Clase 6

jueves, 21 de abril de 2016 19:27

$|z - \lambda w|^2 = (z - \lambda w)\overline{(z - \lambda w)}$

$\cancel{(z - \lambda w)}$

$(z - \lambda w)(\bar z - \bar\lambda \bar w)$

$= z\bar z - z\bar\lambda\bar w - \lambda w \bar z + \lambda\bar\lambda w \bar w$ $u = z\bar\lambda\bar w$

$= |z|^2 + |\lambda|^2|w|^2 - (z\bar\lambda\bar w + \lambda\bar w z)$ $u + \bar u = 2Re(u)$

$= 2Re(z\bar\lambda\bar w)$

$\overline{ab} = \bar a\,\bar b$

$\overline{(a+b)} = \bar a + \bar b$

$r \in R$

$\bar r = r$

$\bar{\bar a} = a$

Cálculo diferencial $f: U \to \mathbb{C}$ $\Omega \subset \mathbb{C}$

Si f admite derivada como función compleja

$(\forall a \in U \ \exists \lim \frac{f(z) - f(a)}{z - a} \ \exists)$

$\Rightarrow$ u y v $f = u + iv$

admiten derivadas parciales de primer orden

y vale $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

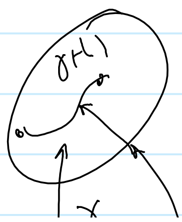
Si $f = u + iv$ y u y v satisfacen Cauchy Riemann

y las derivadas parciales son todas continuas

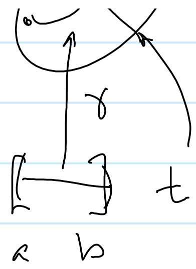
$\Rightarrow$ f admite derivada en los puntos de U

$f: U \to C$ continua $U \subset \mathbb{C}$

$\int f(z)dz$ $\int f$



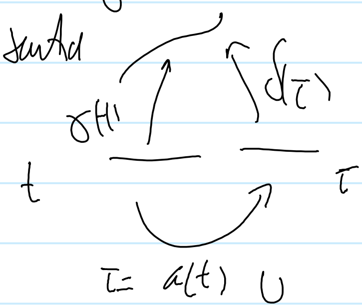
$\int_\gamma f(z)\,dz$ $\qquad \int_\gamma f$



$$\int_a^b f(\gamma(t))\gamma'(t)\,dt$$

1) $\int_\gamma f(z)\,dz$ es independiente de la parametrización siempre que no cambie el sentido

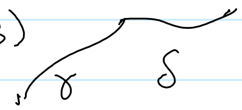
$\gamma \qquad -\gamma$



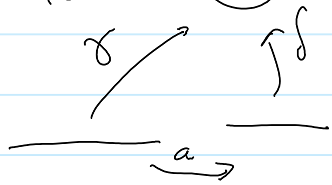
2) $\int_{-\gamma} f = -\int_\gamma f$

$\gamma(t) = \delta(a(t)) \qquad a'(t) > 0$ $\quad U$ $\quad 2$

3)



$\gamma = \delta \circ a$



$$\int_{\gamma\cup\delta} f(z)\,dz = \int_\gamma f(z)\,dz + \int_\delta f(z)\,dz$$

4) $\int_\gamma f'(z)\,dz = f(\gamma(b)) - f(\gamma(a))$ $\quad$ f es analítica y f' es continua

$\gamma : [a,b] \to U \subset \mathbb{C}$

$$\int_\gamma f'(z)\,dz = \int_a^b f'(\gamma(t))\gamma'(t)\,dt = \int_a^b \big(f(\gamma(t))\big)'\,dt$$

$$= f(\gamma(b)) - f(\gamma(a))$$

Obs: $A : [a,b] \to \mathbb{C}$ $\qquad \int_a^b A(x)\,dx \overset{\text{def}}{=} \int_a^b B(x)\,dx + i\int_a^b C(x)\,dx$

