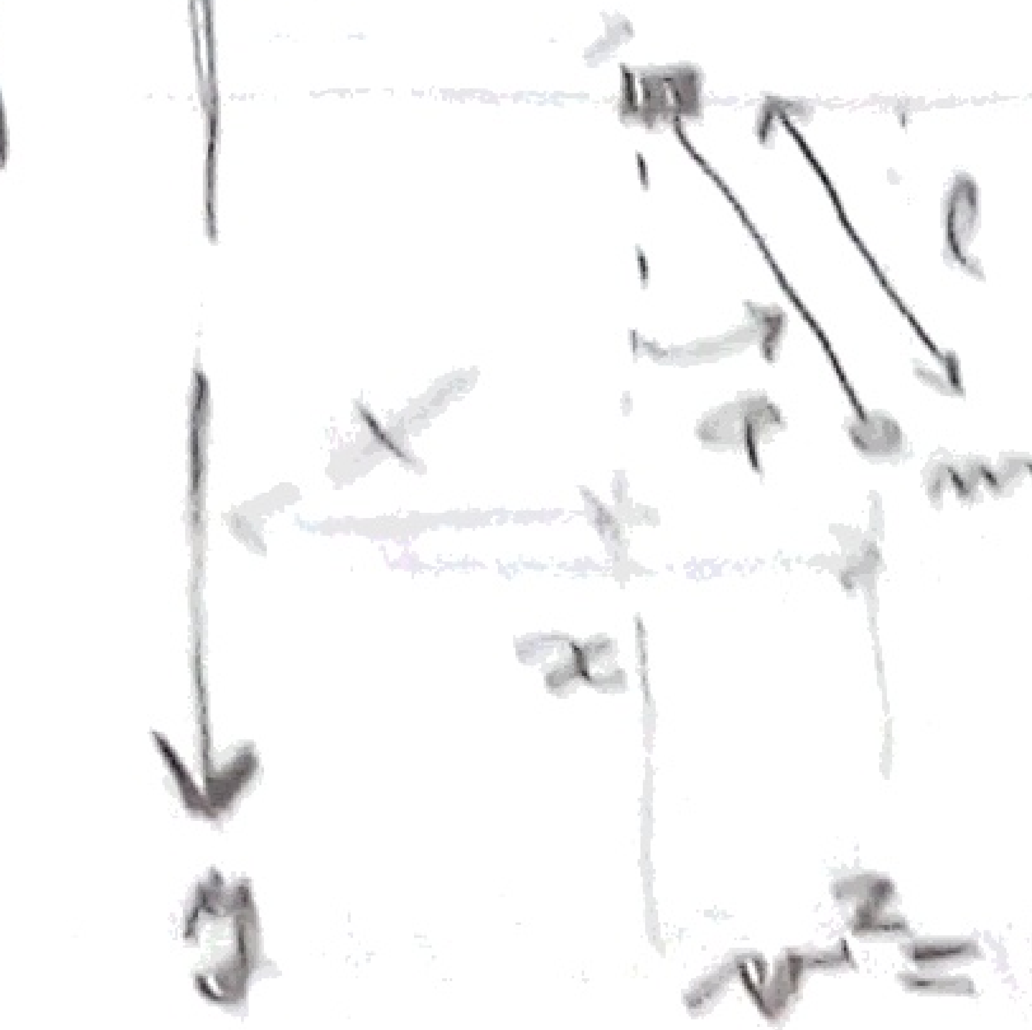
I.1 (1)

$X(t) = a\cos(\gamma t)$



$$\begin{cases} x = X + l\,\mathrm{sen}\,\varphi \\ y = l\cos\varphi \end{cases}$$

$$\begin{cases} \dot x = \dot X + l\dot\varphi\cos\varphi \\ \dot y = -l\dot\varphi\,\mathrm{sen}\,\varphi \end{cases}$$

$$v^2 = \dot x^2 + \dot y^2 = \dot X^2 + l^2\dot\varphi^2 + 2l\dot\varphi\dot X\cos\varphi$$

$$\boxed{L = \frac{ml^2}{2}\dot\varphi^2 + ml\dot\varphi\dot X\cos\varphi + \underbrace{mgl\cos\varphi}_{=(-V)} + \underbrace{\frac{m\dot X^2}{2}}_{=\ f(t)\ \text{irrelevante}}}$$

