$\gamma(t) = Re^{it} \quad t \in [0, 2\pi]$

$$\int_\gamma \frac{P'(z)}{P(z)}\,dz$$

$$P(z) = z^n\left(1 + \frac{a_1}{z} + \dots + \frac{a_n}{z^n}\right)$$

$$\frac{P'}{P} = \frac{d}{dz}(\log P) = \frac{d}{dz}\left(\log z^n + \underbrace{\log\left(1 + \frac{a_1}{z} \dots\right)}\right)$$

$$\int_\gamma \frac{P'}{P} = \int_\gamma \frac{n}{z} + \int_\gamma \underbrace{\frac{\left(1 + \frac{a_1}{z} + \dots + \frac{a_n}{z^n}\right)'}{1 + \frac{a_1}{z} + \dots + \frac{a_n}{z^n}}}$$

denom. $\neq 0$ en el círculo de radio R

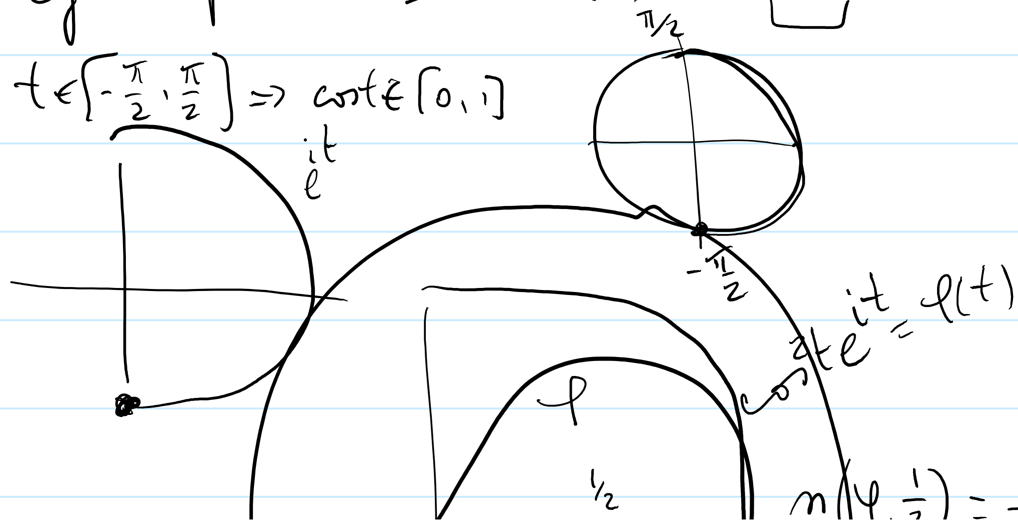
f. hol en ese círculo

$\Longrightarrow$ por Cauchy $\int_\gamma = 0$

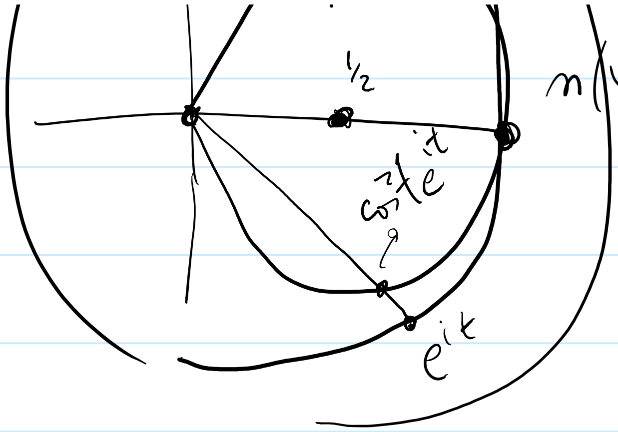
etc.

Ej 10 práctico III $\quad \varphi(t) = \cos^2 t\, e^{it}$

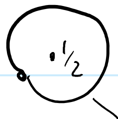
$t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \Rightarrow \cos t \in [0, 1]$



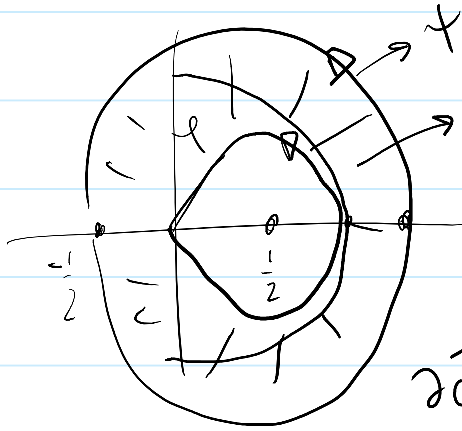
$n\left(\varphi, \frac{1}{2}\right) = \frac{1}{\dots} \left(\frac{dz}{\dots} \right.$



$$n\left(\varphi,\tfrac{1}{2}\right)=\frac{1}{2\pi i}\int\frac{dz}{z-\frac{1}{2}}$$