$A = B + iC$

5) $\boxed{\gamma \text{ cerrado} \qquad \int_\gamma f'(z)\,dz = 0}$

6) Si $f: U \to \mathbb{C}$ continua y $\boxed{\text{acotada}}$ U abierto

$k \in \mathbb{R}$ $\quad |f(z)| \leq k \quad \forall z \in U$

$$\Rightarrow \left|\int_\gamma f\,dz\right| \leq k \,\mathrm{long}(\gamma)$$

Si pensar $\gamma: [a,b] \to \mathbb{C}$; $\gamma(t) = (u(t), v(t))$

$$\ell(\gamma) = \int_a^b \sqrt{u'^2(t) + v'^2(t)}\,dt = \int_a^b |\gamma'(t)|\,dt$$

real

Dem

$$\int_\gamma f(z)\,dz = \int_a^b f(\gamma(t))\gamma'(t)\,dt$$

$$\left|\int_\gamma f(z)\,dz\right| = \left|\int_a^b f(\gamma(t))\gamma'(t)\,dt\right| \leq \int_a^b |f(\gamma(t))\gamma'(t)|\,dt$$

$$\leq \int_a^b |f(\gamma(t))|\,|\gamma'(t)|\,dt \leq k \int_a^b |\gamma'(t)|\,dt$$

$\boxed{\text{Módulo para adentro}}$ $\quad = k\,\ell(\gamma)$

$$f(t) = (u(t) + i v(t))$$

$$\left|\int_a^b u(t)\,dt + i\int_a^b v(t)\,dt\right| \leq \int_a^b |u(t) + i v(t)|\,dt$$

$$\sqrt{\left(\int_a^b u(t)\,dt\right)^2 + \left(\int_a^b v(t)\,dt\right)^2} \leq \int_a^b \left(u^2(t) + v^2(t)\right)^{1/2} dt$$

$$\left(\int^b u(t)\,dt\right)^2 + \left(\int^b v(t)\,dt\right)^2 \leq \left(\int (u^2+v^2)^{1/2} dt\right)^2$$

y

$$\left(\int_a^b u(t)\,dt\right)^2 + \left(\int_a^b v(t)\,dt\right)^2 \le \left(\int_a^b (u^2+v^2)^{1/2}\,dt\right)^2 \quad !!!$$

$\boxed{\text{próxima reunión}}$

$\int_\gamma f(z)\,dz$? $\quad \int_\gamma f(z)\,|dz|$

~~$\gamma'(t)$~~ $\quad \gamma = \delta(a(t))$

$\gamma'(t) = \delta'(a(t))\,\dot a(t)$

$$\int_\gamma f = \int_a^b f(\gamma(t))\,\gamma'(t)\,dt$$

$$\int_a^b f(\gamma(t))\,|\gamma'(t)|\,dt$$

$|\gamma'(t)| = |\delta'(z)|\,|\dot a(t)|$

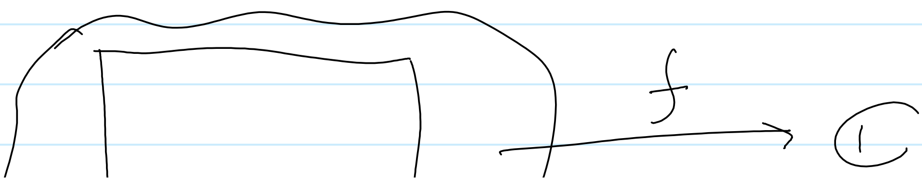
1) $\left|\int_\gamma f(z)\,dz\right| \le \int_\gamma |f|\,|dz| \quad !!!$

