Transformada de Fourier

$$X(j\omega)=\int_{-\infty}^{\infty}x(t)e^{-j\omega t}dt \qquad x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(j\omega)e^{j\omega t}d\omega$$

$$\int_{-\infty}^{\infty}x(t)y^*(t)dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(j\omega)Y^*(j\omega)d\omega$$

$$Im[x(t)]=0 \iff X(j\omega)=X^*(-j\omega)$$

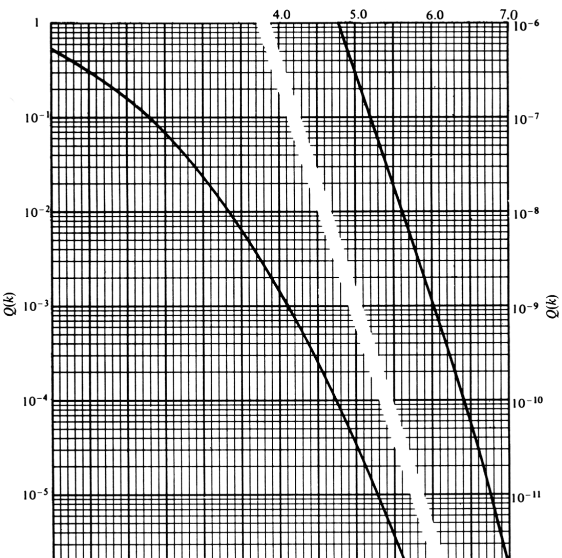
$$x(-t)=x^*(t) \iff Im[X(j\omega)]=0$$

$$Re[x(t)]=0 \iff X(-j\omega)=-X^*(j\omega)$$

$$x(-t)=-x^*(t) \iff Re[X(j\omega)]=0$$

Funciōn	Transformada
$ax(t)+by(t)$	$aX(j\omega)+bY(j\omega)$
$(x*y)(t)$	$X(j\omega)Y(j\omega)$
$x(t)y(t)$	$\frac{1}{2\pi}(X*Y)(j\omega)$
$x^*(t)$	$X^*(-j\omega)$
$X(t)$	$2\pi\, x(-j\omega)$
$x(t-t_d)$	$X(j\omega)e^{-j\omega t_d}$
$x(t)e^{j(\omega_c t+\phi)}$	$X(j(\omega-\omega_c))e^{j\phi}$
$x(t)\cos(\omega_c t)$	$\frac{1}{2}\left(X(j(\omega-\omega_c))+X(j(\omega+\omega_c))\right)$
$x(\alpha t)$	$\frac{1}{\lvert\alpha\rvert}X\left(\frac{j\omega}{\alpha}\right)$
$\frac{d^n x(t)}{dt^n}$	$(j\omega)^n X(j\omega)$
$\int_{-\infty}^{t}x(\lambda)d\lambda$	$\frac{1}{j\omega}X(f)+\pi X(0)\delta(j\omega)$
$t^n x(t)$	$(-j\omega)^{-n}\frac{d^n X(j\omega)}{d\omega^n}$
$\delta(t-t_d)$	$e^{-j\omega t_d}$
$e^{j(\omega_c t+\phi)}$	$2\pi\, e^{j\phi}\delta(\omega-\omega_c)$
$e^{-\pi(at)^2}$	$\frac{1}{a}e^{-\pi(\omega/a)^2}$
$\sum_{k=-\infty}^{\infty}\delta(t-kT)$	${}^{2\pi}/_{T}\sum_{k=-\infty}^{\infty}\delta\left(\omega-{}^{2\pi k}/_{T}\right)$
sign t	${}^{2}/_{j\omega}$
$u(t)$	${}^{1}/_{j\omega}+\pi\delta(\omega)$
$\Pi\left({}^{t}/_{\tau}\right)$	τ sinc $\left({}^{\omega\tau}/_{2\pi}\right)$
$\Lambda\left({}^{t}/_{\tau}\right)$	τ sinc^2 $\left({}^{\omega\tau}/_{2\pi}\right)$
$\left({}^{W}/_{\pi}\right)$ sinc $\left({}^{Wt}/_{\pi}\right)$	$\Pi\left({}^{\omega}/_{2W}\right)$

