3. $3a_{n+1} = 2a_n + a_{n-1}, \quad n \geqslant 1,$
con $a_0 = 7, a_1 = 3.$

$n=1 \quad 3a_2 = \underline{2a_1 + a_0}.$

$3a_n = 2a_{n-1} + a_{n-2}, \quad \forall n \geq 2 \qquad n=2 \quad 3a_2 = 2a_1 + a_0$

$3a_{n+2} = 2a_{n+1} + a_n, \quad n \geq 0$

$a_0 = 7, \quad a_1 = 3, \quad 3a_2 = 2a_1 + a_0 = 13 \Rightarrow a_2 = \frac{13}{3}$

3. $3a_{n+1} = 2a_n + a_{n-1}, \quad n \geqslant 1,$
con $a_0 = 7, a_1 = 3.$

$a_n = r^n$

$(r \neq 0) \qquad 3r^{n+1} - 2r^n - r^{n-1} = 0$

$\boxed{3r^2 - 2r - 1 = 0}$ Ecuación característica

$r = \frac{2 \pm \sqrt{4+12}}{6} = \frac{2 \pm 4}{6} \quad r = 1, \quad r = \frac{-1}{3}.$

Soluciones fundamentales: $a_n = 1^n = 1 \quad ; \quad a_n = \left(\frac{-1}{3}\right)^n.$

solución general: $a_n = A + B \cdot \left(\frac{-1}{3}\right)^n.$

Condiciones iniciales $a_0 = 7, \quad a_1 = 3$

$a_0 = A + B = 7 \quad ; \quad a_1 = A + B \cdot \left(\frac{-1}{3}\right) = 3$

$\begin{cases} A + B = 7 \rightarrow A = 7 - B \rightarrow A = 4 \\ A - \frac{B}{3} = 3 \end{cases}$

$7 - B - \frac{B}{3} = 3 \rightarrow 21 - 3B - B = 9$

$\rightarrow 21 - 9 = 4B$

$\rightarrow 12 = 4B \rightarrow B = 3$

$\boxed{A = 4, \quad B = 3}$ ✓ $\quad \boxed{a_n = 4 + 3 \cdot \left(\frac{-1}{3}\right)^n}$

$A > 0 \quad \sqrt{-A} = \sqrt{-1}\cdot\sqrt{A} = \sqrt{A}\cdot i$

$\sqrt{-4} = 2\cdot i$

4. $a_{n+2} + a_n = 0, \quad n \geqslant 0,$
con $a_0 = 0, a_1 = 3.$

$a_n = r^n \;(r \neq 0) \longrightarrow r^{n+2} + r^n = 0$

$\boxed{r^2 + 1 = 0} \longrightarrow r = \dfrac{0 \pm \sqrt{0-4}}{2} = \dfrac{\pm\sqrt{-4}}{2}$

$= \pm\sqrt{-1} = \pm i$

Sol. fund: $a_n = i^n \;;\; a_n = (-i)^n$.

$\dfrac{1}{i} = -i$

Sol general $a_n = A\cdot i^n + B\cdot(-i)^n$

$a_0 = A + B = 0 \rightarrow B = -A \longrightarrow B = \frac{3}{2} i$

$a_1 = A\cdot i + B(-i) = 3 \rightarrow \underbrace{A\cdot i + (-A)\cdot(-i)}_{A\cdot i} = 3 \Rightarrow 2iA = 3 \Rightarrow A = \dfrac{3}{2i} = -\dfrac{3i}{2}$

$\rightsquigarrow a_n = -\dfrac{3i}{2}\cdot i^n + \dfrac{3i}{2}\cdot\underbrace{(-i)^n}_{i^n \text{ si } n \text{ es par}} \xrightarrow{n \text{ par}} \begin{cases} a_n = 0 & n \text{ par} \\ a_n = 3 \text{ ó } -3 & n \text{ impar.} \end{cases}$

$i = 0\cdot 1 + 1\cdot i$

$a_{n+2} \overset{+0\cdot a_{n+1}}{-} a_n = 0, \quad n \geq 0$
con $a_0 = 0, \; a_1 = 3$

$a_n = r^n, (r \neq 0) \quad r^{n+2} - r^n = 0 \longrightarrow \boxed{r^2 \overset{+0\cdot r}{-} 1 = 0} \longrightarrow r = \pm 1$

Sol fund $a_n = 1^n = 1 \;;\; a_n = (-1)^n$.

