Clase 4

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$$\frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right) \qquad f(x,y) \quad z = x+iy, \quad \bar z = x-iy$$

$$\frac{\partial f}{\partial \bar z} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right) \qquad x = \frac{z+\bar z}{2} \quad y = \frac{z-\bar z}{2i}, \ \frac{\partial f}{\partial \bar z}$$

$$f\left(\frac{z+\bar z}{2}, \frac{z-\bar z}{2i}\right) \longleftarrow \frac{\partial f}{\partial \bar z} = \frac{\partial f}{\partial x}\frac{1}{2} + \frac{\partial f}{\partial y}\frac{1}{2i} = \frac{\partial f}{\partial x}\frac{1}{2} - \frac{i}{2}\frac{\partial f}{\partial y}$$

Ec de Cauchy para $f \longleftrightarrow \frac{\partial f}{\partial \bar z} = 0$

$$\frac{\partial f}{\partial \bar z} = \frac{1}{2}\left(\frac{\partial f}{\partial x} + i\frac{\partial f}{\partial y}\right) \qquad f = u+iv$$

$$= \frac{1}{2}\left(u_x + iv_x + i(u_y + iv_y)\right)$$

$$= \frac{1}{2}\left(u_x - v_y + i(v_x + u_y)\right) = 0$$

$\Updownarrow$

CR (Cauchy Riemann)

$u_x = v_y$, $u_y = -v_x$ $\qquad u_{xx} = v_{yx}$, $u_{yy} = -v_{xy}$

$u_{xx} + u_{yy} = 0$, $v_{xx} + v_{yy} = 0$

$f(x_1, \dots, x_n)$ es $f_{x_1x_1} + \dots + f_{x_nx_n} = 0$ armónica

$\Delta(f) = \sum f_{x_i x_i}$ el laplaciano.

$f = u+iv \qquad \Delta u = \Delta v = 0$ armónicas

$u_x = v_y \quad u_y = -v_x$ CR

u y v son armónicas conjugadas

Nos vamos por toda función armónica es la parte real de una función analítica.

$u(x,y) = \log(x^2+y^2) \qquad u_{xx} + u_{yy} = 0$

$$u_x = \frac{2x}{x^2+y^2} \qquad u_{xx} = \frac{2}{(x^2+y^2)^2}(x^2+y^2 - x2x)$$

$$u_{xx} = \frac{2}{(x^2+y^2)^2}(y^2-x^2) \qquad u_{xx}+u_{yy} = 0$$

$$u_{yy} = \frac{2}{(x^2+y^2)^2}(x^2-y^2)$$

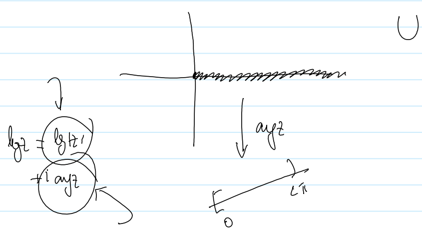
$\log z = \log|z| + i\arg z$?

$\Updownarrow$

z complejo $\quad (\log|z| + i\arg z)$

$$z = e^{\log|z|}e^{i\arg z} = |z|(\cos\alpha + i\sin\alpha)$$

$$\log z = \log\sqrt{x^2+y^2} + i\arg z = \frac{1}{2}\log(x^2+y^2)$$



Funciones holomorfas

Derivada

$f: U \longrightarrow \mathbb{C} = \mathbb{R}^2$, $U \subset \mathbb{C}$

$f: \mathbb{R} \to \mathbb{R}$

$$\lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = f'(a)$$

$f: \mathbb{R}^2 \to \mathbb{R}^2 \qquad d_x f: \mathbb{R}^2 \to \mathbb{R}^2$

$$d_x f(v) = \lim_{t\to 0} \frac{f(x+tv) - f(x)}{t}$$

$\vec v \in \mathbb{R}^2 \qquad f = (u(x,y), v(x,y))$

$$d_x f = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{pmatrix}$$

$f: \mathbb{R}^2 \to \mathbb{R}^2$

$$f(x,y) = u(x,y) + i v(x,y)$$

$$\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$$

Supongamos que todas las derivadas de todos los órdenes existen

f es holomorfa en una región donde está definida si valen las ecuaciones de Cauchy-Riemann

$$\frac{\partial u}{\partial x} \quad \frac{\partial u}{\partial y} \quad \frac{\partial v}{\partial x} \quad \frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Ecuaciones de Cauchy Riemann

$$\begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{pmatrix}$$

$f(z) = z \qquad u(x,y) = x, \quad v(x,y) = y$

$$\frac{\partial u}{\partial x} = 1 \quad \frac{\partial u}{\partial y} = 0, \quad \frac{\partial v}{\partial x} = 0 \quad \frac{\partial v}{\partial y} = 1$$

U abierto de $\mathbb{C}$

$\mathcal{H}(U)$ = conjunto de las funciones de U en $\mathbb{C}$ holomorfas

$\Rightarrow \mathcal{H}(U)$ es un álgebra sobre $\mathbb{C}$.

f es holomorfa $\Rightarrow \frac{1}{f}$ es holomorfa sacando (los ceros de f)

$f = u + iv \qquad f^{-1} = \frac{1}{u+iv} = \frac{u-iv}{u^2+v^2}$

$\frac{1}{z} = \frac{\bar{z}}{|z|^2}$

$U = \frac{u}{u^2+v^2} \qquad V = \frac{-v}{u^2+v^2}$

$U_x = V_y \quad U_y = -V_x$

$U_x = \frac{u_x(u^2+v^2) - u(2uu_x + 2vv_x)}{(u^2+v^2)^2} \qquad \frac{\partial u}{\partial x} = u_x$

$\rightarrow U_x = \frac{u_x(v^2-u^2) - 2uvv_x}{(u^2+v^2)^2} \leftarrow \quad u_y = -v_x$

$\rightarrow V_y = \frac{v_y(u^2+v^2) - v(2uu_y + 2vv_y)}{(u^2+v^2)^2} = -\frac{(v_y(u^2-v^2) + 2uvu_y)}{(u^2+v^2)^2}$

$e^z = e^x(\cos y + i \sin y)$

$u = e^x \cos y \qquad v = e^x \sin y$

$u_x = e^x \cos y \qquad u_y = e^x(-\sin y)$

$v_x = e^x \sin y \qquad v_y = e^x \cos y$

Dos operadores en $\mathcal{H}(U)$

$\frac{\partial}{\partial z} : \mathcal{H}(U) \rightarrow \mathcal{H}(U)$

$\frac{\partial}{\partial \bar{z}} : \mathcal{H}(U) \rightarrow \mathcal{H}(U)$

$\frac{\partial f}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right) f = \frac{1}{2}\frac{\partial f}{\partial x} - i\frac{\partial f}{\partial y}$

$\frac{\partial f}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right) f = \frac{1}{2}\frac{\partial f}{\partial x} + i\frac{\partial f}{\partial y}$

Ejercicio f es holomorfa sii $\frac{\partial}{\partial \bar{z}}(f) = 0$

$f(z) = z$ holomorfa

$e(z) = \bar{z}$ no es holomorfa

$u = x \qquad v = -y$

$u_x = 1 \qquad u_y = 0 \qquad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$v_x = 0 \qquad v_y = -1$

$e^{\bar{z}} \qquad \sqrt{\bar{z}} \qquad z\bar{z} \qquad$ etc