Clase 2

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Hoy haremos ejercicios, calculemos la $\sqrt[3]{i}$

$i = e^{i\pi/2}$ Si $z^3 = i \Rightarrow z = re^{i\alpha}$ donde

$$r^3 e^{3i\alpha} = i = e^{i\pi/2}$$

$e^z \quad z \longrightarrow e^z : \mathbb{C} \longrightarrow \mathbb{C}$

$z = x + iy \quad e^z = e^x e^{iy}$

$e^{iy} = \cos y + i \operatorname{sen} y \quad \Big| \quad e^z = e^x(\cos y + i \operatorname{sen} y)$

$$e^z = \sum_{n=0}^{\infty} \frac{1}{n!} z^n$$

dan lo mismo

$z = x + iy$

$\bar{z} = x - iy$

$e^{iy} = \cos y + i \operatorname{sen} y$

$e^{-iy} = \cos y - i \operatorname{sen} y = e^{\overline{iy}}$

$$\begin{cases} \cos y = \dfrac{e^{iy} + e^{-iy}}{2} \\ \operatorname{sen} y = \dfrac{e^{iy} - e^{-iy}}{2i} \end{cases}$$

$e^z = e^x(\cos y + i \operatorname{sen} y)$

$z = x + iy$

$z' = x' + iy'$

$e^{z+z'} = e^z e^{z'}$

$\cancel{e^{z+z'}} = e^{i(y+y')} = \cos(y+y') + i \operatorname{sen}(y+y')$

$e^{iy} e^{iy'} = (\cos y + i \operatorname{sen} y)(\cos y' + i \operatorname{sen} y')$

$= (\cos y \cos y' - \operatorname{sen} y \operatorname{sen} y') + i(\operatorname{sen} y \cos y' + \cos y \operatorname{sen} y')$

$\cos(y+y') = \cos y \cos y' - \operatorname{sen} y \operatorname{sen} y'$

$\operatorname{sen}(y+y') = \operatorname{sen} y \cos y' + \cos y \operatorname{sen} y'$

$e^{z+z'} = e^z \cdot e^{z'}$ $\qquad \cos 3y$

$(e^{iy})^3 = e^{3iy}$

$(\cos y + i \operatorname{sen} y)^3 = \cos 3y + i \operatorname{sen} 3y$

$\|$

$\cos^3 y + 3\cos^2 y (i \operatorname{sen} y) + 3\cos y (i \operatorname{sen} y)^2 + (i \operatorname{sen} y)^3$

$\cos 3y = \cos^3 y - 3\cos y \operatorname{sen}^2 y \qquad \operatorname{sen}^2 y = 1 - \cos^2 y$

$= \cos^3 y - 3\cos y\,(1-\cos^2 y)$

$= 4\cos^3 y - 3\cos y$

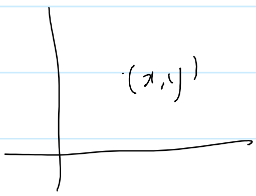
Representación polar de un complejo

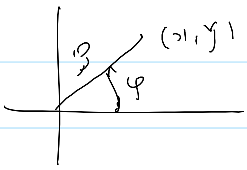
$z = x + iy \qquad i = (0,1) \qquad i^2 = -1$

$\to$ representación cartesiana de z

representación de de Moivre

$\mathbb{R}$





$x = \rho\cos\varphi$

$y = \rho\sin\varphi$

$z = x + iy \qquad z = \rho e^{i\varphi} = \rho\cos\varphi + i\rho\sin\varphi$

$z = \cancel{\rho_{\varphi}} = \rho e^{i\varphi}$

$\varphi = \arctan y/x \qquad \rho = \sqrt{x^2+y^2}$

$z = x + iy \qquad z' = x' + iy' \qquad zz' = \dots\dots$

$z = \rho e^{i\varphi} \qquad z' = \rho' e^{i\varphi'}$

$zz' = \rho\rho' e^{i\varphi} \cdot e^{i\varphi'} = \rho\rho' e^{i(\varphi+\varphi')}$

$z + z' = \rho e^{i\varphi} + \rho' e^{i\varphi'} = \rho'' e^{i\varphi''}$

$z = \rho e^{i\varphi}$

$\bar{z} = \rho e^{-i\varphi}$

$z\bar{z} = |z|^2 = \rho^2$

$\rho = |z|$

$|zw| = |z|\,|w| \qquad z = a + ib \qquad |z|\,|w| = \sqrt{a^2+b^2}\sqrt{c^2+d^2}$

$w = c + id$

$zw = (ac - bd) + i(ad + bc)$

$|zw|^2 = |z|^2|w|^2$

$zw\bar{z}\bar{w} = z\bar{z}w\bar{w}$!!