Cola Gaussiana



Cuando $k>3$ la siguiente es una buena aproximaciōn

$$Q(k)\approx\frac{1}{\sqrt{2\pi}k}e^{-k^2/2}$$

DTFT

$$X(e^{j\theta})=\sum_{n=-\infty}^{+\infty}x[n]e^{-jn\theta} \qquad x[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi}X(e^{j\theta})e^{j\theta n}d\theta$$

$$\sum_{n=-\infty}^{+\infty}x_1[n]x_2^*[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi}X_1(e^{j\theta})X_2^*(e^{j\theta})d\theta$$

$$Im[x[n]]=0 \iff X(e^{-j\theta})=X^*(e^{j\theta})$$

$$x[-n]=x^*[n] \iff Im[X(e^{j\theta})]=0$$

$$Re[x[n]]=0 \iff X(e^{-j\theta})=-X^*(e^{j\theta})$$

$$x[-n]=-x^*[n] \iff Re[X(e^{j\theta})]=0$$

Funciōn	Transformada
$a_1x[n]+a_2y[n]$	$a_1X(e^{j\theta})+a_2Y(e^{j\theta})$
$(x*y)[n]$	$X(e^{j\theta})Y(e^{j\theta})$
$x[n]y[n]$	$\frac{1}{2\pi}\int_{-\pi}^{\pi}X(e^{j\lambda})Y(e^{j(\theta-\lambda)})d\lambda$
$x^*[n]$	$X^*(e^{-j\theta})$
$x[n-n_o]$	$X(e^{j\theta})e^{-jn_o\theta}$
$x[n]e^{jn\theta_o}$	$X(e^{j(\theta-\theta_o)})$
$n\,x[n]$	$j\frac{d\,X(e^{j\theta})}{d\theta}$
$x[-n]$	$X(e^{-j\theta})$
$\delta[n]$	1
$\delta[n-n_o]$	$e^{-jn_o\theta}$
1	$\sum_{k=-\infty}^{+\infty}2\pi\delta(\theta+2\pi k)$
$a^n u[n] \quad (\lvert a\rvert<1)$	$\frac{1}{1-ae^{-j\theta}}$
$u[n]$	$\frac{1}{1-e^{-j\theta}}+\sum_{k=-\infty}^{\infty}\pi\delta(\theta+2\pi k)$
$(n+1)a^n u[n] \quad (\lvert a\rvert<1)$	$\frac{1}{(1-ae^{-j\theta})^2}$
$\frac{r^n\sin\theta_o(n+1)}{\sin\theta_o}u[n], (\lvert r\rvert<1)$	$\frac{1}{1-2r\cos\theta_o e^{-j\theta}+r^2e^{-j2\theta}}$
$\frac{\sin\theta_o n}{\pi n}$	$\sum_k \Pi\left(\frac{\theta+2\pi k}{2\theta_o}\right)$
$x[n]=\begin{cases}1 & 0\le n\le M\\ 0 & \text{otro n}\end{cases}$	$\frac{\sin(\theta(M+1)/2)}{\sin(\theta/2)}e^{-j\theta M/2}$
$e^{j\theta_o n+\phi}$	$\sum_{k=-\infty}^{\infty}2\pi e^{j\phi}\delta(\theta-\theta_o+2\pi k)$

Transformada Z

$$X(z)=\sum_{n=-\infty}^{+\infty}x[n]z^{-n} \qquad x(n)=\frac{1}{2\pi j}\oint_C X(z)z^{n-1}dz$$

donde C es una curva antihoraria en la regiōn de convergencia y que envuelve al origen.