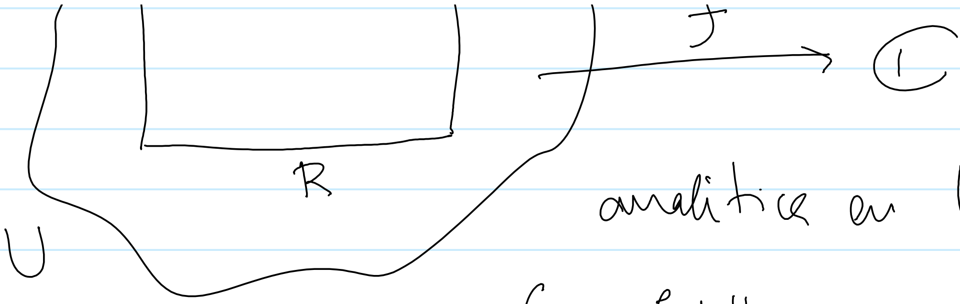
2) $\int_\gamma |dz| = l(\gamma)$

Dem $\quad \int_\gamma |dz| = \int_a^b |\gamma'(t)|\,dt$

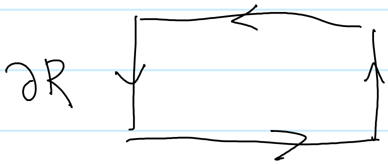
$$\left|\int_a^b f(\gamma(t))\,\gamma'(t)\,dt\right| \le \int_a^b |f(\gamma(t))|\,|\gamma'(t)|\,dt = \int_\gamma |f|\,|dz|$$

Teorema integral de Cauchy



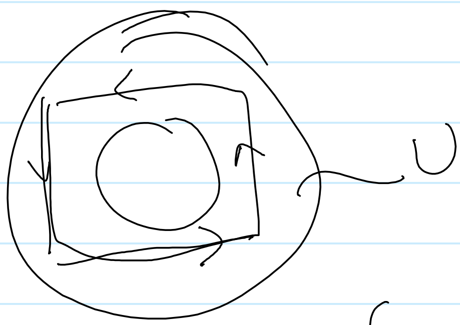


analítica en U

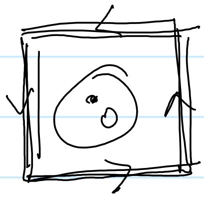


En esas hipótesis

$$\boxed{\int_{\partial R} f(z)\,dz = 0}$$

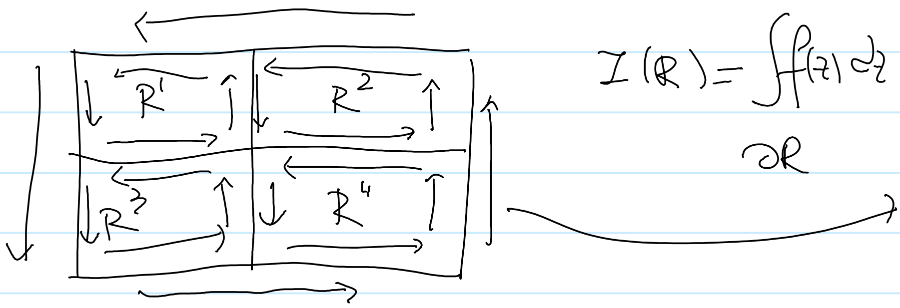


$\frac{1}{z}$



$$2\pi i = \int_{S^1} \frac{dz}{z} = \int_{\partial R} \frac{dz}{z}$$

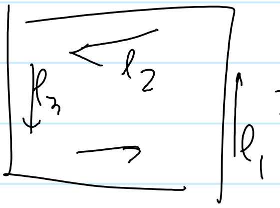
Goursat X|X-XX



$$I(R) = \int_{\partial R} f(z)\,dz$$

$$\int_{\gamma} f + \int_{-\gamma} f = 0$$

$$I(R) = I(R^1) + I(R^2) + I(R^3) + I(R^4)$$



$I(R) = \int_{l_1} f + \int_{l_2} \cdot + \int_{l_3} \cdot + \int$

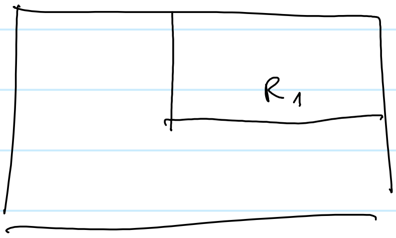
$\exists R^i : \boxed{\frac{1}{4} |I(R)| \le |I(R^i)|}$ si no $i = 1,2,3,4$