$\dot X = -a\gamma\,\mathrm{sen}(\gamma t)$ $\quad \ddot X = -a\gamma^2\cos(\gamma t)$

$$\boxed{L = \frac{ml^2}{2}\dot\varphi^2 - mla\gamma\dot\varphi\,\mathrm{sen}(\gamma t)\cos\varphi + mgl\cos\varphi}$$

$$\frac{\partial L}{\partial\dot\varphi} = ml^2\dot\varphi - mla\gamma\,\mathrm{sen}(\gamma t)\cos\varphi$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\varphi}\right) = ml^2\ddot\varphi - mla\gamma^2\cos(\gamma t)\cos\varphi + mla\gamma\dot\varphi\,\mathrm{sen}(\gamma t)\,\mathrm{sen}\,\varphi$$

$$\frac{\partial L}{\partial\varphi} = +mla\gamma\dot\varphi\,\mathrm{sen}(\gamma t)\,\mathrm{sen}\,\varphi - mgl\,\mathrm{sen}\,\varphi$$

Ec. de Lagrange $\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\varphi}\right) - \frac{\partial L}{\partial\varphi} = 0}$ →

$$\boxed{ml^2\ddot\varphi - mla\gamma^2\cos(\gamma t)\cos\varphi + mgl\,\mathrm{sen}\,\varphi = 0}$$

$$\boxed{\ddot\varphi - \frac{a}{l}\gamma^2\cos(\gamma t)\cos\varphi + \frac{g}{l}\,\mathrm{sen}\,\varphi = 0}$$

$\boxed{\underline{\mathrm{I}}\text{-}2}$ $L = \frac{ml^2}{2}\dot{\varphi}^2 - mla\gamma\dot{\varphi}\,\mathrm{sen}(\gamma t)\cos\varphi + mgl\cos\varphi$

$\rightarrow\ p_\varphi = \frac{\partial L}{\partial\dot{\varphi}} = ml^2\dot{\varphi} - mla\gamma\,\mathrm{sen}(\gamma t)\cos\varphi$

$\rightarrow\ \dot{\varphi} = \frac{p_\varphi + mla\gamma\,\mathrm{sen}(\gamma t)\cos\varphi}{ml^2}$

$p_\varphi\dot{\varphi} = ml^2\dot{\varphi}^2 - mla\gamma\dot{\varphi}\,\mathrm{sen}(\gamma t)\cos\varphi$