$|z|$ si z es real

$z = x + 0i \qquad |z| = \sqrt{x^2+0^2} = \sqrt{x^2} = |x|$

$z = \rho e^{i\varphi} \qquad |z| = |\rho e^{i\varphi}| = |\rho|\,|e^{i\varphi}|$

$\boxed{|e^{i\varphi}| = 1}$ $\qquad \cos\varphi + i\sin\varphi$!!

$\boxed{|e^{i\varphi}| = 1}$ $\cos y + i \sin y$!!

Raíces p-ésimas de un número complejo

$i \longrightarrow$ raíces cuadradas de i

$z = x + iy$

$z = \rho e^{i\varphi}$

$0 \leq \varphi < 2\pi$

Arg z el único

$z : \quad z^2 = i \qquad z = \rho e^{i\varphi}$

$(\rho e^{i\varphi})^2 = i$

φ entre 0 y 2π : $z = \rho e^{i\varphi}$

$$\rho^2 e^{2i\varphi} = e^{i\pi/2} \qquad \begin{cases} \rho^2 = 1 \\ 2\varphi = \pi/2 + 2k\pi \quad k \in \mathbb{Z} \end{cases}$$

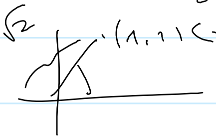
$\rho = 1$

$\varphi = \pi/4 + k\pi$: $\varphi = \pi/4 \quad \sqrt{i} = e^{i\pi/4}$; $\varphi = 5\pi/4 \quad -\sqrt{i} = e^{i5\pi/4}$

Las dos raíces cuadradas de i son $\boxed{\pm e^{i\pi/4}}$

$e^{i\pi/2} = \cos \pi/2 + i \sin \pi/2 = i$

$\sqrt{2}$ $(1,1)$ $1+i$



$e^{i\pi/4}$

$e^{i\pi/4} = \cos \pi/4 + i \sin \pi/4 = \frac{\sqrt{2}}{2}(1+i)$

raíces cuadradas de i

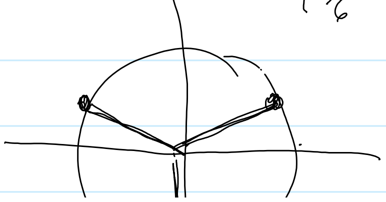
$(1+i)^2 = 1 + 2i + i^2 = 2i$

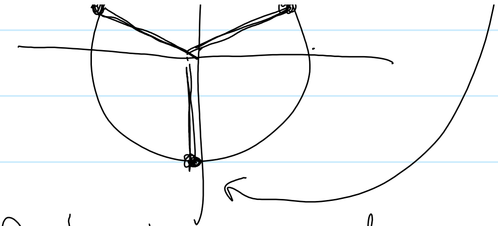
$\left(\frac{1+i}{\sqrt{2}}\right)^2 = i \qquad \sqrt{i} = \pm \frac{1+i}{\sqrt{2}}$

$z : \quad z^3 = i = e^{i\pi/2} \qquad z = \rho e^{i\varphi} \qquad \rho^3 = 1 \rightarrow \rho = 1$

$\rho^3 e^{3i\varphi} = e^{i\pi/2} \qquad 3\varphi = \pi/2 + 2k\pi$

$$\varphi = \pi/6 + \frac{2k\pi}{3} \qquad \begin{cases} \pi/6 & 30 \\ \pi/6 + 2\pi/3 & 150 \\ \pi/6 + 4\pi/3 & 270 \end{cases}$$





Raíces p-ésimas de un número complejo

$w = re^{i\theta}$ $\quad z : z^p = w \quad z = \rho e^{i\alpha}$

$(\rho e^{i\alpha})^p = re^{i\theta}$

$\rho^p e^{ip\alpha} = re^{i\theta} \longrightarrow \rho^p = r$, $\quad p\alpha = \theta + 2k\pi \qquad \rho = r^{1/p}$

$\alpha = \theta/p + \frac{2k\pi}{p}$

θ/p $\quad 0$

$\theta/p + 2\pi/p$ $\quad 1$

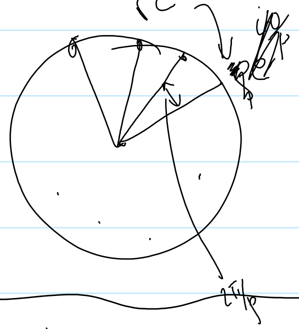
$\theta/p + 4\pi/p$

$\vdots$

$\theta/p + \frac{2(p-1)}{p}\pi$ $\quad p-1$

$\theta/p + 2\pi$

$re^{i\theta/p}$



$e^{i\pi/p}$

$|z + w| \le |z| + |w|$ general ~~$|\sum z_i| \le \sum |z_i|$~~

$\left|\sum_{j=1}^{n} z_j w_j\right|^2 \le \left|\sum_{j=1}^{n} |z_j|^2\right| \cdot \left|\sum_{j=1}^{n} |w_j|^2\right|$ ~~$\mathbb{R}^2$~~