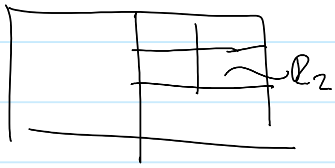
$\frac{1}{4} |I(R)| > |I(R^i)|$

$|I(R)| > |I(R^1)| + |I(R^2)| + |I(R^3)| + |I(R^4)|$

$\frac{1}{4} |I(R)| \le |I(R_1)|$

R



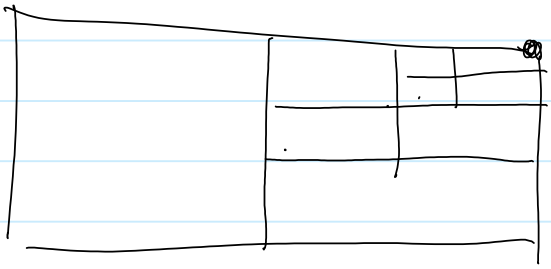


$\frac{1}{4} |I(R_1)| \le |I(R_2)|$

etc

$R \supset R_1 \supset R_2 \supset \cdots \supset R_n \supset \cdots$

$$|I(R_n)| \geq \frac{1}{4^n} |I(R)|$$



Teorema de los rectángulos encajados $\{z_0\} = \bigcap_n R_n$

$\exists$ $z_0 \in R$

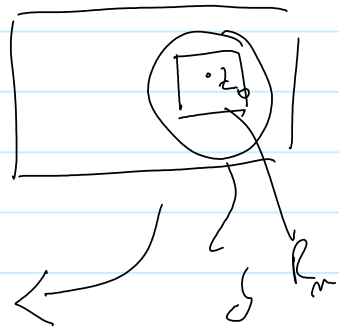
y además $|I(R_n)| \geq \frac{1}{4^n} |I(R)|$

$f(z)$ $f(z_0)$ $f'(z_0)$

$$\frac{\qquad}{z-z_0} = f'(z_0)$$

$$\forall \varepsilon > 0 \ \exists \delta(\varepsilon) > 0 . \quad |z-z_0| < \delta(\varepsilon)$$

$$\left| \frac{f(z)-f(z_0)}{z-z_0} - f'(z_0) \right| < \varepsilon$$



$$\forall \varepsilon > 0 \ \exists \delta > 0 : \quad |z-z_0| < \delta(\varepsilon)$$

$$\Downarrow$$

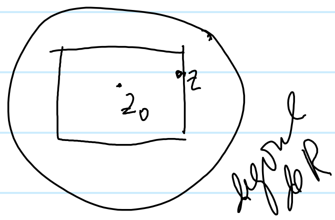
$$(1) \quad |f(z) - f(z_0) - (z-z_0) f'(z_0)| < \varepsilon |z-z_0|$$

$$R_n \subset \{ z : |z-z_0| < \delta \}$$

$$I(R_n) = \int_{\partial R_n} f(z)\, dz = \int_{\partial R_n} \big(f(z) - f(z_0) - (z-z_0) f'(z_0) \big)\, dz$$

$$|I(R_n)| \leq \int_{\partial R_n} |f(z) - f(z_0) - (z-z_0) f'(z_0)|\, |dz|$$

$$\leq \int_{\partial R_n} \varepsilon |z-z_0|\, |dz|$$



$|z-z_0| \leq d_n$ $\quad d_n =$ semidiagonal del rectángulo

$$|I(R_n)| \leq \varepsilon d_n \int_{\partial R_n} |dz| = \varepsilon d_n \, \ell(\partial R_n) \qquad d_n = \frac{1}{2^n} d$$

$$\ell(\partial R_n) = \frac{1}{2^n} \ell(R)$$

$$|I(R_n)| \leq \varepsilon \frac{1}{4^n} d \, \ell(R)$$

$$|I(R_n)| \leq \frac{1}{4^n} \varepsilon \, d \ell \qquad d = \text{diag de } R$$

$$\qquad\qquad\qquad \ell = \text{long de } R$$

$$\vee\!|$$

$$\frac{1}{4^n} |I(R)| \qquad |I(R)| \leq \varepsilon d \ell \implies \text{como vale } \forall \varepsilon$$

$$I(R) = 0$$

QED