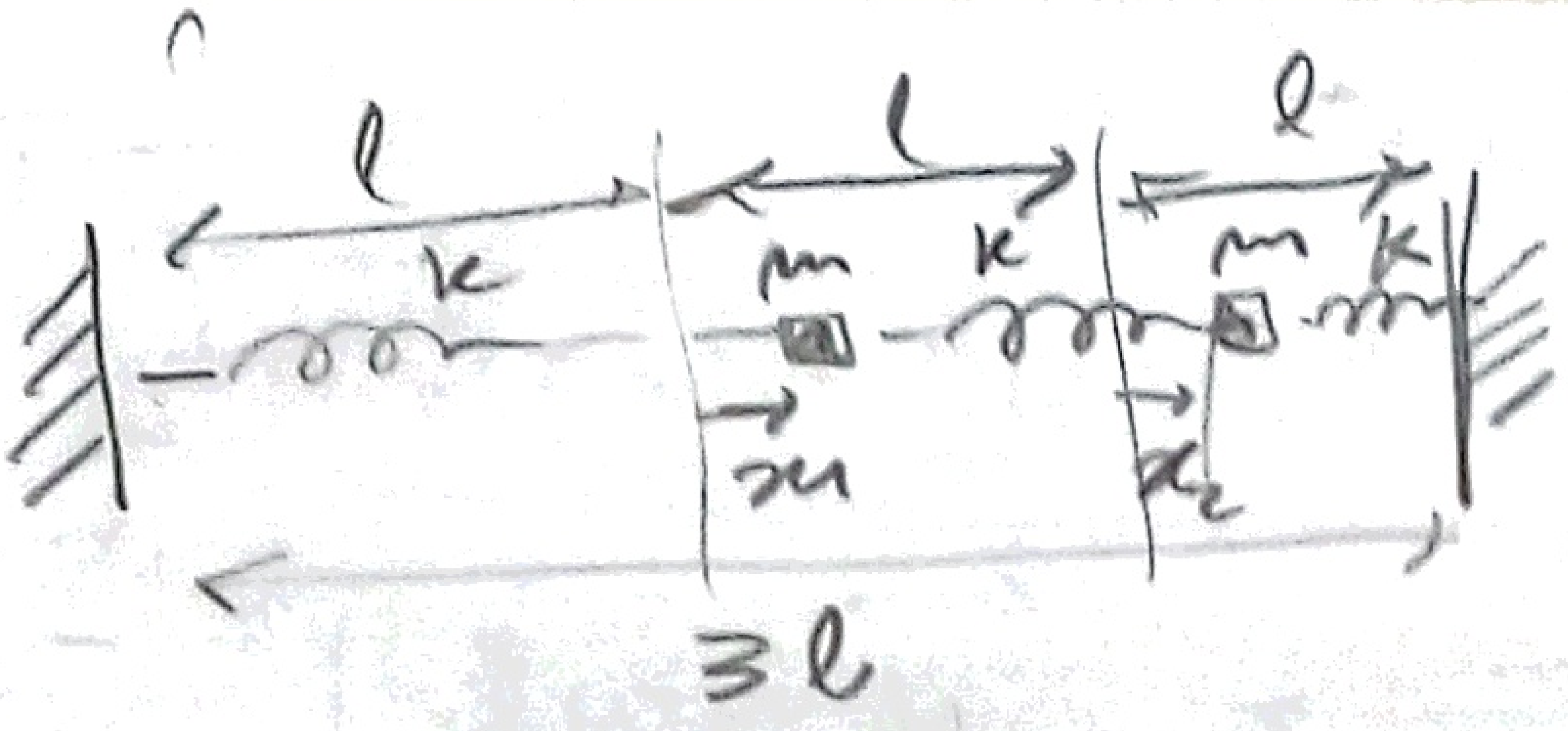
$H = p_\varphi\dot{\varphi} - L = \frac{ml^2}{2}\dot{\varphi}^2 - mgl\cos\varphi \Longrightarrow$

$\Rightarrow \boxed{H(\varphi, p_\varphi) = \frac{[p_\varphi + mla\gamma\,\mathrm{sen}(\gamma t)\cos\varphi]^2}{2ml^2} - mgl\cos\varphi}$

¿Se conserva H? $\frac{dH}{dt} = \frac{\partial H}{\partial t} \neq 0$

↳ H NO se conserva, ya que depende explícitamente del tiempo.

$\boxed{\text{I-3}}$



②

$$V = \frac{k x_1^2}{2} + \frac{k x_2^2}{2} + \frac{k}{2}(x_2 - x_1)^2 =$$

$$= \frac{k}{2}\left\{ 2x_1^2 + 2x_2^2 - 2x_1 x_2 \right\}$$

$$\mathbb{K} = k\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 2k & -k \\ -k & 2k \end{pmatrix}$$

$$T = \frac{m}{2}(\dot{x}_1^2 + \dot{x}_2^2) \rightarrow \mathbb{M} = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

$$\det(\mathbb{K} - \omega^2 \mathbb{M}) = \begin{vmatrix} 2k - \omega^2 m & -k \\ -k & 2k - \omega^2 m \end{vmatrix} =$$

$$= m^2 \begin{vmatrix} 2\omega_0^2 - \omega^2 & -\omega_0^2 \\ -\omega_0^2 & 2\omega_0^2 - \omega^2 \end{vmatrix} = (2\omega_0^2 - \omega^2)^2 - \omega_0^4 = 0$$