Sol general $a_n = A + B\cdot(-1)^n$

$a_0 = A + B = 0 \rightarrow A = -B \rightarrow A = \frac{3}{2}$

$a_1 = A - B = 3 \rightarrow -B - B = 3$

$-2B = 3$

$\boxed{B = -\frac{3}{2}}$

Sol $\boxed{a_n = \frac{3}{2} - \frac{3}{2}(-1)^n}$.

6. $a_n - 6a_{n-1} + 9a_{n-2} = 0, \quad n \geqslant 2,$
con $a_0 = 5, a_1 = 12.$

$a_n = r^n \qquad r^n - 6r^{n-1} + 9r^{n-2} = 0$

$(r \neq 0) \qquad \to r^2 - 6r + 9 = 0 \longrightarrow r = 3$ raiz doble.

Vamos a ver en teórico que esta solución sirve cuando hay una raíz doble.

Soluciones fundamentales: $a_n = 3^n$, $a_n = n \cdot 3^n$

Sol gral $a_n = A \cdot 3^n + B \cdot n \cdot 3^n$. $\quad (a_0 = A \quad , a_1 = 3A + 3B)$

Cond iniciales . . . . . $a_0 = 5$

$a_1 = 12$

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1. $a_n = 5a_{n-1} - 6a_{n-2}, \quad n \geqslant 2,$
   con $a_0 = 1, a_1 = 3.$ ← lo hicimos 2 raíces ≠
2. $2a_{n+2} - 11a_{n+1} + 5a_n = 0, \quad n \geqslant 0,$
   con $a_0 = 2, a_1 = -8.$ ← 2 raíces reales ≠
3. $3a_{n+1} = 2a_n + a_{n-1}, \quad n \geqslant 1,$
   con $a_0 = 7, a_1 = 3.$ ← 2 raíces ≠
4. $a_{n+2} + a_n = 0, \quad n \geqslant 1,$
   con $a_0 = 0, a_1 = 3.$ ← complejo
5. $a_{n+2} + 4a_n = 0, \quad n \geqslant 1,$
   con $a_0 = a_1 = 1.$ ← complejo
6. $a_n - 6a_{n-1} + 9a_{n-2} = 0, \quad n \geqslant 2,$
   con $a_0 = 5, a_1 = 12.$ ← 1 raiz doble (real)
7. $a_n + 2a_{n-1} + 2a_{n-2} = 0, \quad n \geqslant 2,$
   con $a_0 = 1, a_1 = 3.$ ← complejo.
8. $a_{n+2} + ba_{n+1} + ca_n = 0, \quad n \geqslant 0,$
   con $a_0 = 0, a_1 = 1, a_2 = 4$ y $a_3 = 37$, y siendo b y c constantes desconocidas.

$$\begin{cases} a_2 + b a_1 + C a_0 = 0 \\ a_3 + b a_2 + C a_1 = 0 \end{cases}$$

de tórico

1. $a_{n+1} - 1.5a_n = 0, \quad n \geqslant 0.$

2. $a_n - na_{n-1} = 0, \quad n \geqslant 1.$

coeficiente NO constante

$a_n = n\, a_{n-1} = n\cdot(n-1)\, a_{n-2} = n\cdot(n-1)(n-2)\, a_{n-3} = \cdots$

$a_n = n!$ es una solución $\rightsquigarrow$ $\boxed{A \cdot n!}$ es una solución

3. $na_n - (n-1)a_{n-1} = 0, \quad n \geqslant 2.$

4. $a_n/a_{n-1}^p = 2$, siendo $a_0 = 1$, p positivo diferente de 1.

$n \cdot a_n = (n-1)\, a_{n-1} = (n-2)\, a_{n-2}.$

$\boxed{n \cdot a_n = A \quad \forall n}$

$\boxed{a_n = ?}$

$a_n = 2\, a_{n-1}^{p}$

$a_0 = 1, \quad a_1 = 2, \quad a_2 = 2 \cdot 2^p = 2^{p+1}, \quad a_3 = 2 \cdot (2^{p+1})^p = 2^{p^2+p+1}$

$a_4 = 2 \cdot \left(2^{p^2+p+1}\right)^p = 2^1 \cdot 2^{p(p^2+p+1)} = 2^{p^3+p^2+p+1}$