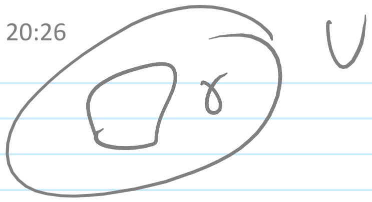
Clase 12

jueves, 12 de mayo de 2016 20:26

$f: U \to C$

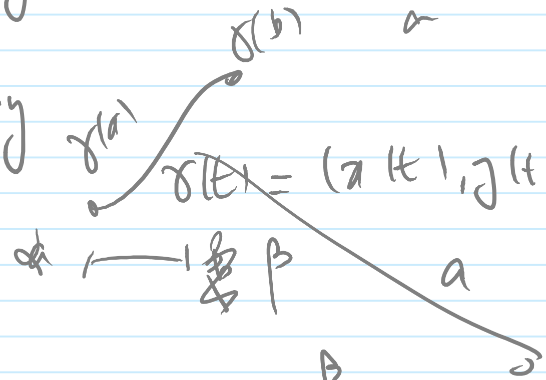


$a, b: R^2 \to R$

$$\int_a^b f(x)\,dx = \int_a^b f$$

$a\,dx + b\,dy$

$$\int_\gamma a\,dx + b\,dy$$



$\gamma(t) = (x(t), y(t))$

$\alpha \longmapsto \beta$

a

R

$$= \int_\alpha^\beta a(\gamma(t))\,x'(t)\,dt + \int_\alpha^\beta b(\gamma(t))\,y'(t)\,dt$$

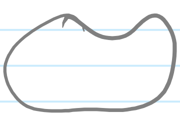
$(a(x,y), b(x,y)) = \vec{v}(x,y)$

$$\int_\gamma a\,dx + b\,dy = \int_\alpha^\beta \vec{v}(\gamma(t)) \cdot \gamma'(t)\,dt$$

Formula de Green

$$\int_\gamma a\,dx + b\,dy = \int_R (b_x - a_y)\,dx\,dy$$

$f(z) = f(x+iy)$



$R^2 = C$

$$\int_\gamma f(z)\,dz = \int_\alpha^\beta f(\gamma(t))\,\gamma'(t)\,dt$$

$f = u + iv$

γ $\quad\alpha$ $\qquad\qquad f = u + iv$

$\gamma(t) = x(t) + i\,y(t)$

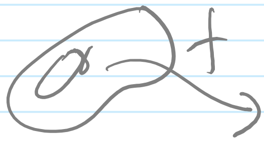
$$\int_\alpha^\beta \big(u(\gamma(t)) + i\,v(\gamma(t))\big)\big(x'(t) + i\,y'(t)\big)\,dt$$

$$= \int_\alpha^\beta (u x' - v y')\,dt + i \int_\alpha^\beta (v x' + u y')\,dt$$

$$= \int_\gamma (u\,dx - v\,dy) + i \int_\gamma (v\,dx + u\,dy)$$

$$\int_\gamma f(z)\,dz = \int_\gamma (u\,dx - v\,dy) + i \int_\gamma (v\,dx + u\,dy)$$

$\int_\gamma f(z)\,dz = 0$



$\int_\gamma u\,dx - v\,dy = 0$

$\int_\gamma v\,dx + u\,dy = 0$

} Teorema de Green

1ª versión

$$\int_\gamma u\,dx - v\,dy = \int_R (-v_x - u_y)\,dx\,dy$$

$\partial R = \gamma$

$$\int_\gamma v\,dx + u\,dy = \int_R (u_x - v_y)\,dx\,dy$$

CR $\Downarrow$ $u_x = v_y$, $u_y = -v_x$

$f(x+iy) = u + iv$ es analít. si $\begin{cases} u_x = v_y \\ u_y = -v_x \end{cases}$

$f(x+iy) = \underbrace{x^2-y^2+i2xy}_{z^2}$ es analítica por C-R $\quad (z = x+iy)$

1. $h = x^2+2x-iy$ $\begin{cases} u = x^2+2x \\ v = -y \end{cases}$ $\quad$ CR: $\begin{cases} u_x - v_y = 2x+2+1 = 2x+3 \\ u_y + v_x = 0 \checkmark \end{cases}$

CR se cumple a lo largo de la recta $\{2x+3=0\}$ que no ~~contiene~~ tiene interior $\Rightarrow$ h no es analít. en ningún abierto del plano complejo

2 - $f = x + ay + i(bx+cy)$ $\begin{cases} u = x+ay \\ v = bx+cy \end{cases} \Rightarrow \begin{cases} u_x - v_y = 1-c = 0 \\ u_y + v_x = a+b = 0 \end{cases}$

f es derivable sii $c=1$ y $a+b=0 \Rightarrow b=-a$

O sea $f = x+ay+i(-ax+y) = \underbrace{x+iy}_{z} + a\underbrace{(y-ix)}_{-iz}$

$z = x+iy$

$-iz = +y-ix$

Con resumen:

$$f = z - aiz = (1-ai)z$$

$$g = \cos x\,(\operatorname{ch} y + a \operatorname{sh} y) + i \operatorname{sen} x\,(\operatorname{ch} y + b \operatorname{sh} y)$$

$$\left[e^z = e^{x+iy} = e^x \cdot e^{iy} = e^x[\cos y + i \operatorname{sen} y] \right]$$

$\begin{cases} u = \cos x\,(\operatorname{ch} y + a \operatorname{sh} y) \\ v = \operatorname{sen} x\,(\operatorname{ch} y + b \operatorname{sh} y) \end{cases}$

$\begin{cases} u_x = -\operatorname{sen} x\,(\operatorname{ch} y + a \operatorname{sh} y) \\ u_y = \cos x\,(\operatorname{sh} y + a \operatorname{ch} y) \end{cases}$ $\quad$ " sii $a=b=1$

$\begin{cases} v_y = \operatorname{sen} x\,(\operatorname{sh} y + b \operatorname{ch} y) \\ v_x = \cos x\,(\operatorname{ch} y + b \operatorname{sh} y) \end{cases}$

$\operatorname{ch} y = \frac{1}{2}(e^y + e^{-y}) \qquad \operatorname{ch}' y = \operatorname{sh} y$

$\operatorname{sh} y = \frac{1}{2}(e^y - e^{-y}) \qquad \operatorname{sh}' y = \operatorname{ch} y$

$u_x = v_y$ sii $a=b=1$

$u_y = -v_x$ sii $a=b=1$

} por lo tanto g es derivable sii $a=b=1$ o sea:

$$g = -\cos x\,(\text{ch}\,y + \text{sh}\,y) + i\,\text{sen}\,x\,(\text{ch}\,y + \text{sh}\,y)$$

$$= \underbrace{(\text{ch}\,y + \text{sh}\,y)}_{e^{y}}\underbrace{(-\cos x + i\,\text{sen}\,x)}_{-e^{-ix}}$$

$e^{-ix} = \cos x$ [redacted] $i\,\text{sen}\,x$

$-e^{-ix} = -\cos x + i\,\text{sen}\,x$

$$g = -e^{y-ix}$$

$z = x + iy$

$iz = -y + ix$

$-iz = y - ix$

$$\Rightarrow \boxed{g = -e^{-iz}}$$