Secuencia	Transformada Z	ROC
$ax[n]+by[n]$	$aX(z)+bY(z)$	contiene $R_x\cap R_y$
$x[n-n_o]$	$z^{-n_o}X(z)$	R_x, quizā ± 0 ō ∞
$z_o^n x[n]$	$X(z/z_o)$	$\lvert z_o\rvert R_x$
$n^k x[n]$	$\left(-z\frac{d}{dz}\right)^k X(z)$	R_x, quizā ± 0 ō ∞
$x^*[n]$	$X^*(z^*)$	R_x
$x[-n]$	$X(1/z)$	$1/R_x$
$(x*y)[n]$	$X(z)Y(z)$	contiene $R_x\cap R_y$
$\delta[n]$	1	$\forall z$
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert>\lvert a\rvert$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert<\lvert a\rvert$
$\delta[n-n_o]$	z^{-n_o}	$\forall z$ excepto 0 ō ∞
$\cos(\omega_o n)u[n]$	$\frac{1-z^{-1}\cos\omega_o}{1-2z^{-1}\cos\omega_o+z^{-2}}$	$\lvert z\rvert>1$
$\sin(\omega_o n)u[n]$	$\frac{z^{-1}\sin\omega_o}{1-2z^{-1}\cos\omega_o+z^{-2}}$	$\lvert z\rvert>1$

Transformada Z unilateral: $\begin{cases}X(z)=\sum_{n=0}^{+\infty}x[n]z^{-n}\\ x[n]\longleftrightarrow X_u(z)\end{cases}$

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Algunas funciones ūtiles

$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-m)^2}{2\sigma^2}}$	Distribucīon de Gauss
$Q(k) = \frac{1}{\sqrt{2\pi}} \int_k^{\infty} e^{-\lambda^2/2} d\lambda$	Cola Gaussiana
$\text{sinc } t = \frac{\sin \pi t}{\pi t}$	Sinc
$\text{sign } t = \begin{cases} 1 & t > 0 \\ -1 & t < 0 \end{cases}$	Signo
$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$	Escalōn
$\Pi\left(\frac{t}{\tau}\right) = \begin{cases} 1 & \lvert t\rvert < \frac{\tau}{2} \\ 0 & \lvert t\rvert > \frac{\tau}{2} \end{cases}$	Rectāngulo
$\Lambda\left(\frac{t}{\tau}\right) = \begin{cases} 1 - \frac{\lvert t\rvert}{\tau} & \lvert t\rvert < \tau \\ 0 & \lvert t\rvert > \tau \end{cases}$	Triāngulo

Identidades trigonomētricas

$e^{j\theta} = \cos\theta + j\sin\theta$

$\cos\theta = {}^1\!/_2\,(e^{j\theta} + e^{-j\theta})$

$\sin\theta = {}^1\!/_{2j}\,(e^{j\theta} - e^{-j\theta})$

$\cos\theta = \sin(\theta + 90^\circ)$

$\sin\theta = \cos(\theta - 90^\circ)$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = {}^1\!/_2\,(1 + \cos 2\theta)$

$\cos^3\theta = {}^1\!/_4\,(3\cos\theta + \cos 3\theta)$

$\sin^2\theta = {}^1\!/_2\,(1 - \cos 2\theta)$

$\sin^3\theta = {}^1\!/_4\,(3\sin\theta - \sin 3\theta)$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = (\tan\alpha \pm \tan\beta) \,/\, (1 \mp \tan\alpha\tan\beta)$

$\sin\alpha\sin\beta = {}^1\!/_2\,\cos(\alpha - \beta) - {}^1\!/_2\,\cos(\alpha + \beta)$

$\cos\alpha\cos\beta = {}^1\!/_2\,\cos(\alpha - \beta) + {}^1\!/_2\,\cos(\alpha + \beta)$

$\sin\alpha\cos\beta = {}^1\!/_2\,\sin(\alpha - \beta) + {}^1\!/_2\,\sin(\alpha + \beta)$

Fōrmula de Poisson

$$\sum_{k=-\infty}^{+\infty} e^{jk\alpha t} = \frac{2\pi}{\alpha} \sum_{k=-\infty}^{+\infty} \delta\left(t - k\frac{2\pi}{\alpha}\right)$$

Fōrmula 1

$$e^{j\pi} + 1 = 0$$

Serie Geomētrica

$$\sum_{k=N_1}^{N_2} \alpha^k = \frac{\alpha^{N_1} - \alpha^{N_2+1}}{1 - \alpha} \quad \text{si } N_2 > N_1 \text{ y } \alpha \neq 1$$

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