$z = x + iy \qquad z + w = (x + x') + i(y + y')$

$w = x' + iy' \qquad \left\|\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} x' \\ y' \end{pmatrix}\right\| \le \left\|\begin{pmatrix} x \\ y \end{pmatrix}\right\| + \left\|\begin{pmatrix} x' \\ y' \end{pmatrix}\right\|$

Vejencias

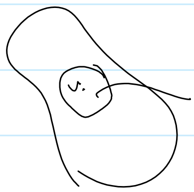
Topología $\mathbb{C}$ es la de $\mathbb{R}^2$

- abierto
- cerrado
- compacto
- conexo

Abierto $\mathbb{R}^2 \supset U$

$\forall u \in U$ existe $B(u, r_u)$

$= \{x \in \mathbb{R}^2 : d(x,u) < r_u\}$

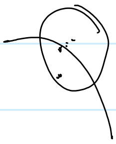


$B(u, r_u)$ $\quad C$ es cerrado $\Leftrightarrow \mathbb{R}^2 \setminus C$ es abierto

Ab V (int V) $\quad \cap$ Compacto $\mathbb{R}^2$

abierto

$\partial V \quad V \quad \text{int}\, V$



$D \quad \overline{D}$

$\partial D = \overline{D} - \text{int}(D)$

Compacto $\mathbb{R}^2$

→ cerrado y acotado

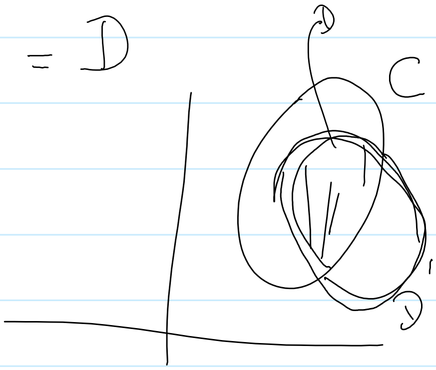
⟷ toda sucesión en C admite una sucesión parcial (o subsucesión) convergente

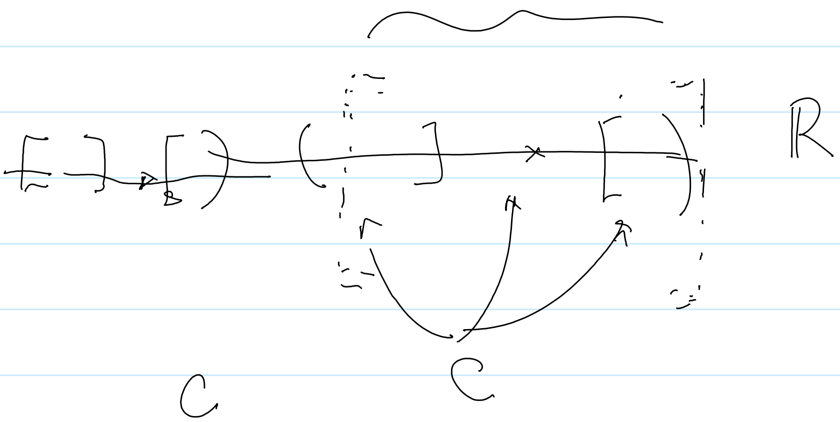
C desconexo si si admite una descomposición $C = C_1 \cup C_2$, C_1, C_2 cerrados "en C"

⟷ una descomposición $C = U_1 \cup U_2$ → U_1, U_2 "abiertos en C"

$C \supset D$ → es cerrado en C

$\mathbb{R}^2$ si $\exists\, D'$ cerrado en $\mathbb{R}^2$: $D' \cap C = D$

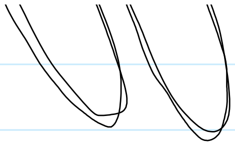




C

$\mathbb{R}^2$

→ unión de 2 abiertos C disjuntos



unión de C disjuntos que no necesariamente son abiertos en $\mathbb{R}^2$

C es arco conexo si $\forall\, c_1$ y $c_2 \in C$ existe un arco $\gamma : [0,1] \to C$.

$\gamma(0) = c_1$ y $\gamma(1) = c_2$

En $\mathbb{R}^2$ si U es abierto conexo y arco conexo coinciden.