$z=\frac{1}{2}+e^{it}r$

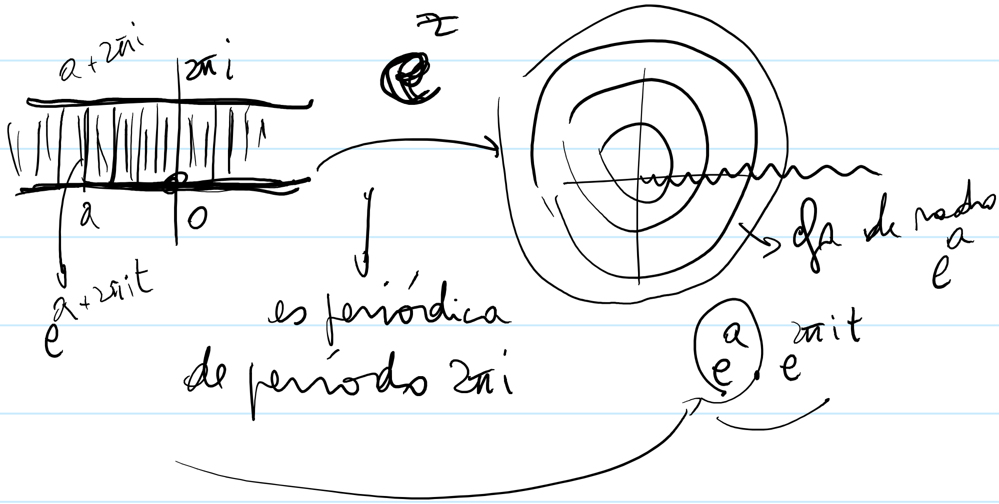


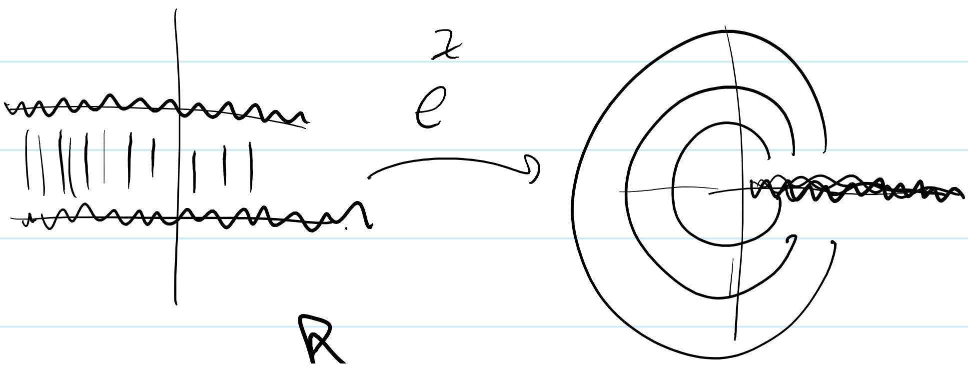
C

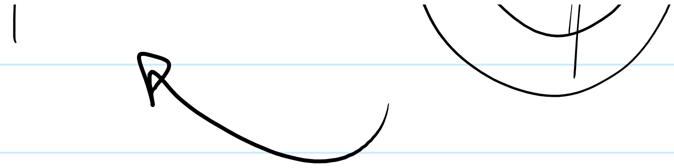
$\partial C=\psi-\varphi$

$$\int_{\partial C}\frac{dz}{z-\frac{1}{2}}=0\to\int_{\varphi}=\int_{\psi}=2\pi i$$

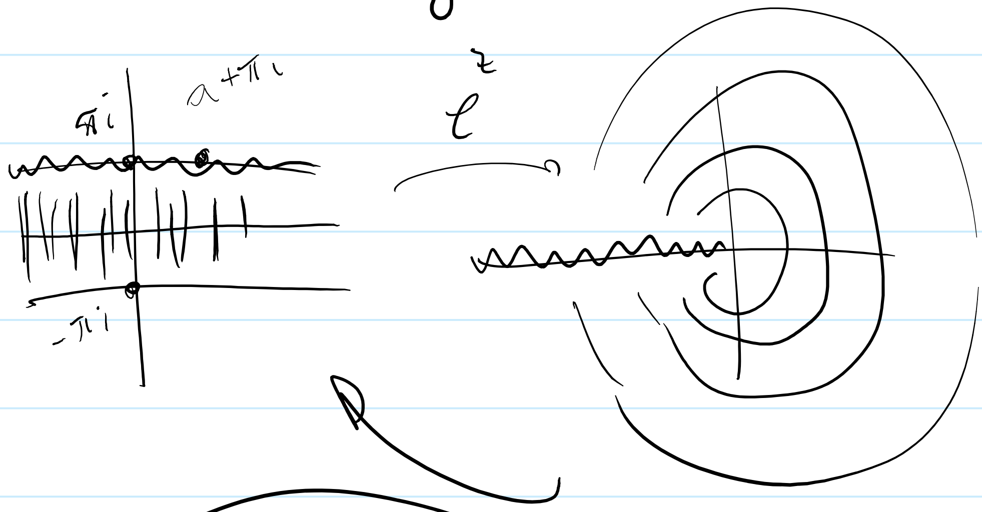
$$\Longrightarrow\quad n\left(\varphi,\tfrac{1}{2}\right)=\frac{1}{2\pi i}\int_{\varphi}\frac{dz}{z-\frac{1}{2}}=1$$