$\omega_0^2 = \frac{k}{m}$

$$\rightarrow 4\omega_0^4 + \omega^4 - 4\omega_0^2\omega^2 - \omega_0^4 = 0$$

$$\omega^4 - 4\omega_0^2\omega^2 + 3\omega_0^4 = 0$$

$$\omega^4 - 4\omega_0^2\omega^2 + 3\omega_0^4 = 0 \rightarrow$$

$$\omega^2 = \frac{4\omega_0^2 \pm \sqrt{16\omega_0^4 - 12\omega_0^4}}{2} =$$

$$\boxed{\omega^2 = 2\omega_0^2 \pm \omega_0^2 = \begin{cases} 3\omega_0^2 \\ \omega_0^2 \end{cases}}$$

Modos Normales

I $\boxed{\omega_I = \omega_0}$

$$\begin{pmatrix} \omega_0^2 & -\omega_0^2 \\ -\omega_0^2 & \omega_0 \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = 0$$

$$\rightarrow \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = 0 \rightarrow \begin{array}{l} A_1 - A_2 = 0 \\ -A_1 + A_2 = 0 \end{array}$$

$$\boxed{A_1 = A_2}$$

$$\boxed{\begin{array}{l} x_1^{(I)}(t) = A_I \cos\left(\sqrt{\frac{k}{m}}\, t + \phi_I\right) \\ x_2^{(I)}(t) = A_I \cos\left(\sqrt{\frac{k}{m}}\, t + \phi_I\right) \end{array}}$$

II $\boxed{\omega_{II} = \sqrt{3}\,\omega_0}$

$$\begin{pmatrix} -\omega_0^2 & -\omega_0^2 \\ -\omega_0^2 & -\omega_0^2 \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\rightarrow \boxed{A_1 = -A_2}$$

$$\boxed{\begin{array}{l} x_1^{(II)}(t) = A_{II} \cos\left(\sqrt{\frac{3k}{m}}\, t + \phi_{II}\right) \\ x_2^{(II)}(t) = -A_{II} \cos\left(\sqrt{\frac{3k}{m}}\, t + \phi_{II}\right) \end{array}}$$

③

II.1 $S=\int_{t_i}^{t_f} L\,dt \rightarrow \delta S=\int_{t_i}^{t_f}\left\{\frac{\partial L}{\partial \dot q}\delta\dot q+\frac{\partial L}{\partial q}\delta q\right\}dt$

$\frac{\partial L}{\partial \dot q}\delta\dot q=\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\delta q\right)-\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\right)\delta q \rightarrow$

$\boxed{\delta\dot q=\frac{d}{dt}(\delta q)}$

$\delta S=\int_{t_i}^{t_f}\left\{\frac{\partial L}{\partial q}-\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\right)\right\}\delta q\,dt+$

$+\underbrace{\int_{t_i}^{t_f}\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\delta q\right)dt}_{=\left(\frac{\partial L}{\partial \dot q}\delta q\right)\Big|_{t_i}^{t_f}}$ $\rightarrow$ $\delta S=0$,

um $\delta q(t_i)=\delta q(t_f)=0$

implica

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\right)-\frac{\partial L}{\partial q}=0}$$

II.2 Forma canónica $S=\int_{t_i}^{t_f} L\,dt$

$S'=\int_{t_i}^{t_f} L'\,dt=\underbrace{\int_{t_i}^{t_f} L\,dt}_{=S}+\underbrace{\int_{t_i}^{t_f}\frac{df(q,t)}{dt}\,dt}_{=(f(q,t))\Big|_{t_i}^{t_f}} \Rightarrow$

$$\boxed{S'=S+f(q(t_f),t_f)-f(q(t_i),t_i)} \Rightarrow$$

$$\Rightarrow \delta S' = \delta S + \underbrace{\delta f(q(t_i), t_i) - \delta f(q(t_f), t_f)}_{=0 \quad \text{si } \delta q(t_i) = \delta q(t_f) = 0}$$

La condición

$\delta S = 0$ es equivalente a $\delta S' = 0$

$\Rightarrow$ las ecuaciones de Lagrange, equivalentes a dicha condición, son los mismos en ambos casos.