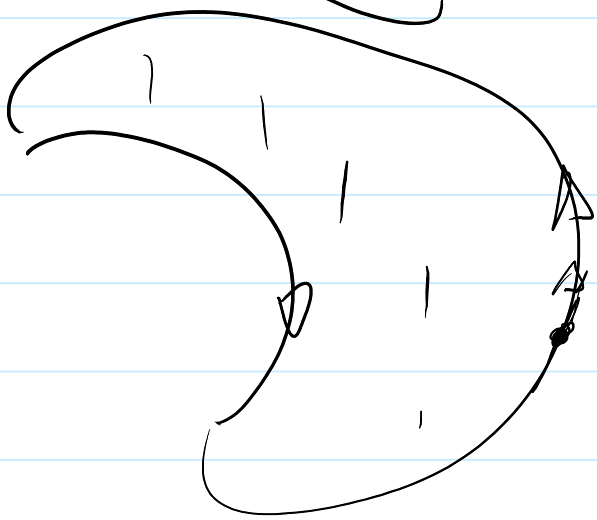


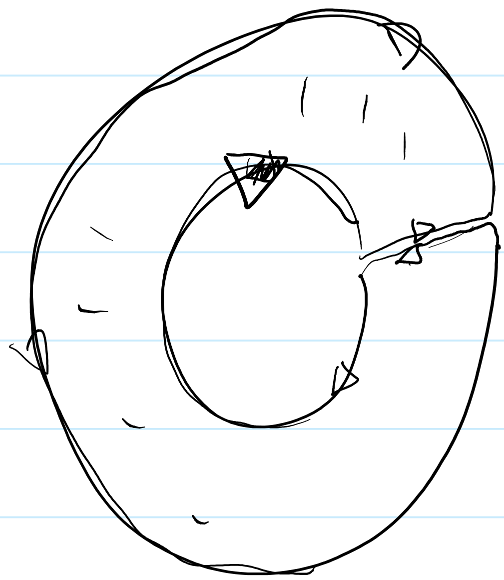




$\log z$







$$f(z) = e^z\left(\frac{1}{z} + \frac{a}{z^3}\right) = u + iv = U_x + iV_x$$

$$\left(\frac{e^z}{z}\right)' = e^z\left(\frac{1}{z} \quad \frac{1}{z^2}\right)$$

$$\left(\frac{e^z}{z^2}\right)' = e^z\left(\frac{1}{z^2} - \frac{2}{z^3}\right)$$

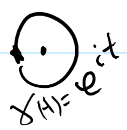
$$\Big[\underbrace{e^z\left(\frac{1}{z} + \frac{1}{z^2}\right)}_{G}\Big]' = e^z\left(\frac{1}{z} - \frac{2}{z^3}\right)$$

$$F'(z) = e^z\left(\frac{1}{z} + \frac{a}{z^3}\right)$$

$$(F - G)' = \frac{e^z(a+2)}{z^3}$$

Si $a+2 \neq 0 \Rightarrow H = \frac{1}{a+2}(F - G)$ es tal que $H' = \frac{e^z}{z^3}$

$$\Rightarrow \int_\gamma \frac{e^z}{z^3}\,dz = 0 \quad \times \quad \Rightarrow a+2 = 0$$



$a = -2 \Rightarrow$ $\quad \gamma(t) = e^{it} \quad 0 \leq t \leq 2\pi$

$$\int_\gamma \frac{e^z}{z}\,dz = \int_\gamma \left(\frac{1}{z} + \underbrace{\frac{e^z - 1}{z}}_{\text{sing. evit.}}\right)dz = \int_\gamma \frac{dz}{z} = 2\pi i$$

Cauchy $\quad \cancel{\frac{n}{2\pi i}} \int \frac{f(z)}{z^{n+1}}\,dz$

Cauchy gen. $\cancel{\frac{n!}{2\pi i}} \int_\gamma \frac{f(z)}{z^{n+1}}\,dz$

$$\int \frac{e^z}{z^3}\,dz \qquad e^z = 1 + z + \frac{z^2}{2} + z^3\varepsilon$$